Bose-Einstein Condensation of Dark Matter Axions

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Texas Cosmology Network Meeting
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I'll argue:

The Dark Matter is Axions

based on arXiv: 0901.1106

with Qiaoli Yang

How so?

- axions differ from ordinary CDM because they form a BEC, and the axion BEC continues to rethermalize, tracking the lowest energy state.

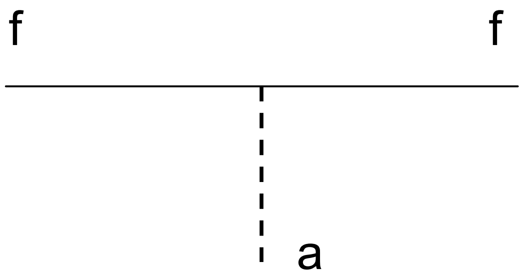
- axion BEC appears to solve two puzzles with CDM
  1) evidence for caustic rings of dark matter implies that the dark matter falls onto galactic halos with ``net overall rotation", implying $\vec{!} \quad \vec{v} \# 0$ whereas CDM predicts $\vec{!} \quad \vec{v} = 0$,
  2) unexpected alignments among CMBR multipoles .

Outline

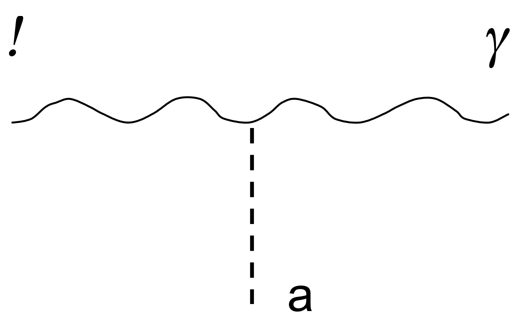
- Review of axion properties
- Cold dark matter axions form a rethermalizing Bose-Einstein condensate
- Axion BEC differs from ordinary CDM
- Compare CDM and axion BEC density perturbations with observations in three arenas

	CDM	axion BEC
1. upon entering the horizon		
2. in the linear regime within the horizon		
3. in the non-linear regime		

$$m_a \quad 6\ \text{eV}\ \frac{10^6\ \text{GeV}}{f_a}$$



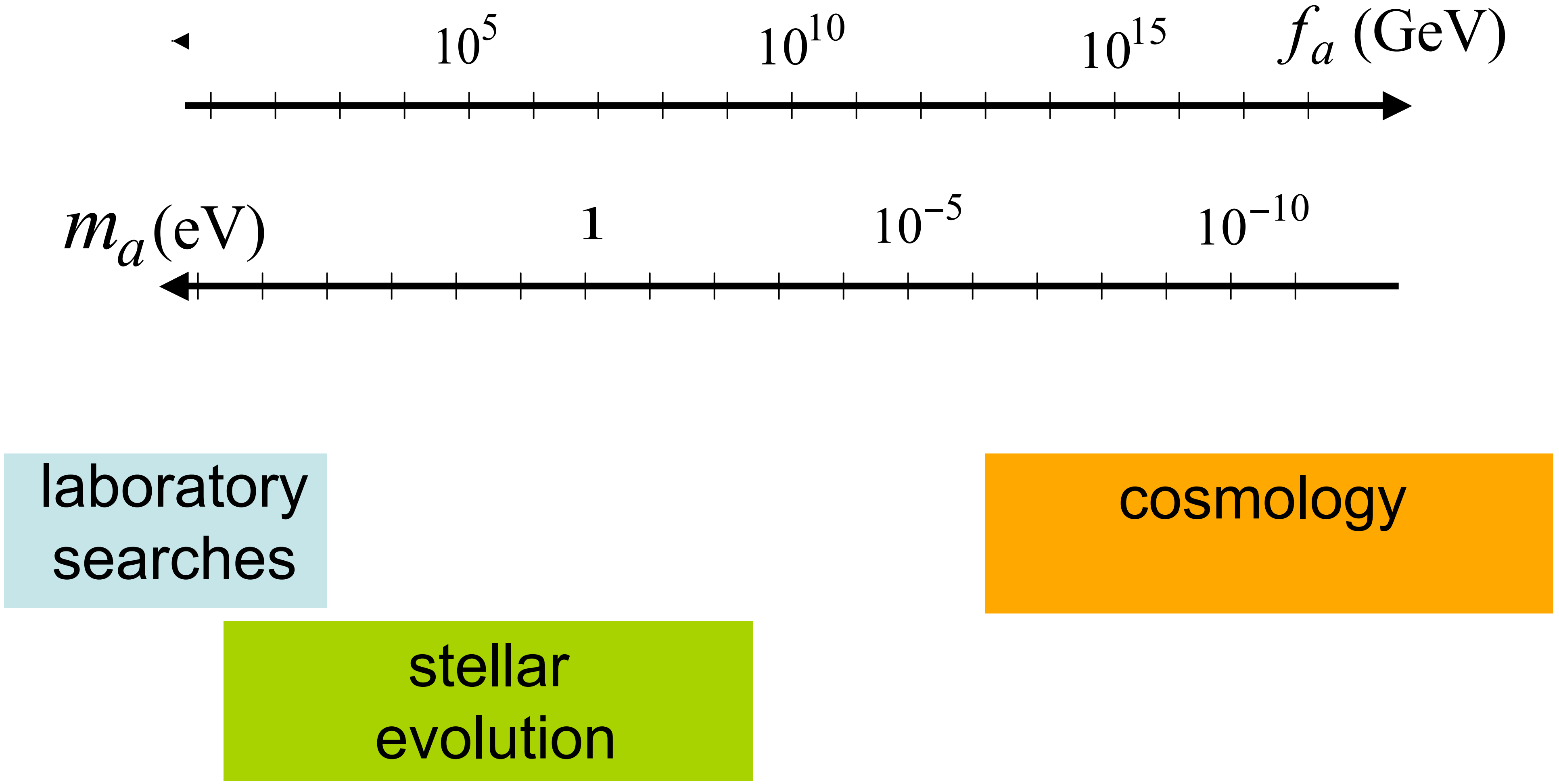
$$L_{a\bar{f}f} = i\, g_f\, \frac{\varphi}{f_a}\, \overline{f}\, \gamma_5\, f$$



