The Dynamics of Disk-Planet and Ring-Satellite Interactions

Joseph M. Hahn
Space Science Institute (SSI)

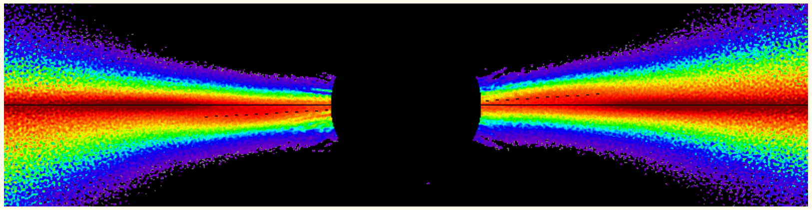
- research scientist with SSI
  - HQ in Boulder, CO
  - work from home office in NW Austin

Current Research Activities

- **circumstellar debris disks**
  - will address the asymmetries that are routinely seen in these disks
- **Saturn's rings**
  - has *many* interesting dynamical problems that are still unsolved

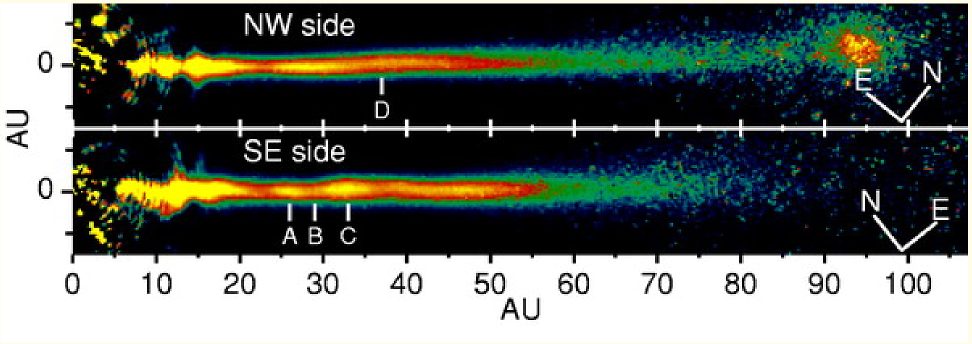
Circumstellar Debris Disks

- dusty disks extend $r \sim$ hundreds of AU



β Pic image from Golimowski et al (2006)

- almost always asymmetric,
  with one ansa brighter by factor ~ 1.5

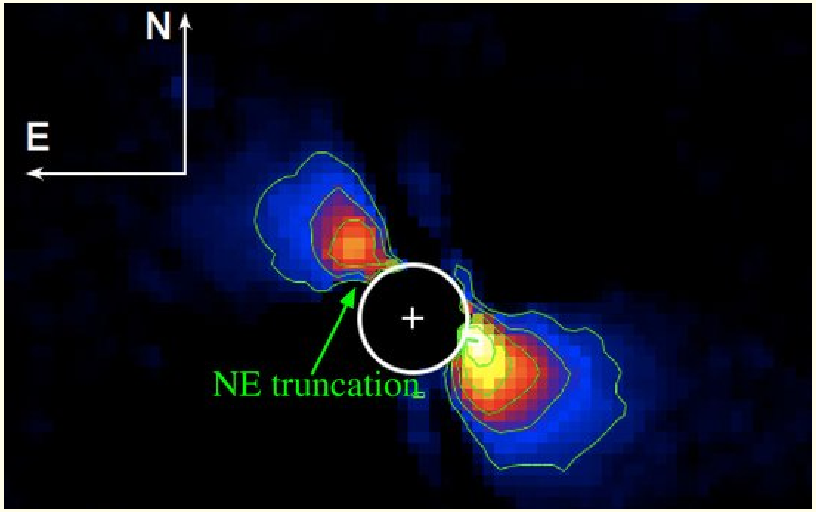


AU Mic image from Krist et al (2005)

- due to unseen embedded planets?
  - probably not–planetary disturbances are likely too localized

- due to passing stars?
  - possible, but statistically unlikely (Kalas et al)

- due to gas drag exerted by ISM ?
  - very possible...see Debes et al (2009) and their model for HD 32297



HD 32297 image from Mawet et al (2009)

- will give my simple explanation for disk asymmetries in a later slide...