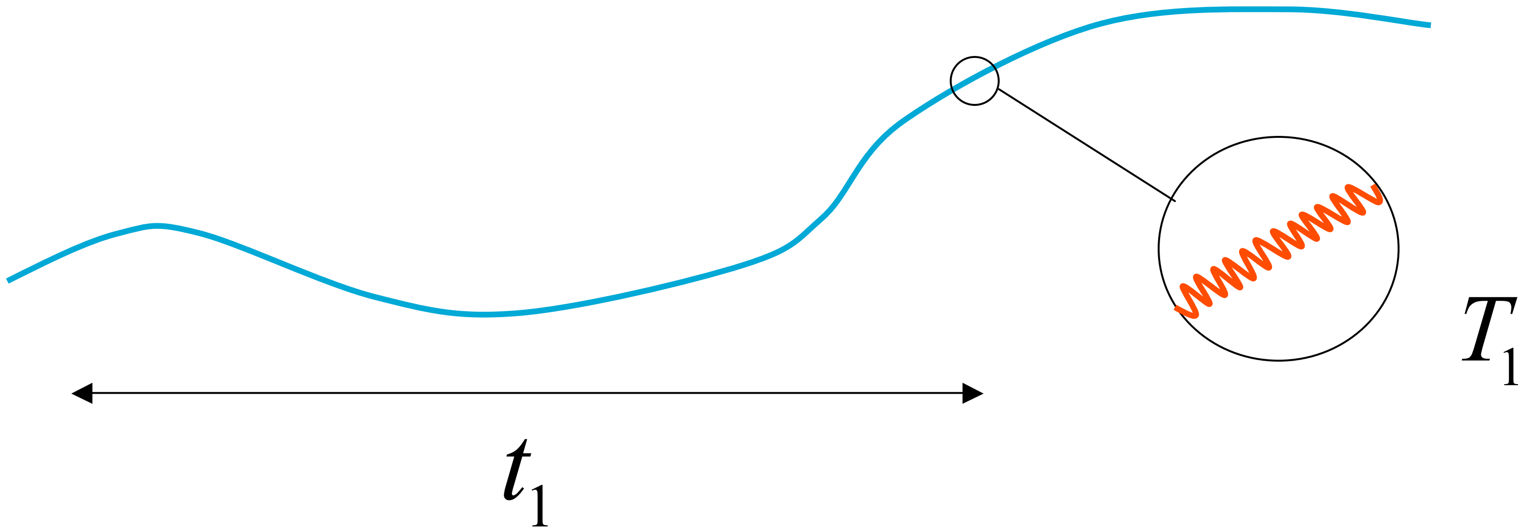
$$L_{a\gamma\gamma} = g_\gamma\, \frac{\alpha}{\pi}\, \frac{\varphi}{f_a}\, \vec{E}\cdot\vec{B}$$

g_γ = 0.97 in KSVZ model
0.36 in DFSZ model

The remaining axion window



There are two axion populations: hot and cold.

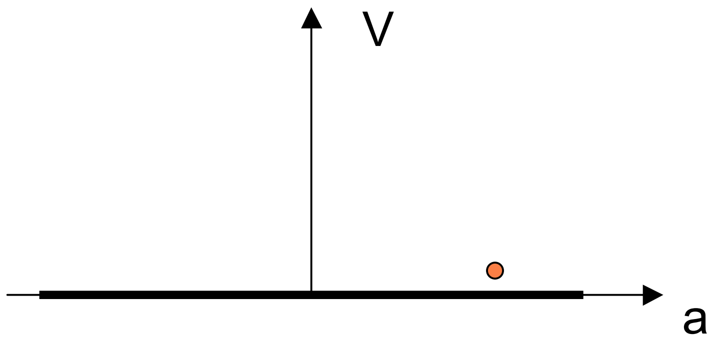


When the axion mass turns on, at QCD time,

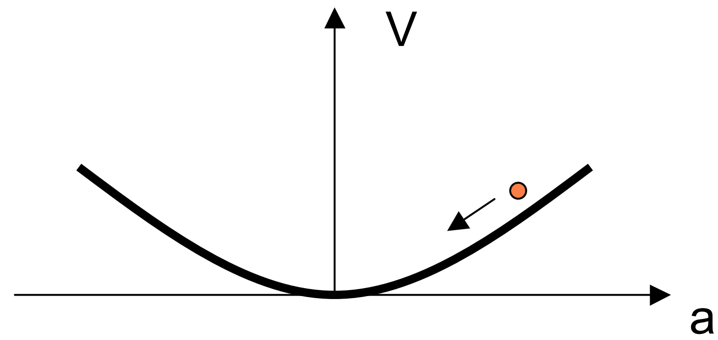
$$T_1 \quad 1 \text{ GeV} \qquad\qquad t_1 \quad 2\cdot 10^{-7} \text{ sec}$$

$$p_a(t_1) = \frac{1}{t_1} \quad 3\cdot 10^{-9} \text{ eV}$$

Axion production by vacuum realignment



$$T \geq 1\,\mathrm{GeV}$$



$$T \leq 1\,\mathrm{GeV}$$

$$n_a(t_1) \quad \frac{1}{2} m_a(t_1)\,\varphi(t_1)^2 \quad \frac{1}{2t_1} f_a{}^2\,\theta(t_1)^2$$

initial misalignment angle

$$\rho_a(t_0) \quad m_a\, n_a(t_1)\left(\frac{a_1}{a_0}\right)^3 \propto m_a{}^{-\frac{7}{6}}$$

Cold axion properties

- number density

$$n(t) \quad \frac{4 \cdot 10^{47}}{\mathrm{cm}^3} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{\frac{5}{3}} \left(\frac{a(t_1)}{a(t)} \right)^3$$

- velocity dispersion

$$\delta \mathrm{v}(t) \quad \frac{1}{m_a\, t_1} \frac{a(t_1)}{a(t)}$$

if decoupled

- phase space density

$$\mathbf{N} \quad n(t) \frac{(2\pi)^3}{\frac{4\pi}{3} (m_a\, \delta \mathrm{v})^3} \quad 10^{61} \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{\frac{8}{3}}$$

The cold axions thermalize and therefore form a BEC

$$\mathcal{L}_a = \ldots + \frac{1}{4!}\frac{m^2}{f^2}\varphi^4 + \ldots$$

$$\sigma_0 = \frac{1}{64\pi}\frac{m^2}{f^4} \quad 1.5\times 10^{-105}\ \mathrm{cm}^2 \left(\frac{m}{10^{-5}\ \mathrm{eV}}\right)^6$$

$$n(t)\ \sigma_0\ \delta \mathrm{v}(t) \ll H(t)$$

but …

At high phase space density $(\mathbf{N} \quad 10^{61})$

scattering rate

$$\Gamma_{scatt} \qquad n\ \sigma_0\ \delta v\ \mathbf{N}^{2}$$

thermalization rate

$$\Gamma_{relax} \qquad n\ \sigma_0\ \delta v\ \mathbf{N}$$

$$\Gamma_{relax}(t_1) \qquad H(t_1)$$

D. Semikoz
and I. Tkachev,
1995, 1997

A critical aspect of axion BEC phenomenology is whether the BEC continues to thermalize after it has formed.

Axion BEC means that almost all axions go to one state.

However, only if the BEC continually rethermalizes does the axion state track the lowest energy state.

After t_1 the thermalization rate due to self-interactions is

$$\Gamma_\lambda \qquad \lambda \; n \; m^{-2}$$

$$\Gamma_\lambda / H \qquad O(1)$$ at time t_1

$$\Gamma_\lambda(t) / H(t) \propto t \, a(t)^{-3} \propto a(t)^{-1}$$

Self-interactions are insufficient to rethermalize axion BEC after t1 even if they cause axion BEC at t1.

However, the thermalization rate due to gravitational interactions

$$\Gamma_g \qquad G\, n\, m^2\, l^2$$

with $l = (m\,\delta \mathrm{v})^{-1}$

$$\Gamma_g / H \qquad 4 \cdot 10^{-8} \left(\frac{f}{10^{12}\ \mathrm{GeV}} \right)^{\frac{2}{3}}$$

at time t_1

$$\Gamma_g(t) / H(t) \propto t\, a(t)^{-1} \propto a(t)$$

Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

$$T_\gamma \quad 100\,\mathrm{eV}\left(\frac{f}{10^{12}\ \mathrm{GeV}}\right)^{\frac{1}{2}}$$

After that

$$\delta\mathrm{v} \quad \frac{1}{m\,t}$$

$$\Gamma_g(t)/H(t) \propto t^3\,a(t)^{-3}$$

axion BEC

$$D_\mu D^\mu \varphi(x) = g^{\mu\nu}[\partial_\mu \partial_\nu - \Gamma^\lambda{}_{\mu\nu} \partial_\lambda]\varphi(x) = m^2 \varphi(x)$$

$$\varphi(x) = \Sigma_\alpha [a_\alpha \Phi_\alpha(x) + a_\alpha{}^\dagger \Phi_\alpha{}^*(x)]$$

Except for a tiny fraction, all axions are in the same state $(\alpha = 0)$

$$| N > = \frac{1}{\sqrt{N!}} (a_0{}^\dagger)^N | 0 >$$

N is the number of axions

$$< N | T_{\mu\nu} | N > = N[\partial_\mu \Phi_0{}^* \partial_\nu \Phi_0 + \partial_\nu \Phi_0{}^* \partial_\mu \Phi_0$$
$$+ g_{\mu\nu}(-\partial_\lambda \Phi_0{}^* \partial^\lambda \Phi_0 - m^2 \Phi_0{}^* \Phi_0)]$$

$$\Phi_0 = \frac{A}{a(t)^{\frac{3}{2}}} e^{-imt}$$

in a homogeneous and isotropic space-time.