The Birth Ring

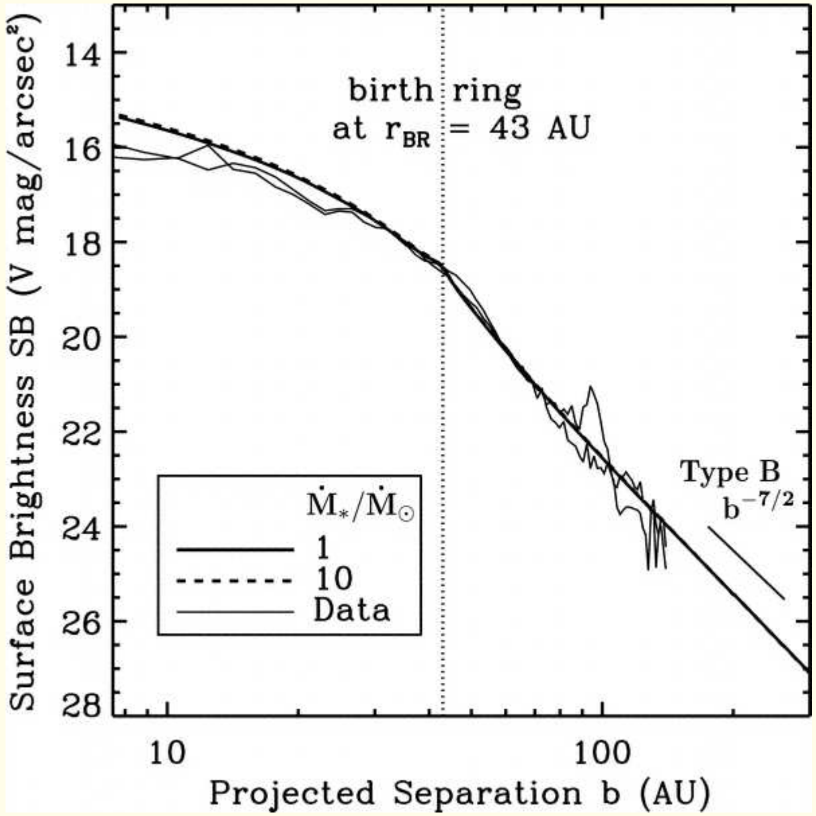
- a debris disk is thought to be generated by a birth ring (BR) of planetesimals that collide and generate dust

- radiation pressure (RP) lofts small 1–10μm grains into wide orbits out to $r \sim 10^{2 \text{ to } 3}$ AU

$$a = \frac{1-\beta}{1-2\beta} a_{ring} \quad \text{and} \quad e = \frac{\beta}{1-\beta}$$

$$\text{where } \beta = \frac{\text{RP}}{\text{gravity}} \sim \frac{0.5\ \mu\text{m}}{\text{grain radius}}$$

when orbiting a sunlike star



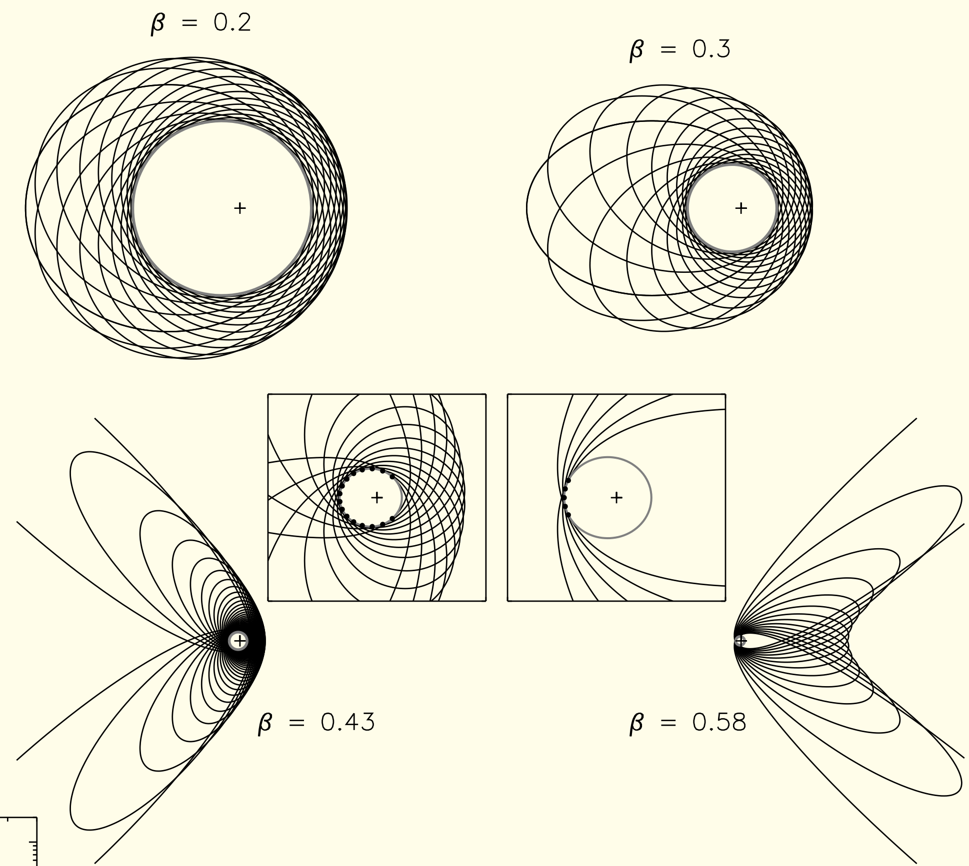
model for AU Mic by Strubbe & Chiang (2006)

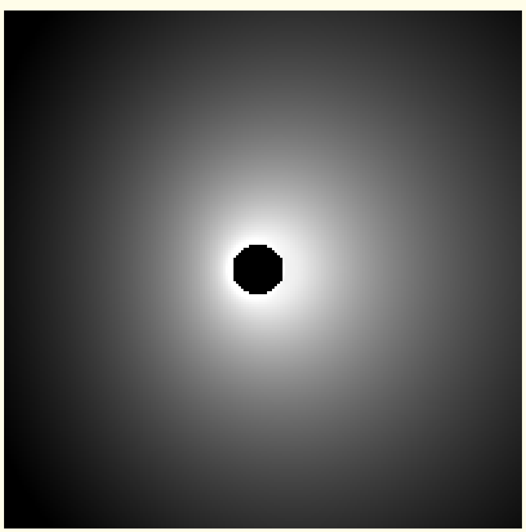
- meanwhile, Poynting-Robertson (PR) drag causes some grains to drift inwards of BR

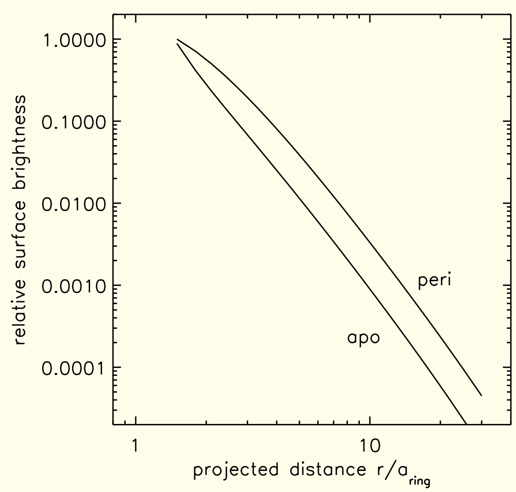
- the BR lies where the disk's surface brightness profile breaks

How Do You Make the Disk Asymmetric?

- easy...make the BR eccentric
- dust grains manufactured near BR's periapse are launched into wider & eccentric orbits, due to BR's faster orbital speed
- dust streamlines are concentrated near BR's periapse, increasing disk's surface brightness there







Dust-Dust Collisions

- previous calculations ignored collisions among dust grains

- collisions are important, steepen the disk's dust size distribution
  - smaller grains are longer lived, which increases the disk's asymmetry