In Minkowski space-time

Let $$\Phi_0(x) = e^{-imt}\,\Psi(x)$$

then $$i\,\partial_t\,\Psi = -\frac{1}{2m}\nabla^2\,\Psi$$

for non - relativistic motion

Let $$\Psi(\vec{x},t) = \frac{1}{\sqrt{2mN}}\,B(\vec{x},t)\,e^{i\beta(\vec{x},t)}$$

then $$T_{00}(\vec{x},t) \equiv \rho(\vec{x},t) = m\left(B(\vec{x},t)\right)^2$$

$$T_{0j} \equiv -\rho \, \mathrm{v}_j = -B^2 \, \partial_j \beta$$

hence $$\vec{\mathrm{v}}(\vec{x},t) = \frac{1}{m}\vec{\nabla}\beta(\vec{x},t)$$

$$T_{jk} = \rho \, \mathrm{v}_j \, \mathrm{v}_k + \frac{1}{4m^2}\left(\frac{1}{\rho}\partial_j\rho \; \partial_k\rho - \delta_{jk}\nabla^2\rho\right)$$

↑

stresses related to the Heisenberg uncertainty principle tend to homogenize the axion BEC

We have

$$\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{\mathrm{v}}) = 0$$

and

$$\partial_t \vec{\mathrm{v}} + (\vec{\mathrm{v}} \cdot \vec{\nabla}) \vec{\mathrm{v}} = -\vec{\nabla} q$$

with

$$q = -\frac{1}{2m^2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}$$

To recover CDM, let m go to infinity

In the linear regime, within the horizon,

axion BEC density perturbations obey

$$\partial_t^{\,2}\,\delta(\vec{k},t) + 2H\partial_t\delta(\vec{k},t) - \left(4\pi G\rho_0 - \frac{k^4}{4m^2a^4}\right)\delta(\vec{k},t) = 0$$

Jeans' length

$$\ell_{\rm J} = \left(16\pi\, G\,\rho\, m^2\right)^{\frac{1}{4}} = 1.02\cdot 10^{14}\ {\rm cm}\left(\frac{10^{-5}\ {\rm eV}}{m}\right)^{\frac{1}{2}}\left(\frac{10^{-29}\ {\rm g/cc}}{\rho}\right)^{\frac{1}{4}}$$

In the linear regime within the horizon, axion BEC and CDM are indistinguishable on all scales of observational interest,

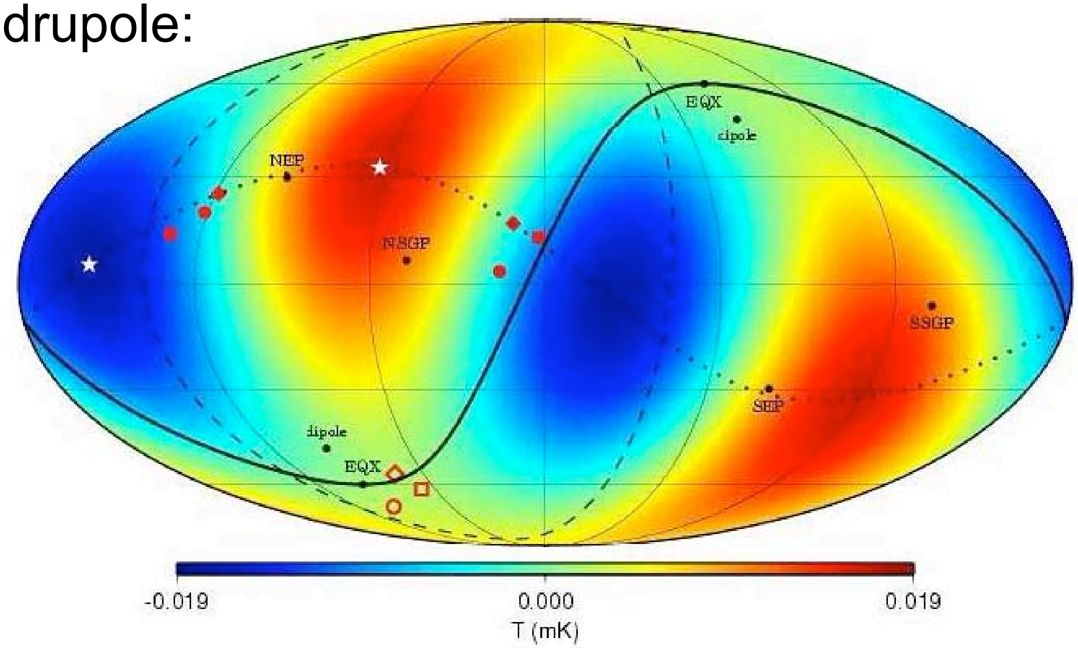
but

axion BEC differs from CDM

in the non-linear regime &

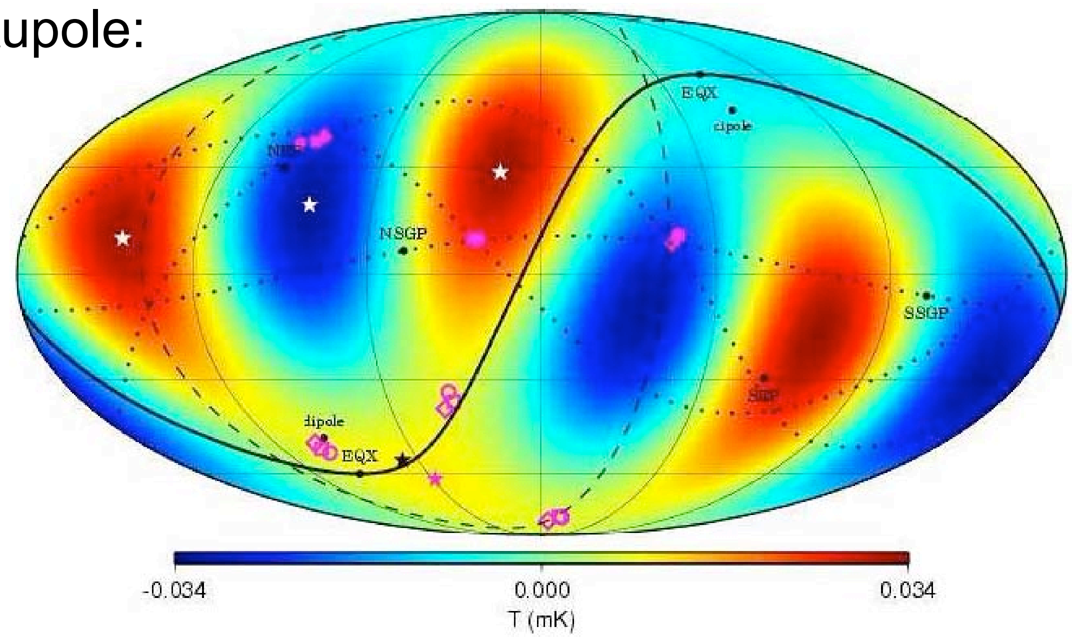
upon entering the horizon

quadrupole:



CMB multipoles are aligned

octupole:



M. Tegmark,
A. de Oliveira-Costa
A. Hamilton, 2003

C. Copi
D. Huterer
D. Schwarz
G. Starkman, 2006

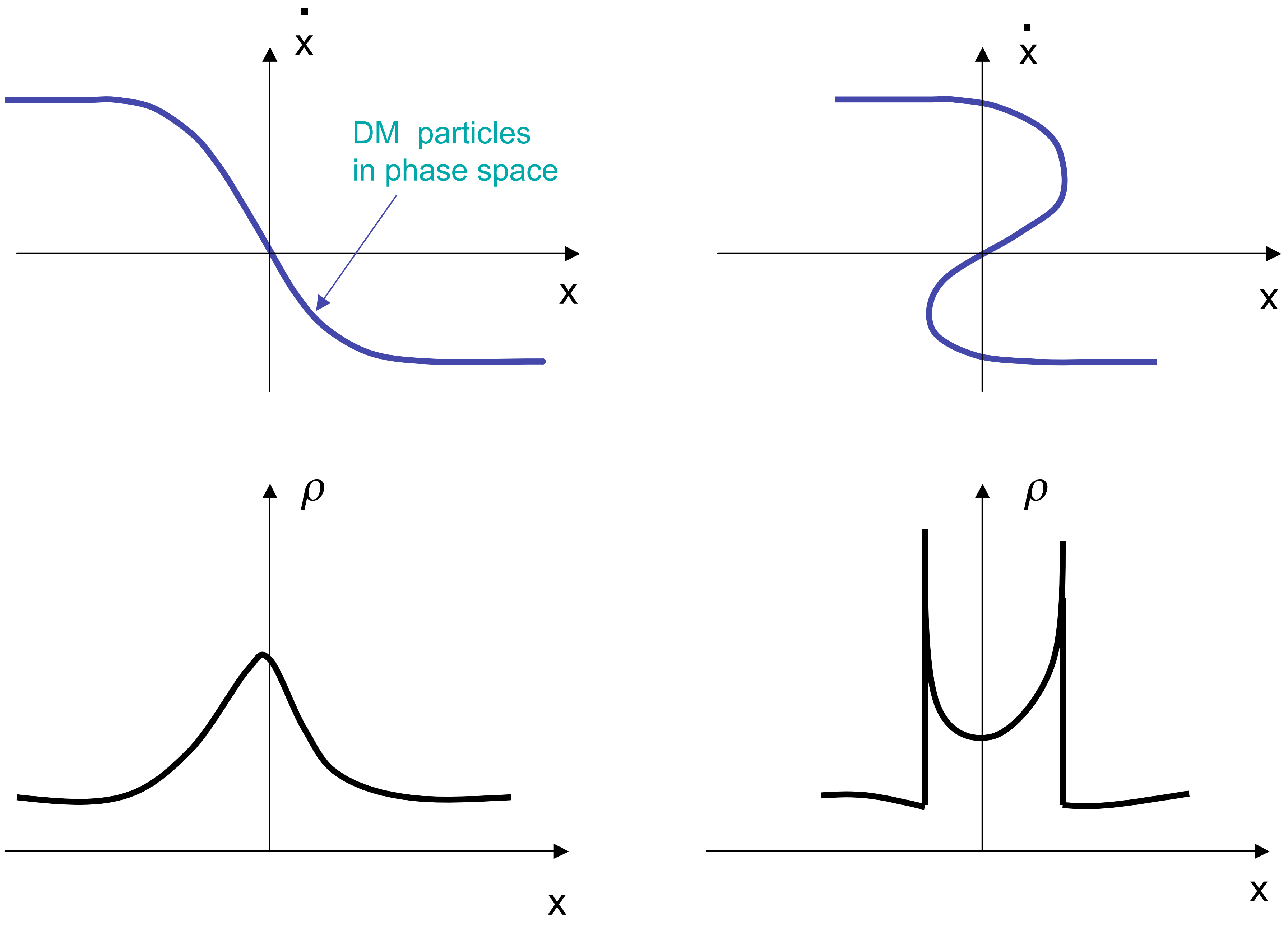
Upon entering the horizon

CDM density perturbations evolve linearly

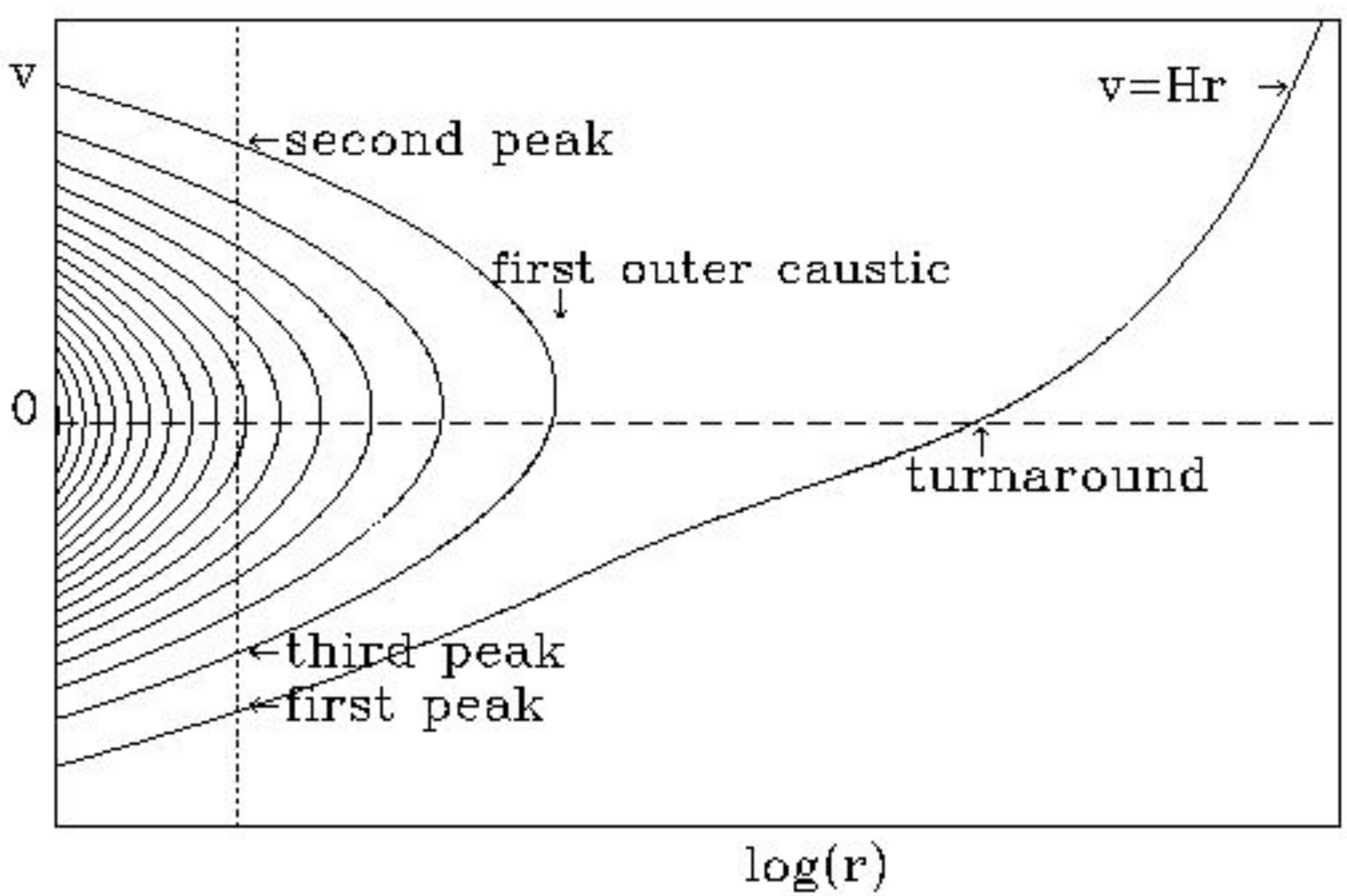
the density perturbations in the axion BEC evolve non-linearly because the axionBEC rethermalizes upon entering the horizon.

axion BEC provides a possible mechanism for the alignment of CMBR multipoles through the ISW effect.