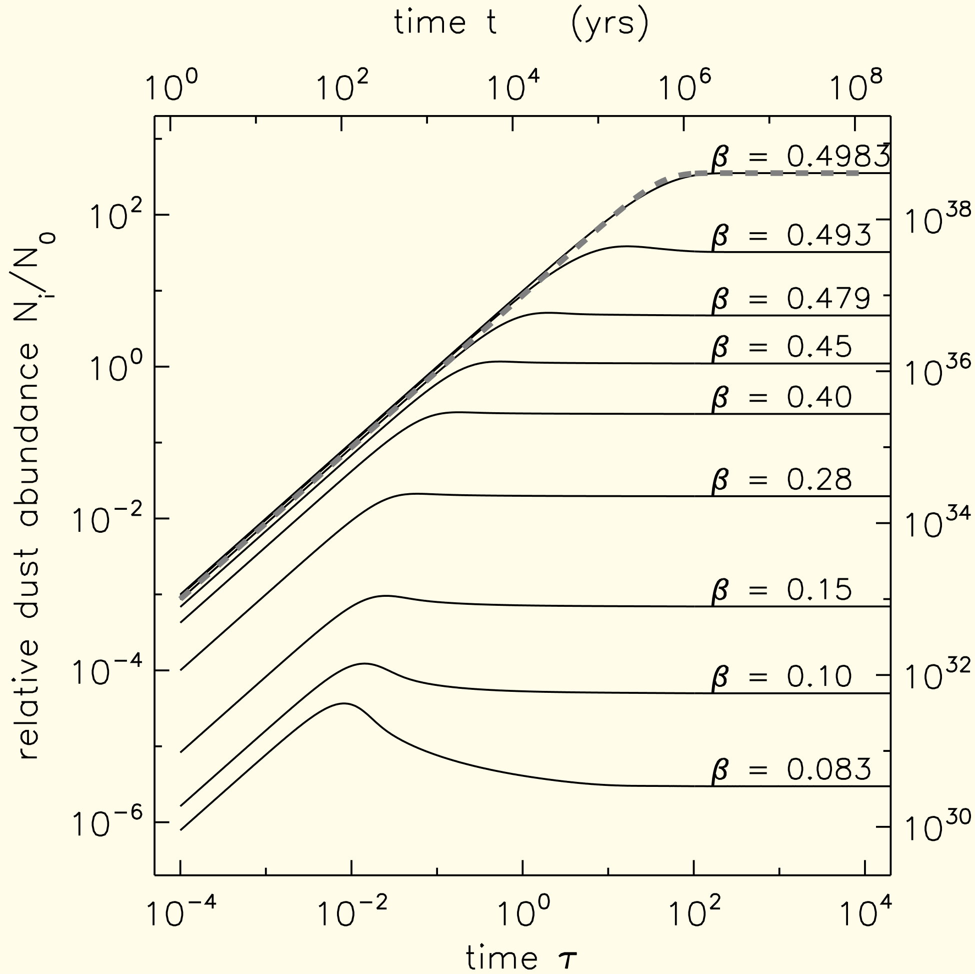
- I will account for collisions by 'quantizing' the problem—
  assume the BR is the source of numerous discreet dusty streamlines (orbits)
  in the disk, each characterized by $\beta, a(\beta), e(\beta), \tilde{\omega}$
  - quantization allows me to avoid slow Monte Carlo simulations,
    and really slow Nbody simulations
  - also replaces tedious 3D integrals over the disk volume with simple sums

- inspect every site where pairs of streamlines cross, and calculate a collision probability density α_{ij}

- this provides rate equation for the dust abundance $N_i(t)$ in each streamline i:

$$\frac{dN_i}{dt} = \text{dust production in BR} - \text{collisional destruction}$$

$$= p_i - N_i \sum_{j \neq i} \alpha_{ij} N_j$$



solar star, $a_r = 50$ AU, $\dot{M}_d = 10^{23}$ gm/yr

- this coupled system of equations is easily solved for streamline populations $N_i(t)$

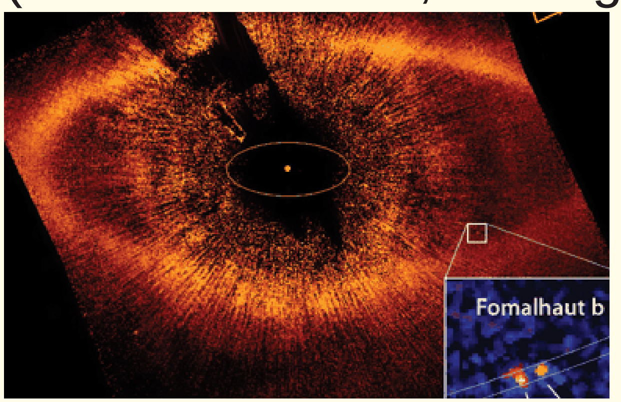
- this system of equations is *scale invariant*, which is very handy
solve then once, then rescale the results for any a_{ring}, $L_\star$, $M_\star$, etc.

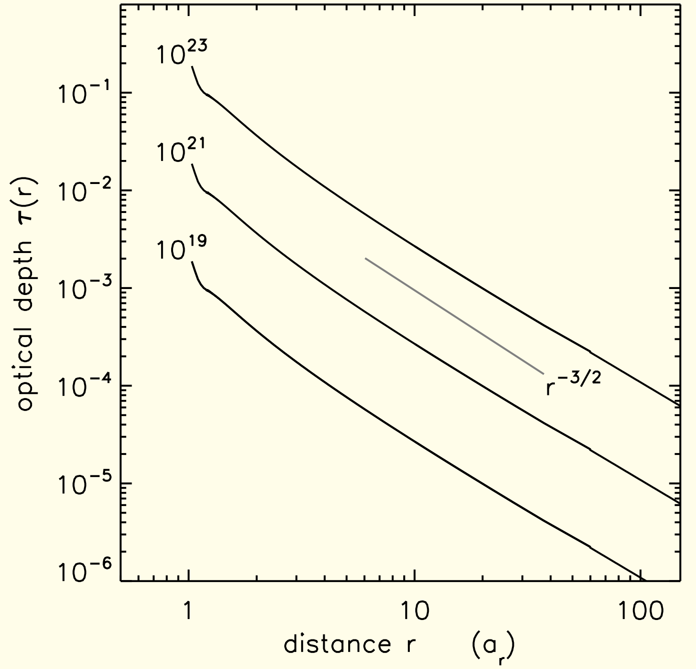
- results are mostly sensitive to the BR's dust production rate $\dot{M}_d$

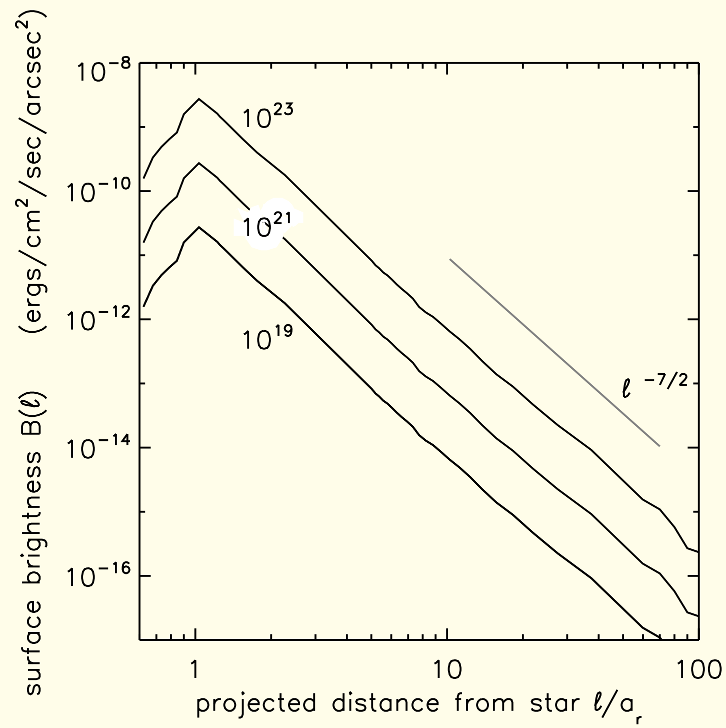
- comparing surface brightness of β Pic and AU Mic to simulations, I should be able to extract $\dot{M}_d$ and total disk mass M_d...almost there...

- comparing simulated & observed asymmetries will also provide the unseen BR's eccentricity

- final task: include planetary perturbations & infer planet masses from disk observations, such as Fomalhaut, for example (Kalas et al 2008, Chiang et al 2009)

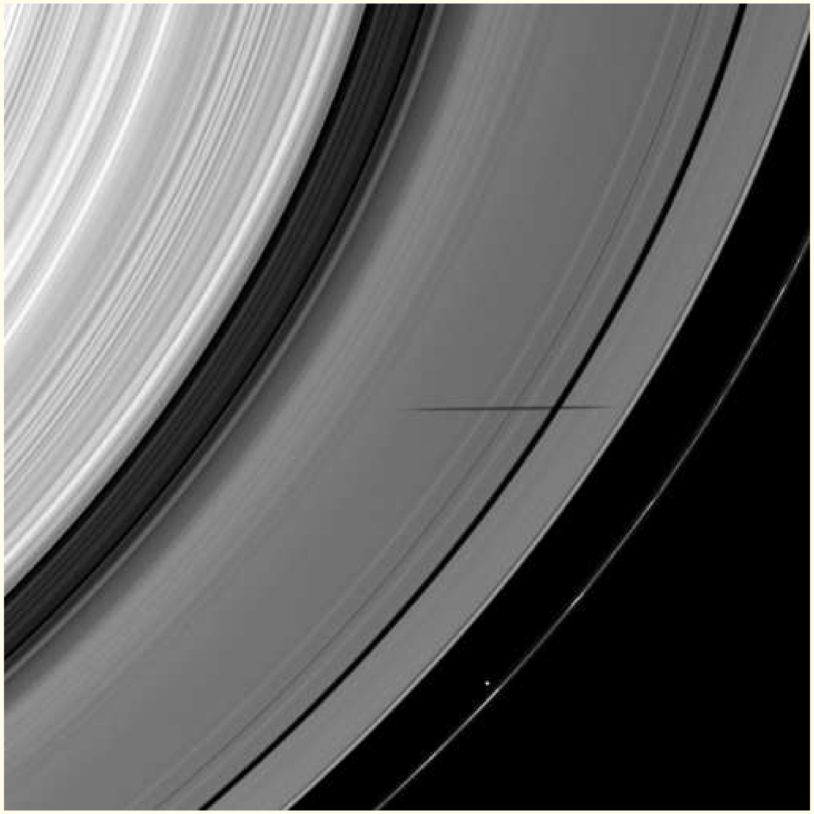






Saturn's Rings: an interesting & complex disk-perturber system

- Saturn has 2 broad, dense rings
  - outer B ring is confined by $m = 2$ ILR (eg, 2:1 MM resonance) with Mimas
  - outer A ring is confined by $m = 7$ ILR w/ coorbitals Janus & Epimetheus



press release image from CICLOPS website

- a naive solution to 3-body problem provides motion of particles orbiting at the ring-edge:

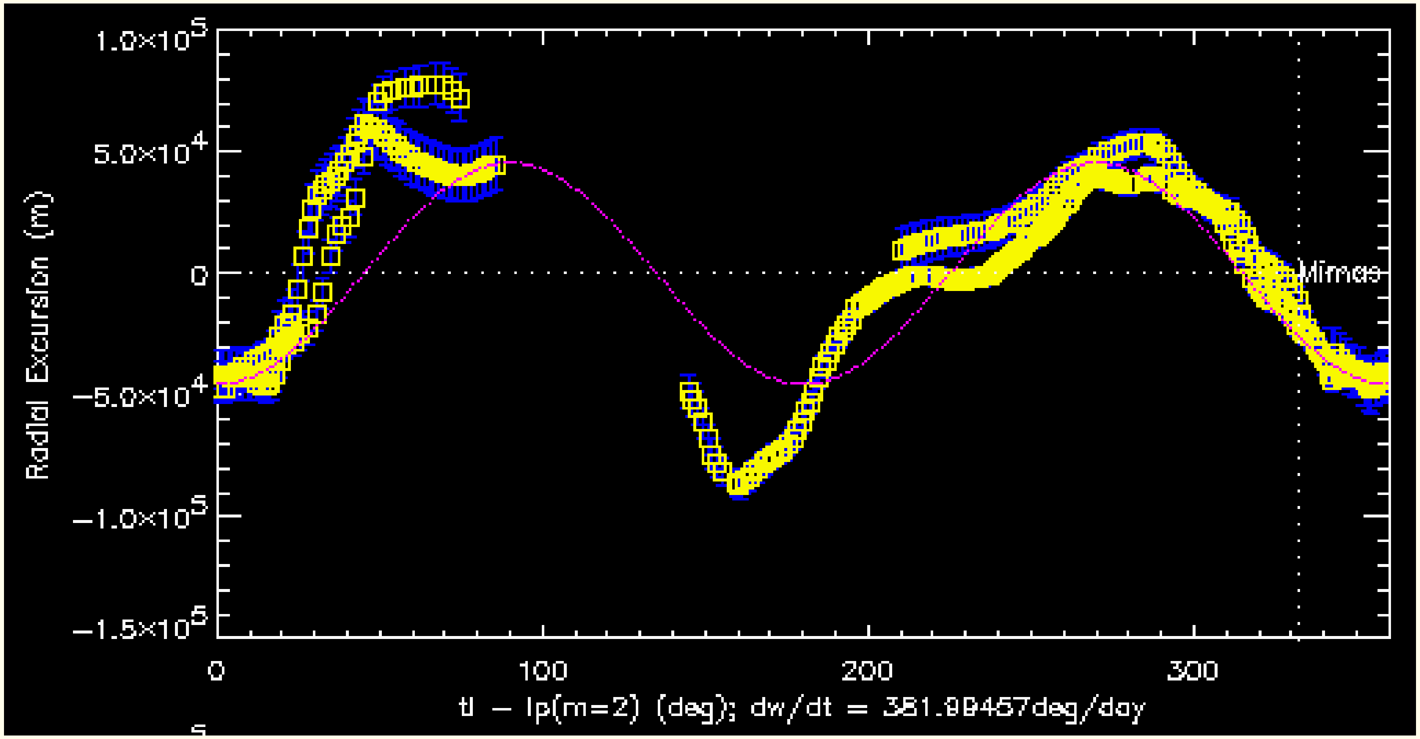
$$r(\theta) - a \quad = \quad R\cos[m(\theta - \theta_s - \tilde{\omega})]$$

which is an m-lobed pattern that corotates with the satellite's longitude $\theta_s(t)$

- accounting for the ring's mass & gravity suggests that the ring's epicyclic amplitude R would be sensitive to ring's σ

- this is an interesting possibility, since the surface density σ for the B ring (Saturn's biggest & most massive ring) is completely unknown