DM forms caustics in the non-linear regime



Phase space structure of spherically symmetric halos



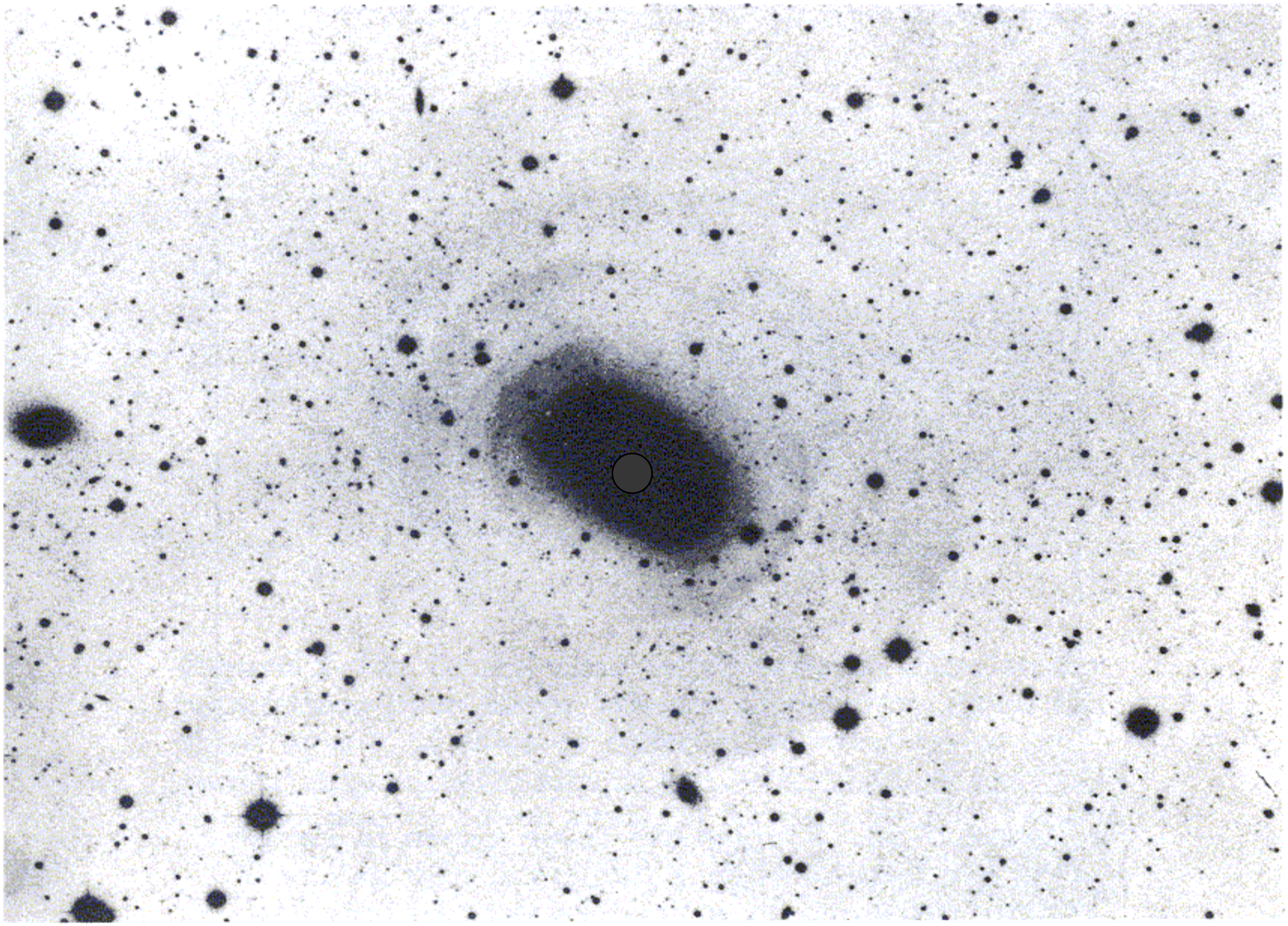


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)