- also, Cassini is there, busily measuring R for many of Saturn's rings



from Spitale & Porco (2006)

- Cassini observations clearly show the expected $m = 2$ pattern with an amplitude $R_{obs} = 45$km that is forced by Mimas

- but the B ring-edge also shows other structure (possibly $m = 1$ and $m = 3$ patterns, too) whose origin is unclear (Spitale et al 2009, Hedman et al 2009)

How to solve this problem (abridged)

- treat the ring as if composed of numerous nested streamlines (orbits) whose shapes and orientations are functions of semimajor axis a:

$$\text{radial displacement} \quad r(a,\theta) - a = R(a)\cos m[\theta - \theta_s - \tilde{\omega}(a)]$$

- Newton's second law of motion (NII) for a single ring particle in any one streamline is

$$\ddot{\mathbf{r}} = -\nabla(\Phi_{Saturn} + \Phi_{satellite}) + \mathbf{a_g} + \mathbf{a_p} + \mathbf{a}_\nu$$

- all perturbing accelerations are functions of three unknown $R(a), \tilde{\omega}(a), \sigma(a)$, plus three input parameters σ_∞, c, ν

- Fourier expand the perturbing accelerations, keep only the resonant terms
  - NII is 2D, which provides two DEQs
  - equilibrium also requires the ring's viscous torque to balance the satellite's gravitational torque, which provides 3^{rd} DEQ

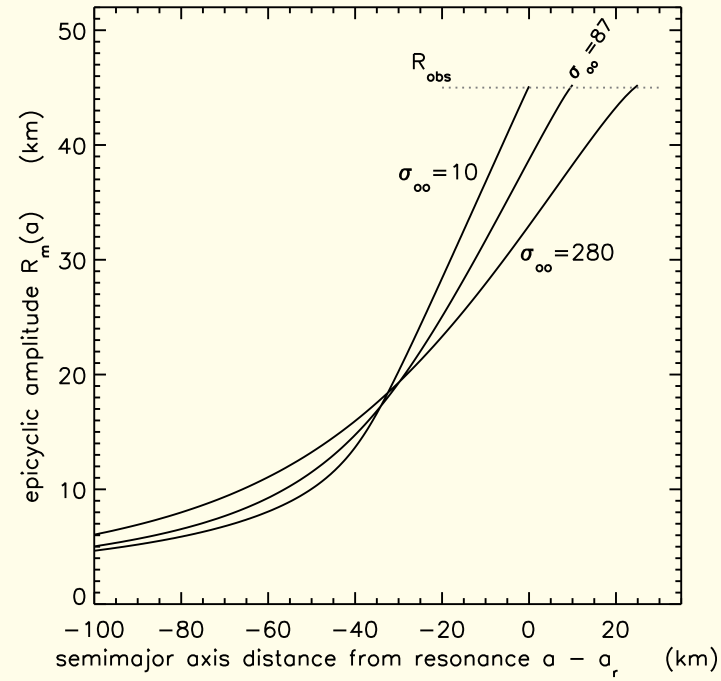
- result: 3 complicated NL DEQs for 3 unknown $R(a), \tilde{\omega}(a), \sigma(a)$... see Hahn et al (2009) for the gruesome details

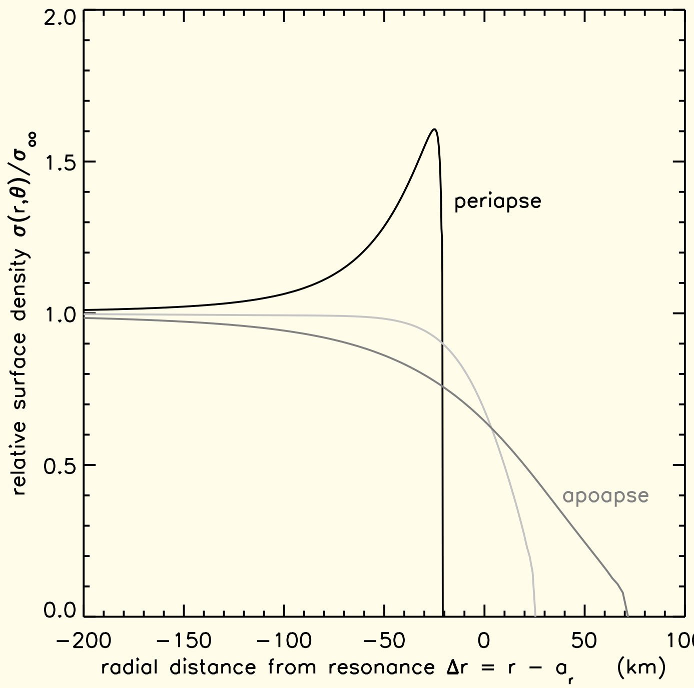
Results

- outcome depends on whether the ring's outer edge lies just interior or exterior to Mimas' $m = 2$ resonance

- analysis of Voyager observations by Porco et al (1984) puts the ring-edge at 24 ± 10 km *beyond* the resonance
  - this suggests B ring has $\sigma \sim 280$ gm/cm^2 (about $6\times$ A ring)
  - Joe Spitale will eventually provide me with a more accurate measurement of the ring-edge's a, which will then allow me to refine my measurement of σ there

- the model also predicts the ring's σ-variations with longitude θ, which are due to Mimas' radial perturbations there





but the model has one curious problem...

- This model seems very promising, but...
  balancing the ring's viscous torque against the satellite's grav' torque is difficult, and is only possible when the ring's bulk viscosity ν_b is $\gg$ shear viscosity ν_s
  - which is weird

- note that the model assumes the ring is compressible,
  and it employs standard hydrodynamic viscosity (L&L's *Fluid Mechanics*):

$$
\begin{aligned}
a_\nu^r &= \frac{1}{\sigma}\frac{\partial}{\partial r}\left[\left(\frac{4}{3}\nu_s + \nu_b\right)\sigma\frac{\partial v_r}{\partial r}\right] \\
a_\nu^\theta &= \frac{1}{\sigma}\frac{\partial}{\partial r}\left[\nu_s\sigma\frac{\partial v_\theta}{\partial r}\right]
\end{aligned}
$$

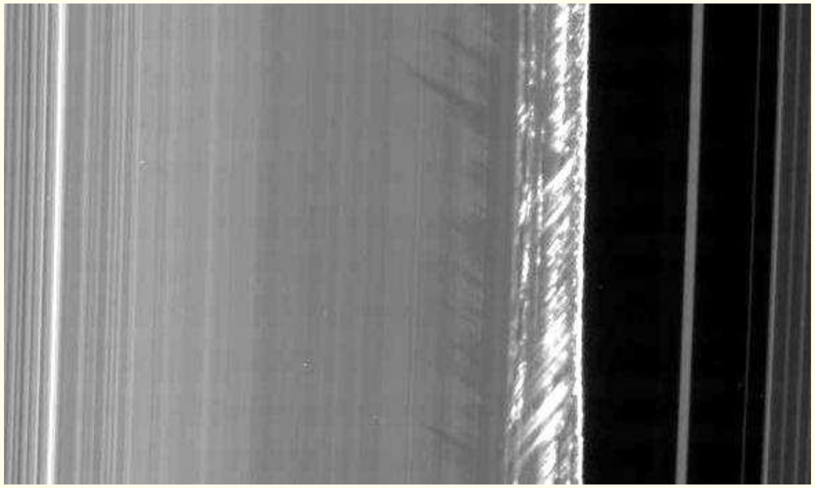
- these assumptions work great for studies of damped spiral density waves that are seen all across Saturn's A ring

- however a recent Cassini image now has me questioning my assumptions...

Springtime for Saturn

- vernal equinox at Saturn was August 11
  - Sun's illumination is nearly coplanar with the rings
  - any small vertical structure in the ring will cast long shadows

- my speculative read of this B ring image:
  - Mimas' radial perturbation shoves ring matter inwards
  - but the ring is crowded, so particles are displaced vertically, which forms ragged mountains of particles along ring-edge
  - ring particles also tumble downhill (avalanches), creating bright dust trails that shear due to orbital motion



July 26 closeup of outer B ring,
from Cassini raw image archive

- this vertical tumbling due to Mimas' push-pull of the ring is probably very lossy...
  - so I need to revise model, account for ring's z-motions, perhaps w/ incompressible EOS
  - hopefully this additional dissipation due to ring's z-motions will help solve the torque-balance problem

Joe Hahn, planetary dynamicist

- am always interested in meeting new potential collaborators who want to work on topics of mutual interest

- currently working on debris disks and planetary rings
  - I would be thrilled to work with anyone who wants to perform hydrodynamic simulations of rings. Nobody is doing this, so this subfield is wide open!

- I am also interested in the dynamics of the early outer Solar System
  - planet migration driven by planetesimal scattering, the resonant structure of the Kuiper Belt, the formation of Oort Cloud, etc
  - career goal: to find a more plausible alternative to the Nice model

- I can think of lots of cool projects that are worthy of a PhD thesis
  - promising grad students interested in planetary dynamics should look me up!
  - would also be interested in raising $ to support a postdoc to work on topics of mutual interest