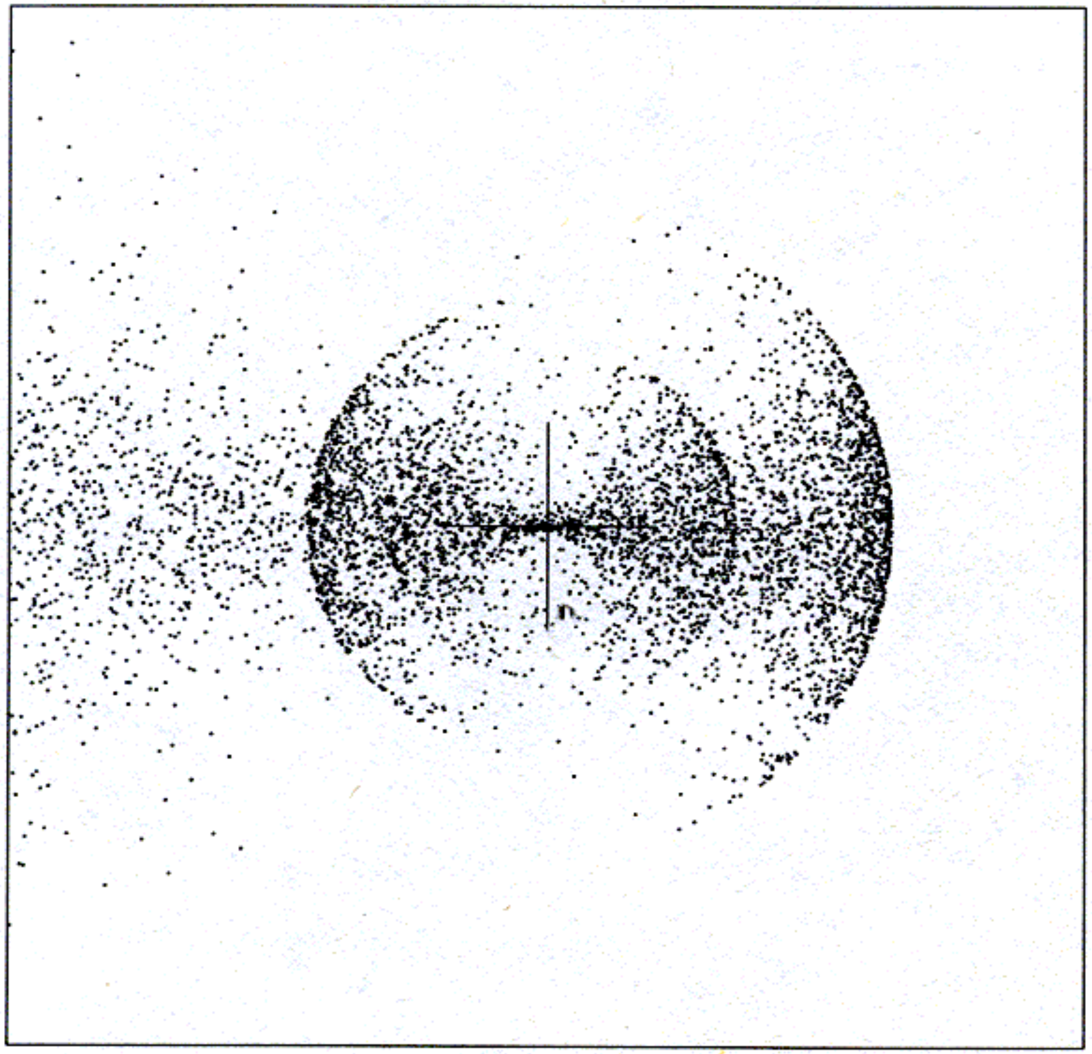
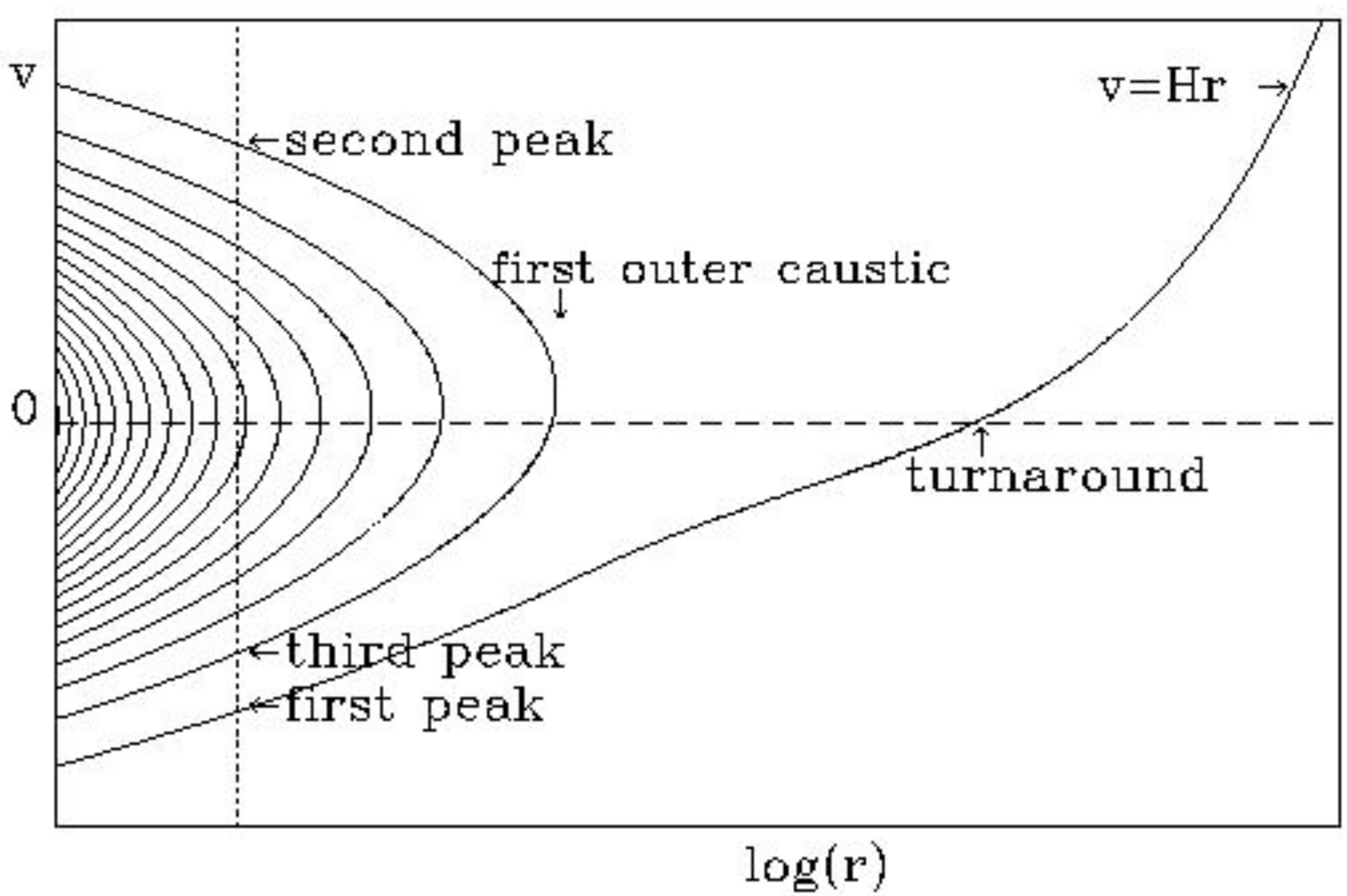


Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)

Phase space structure of spherically symmetric halos



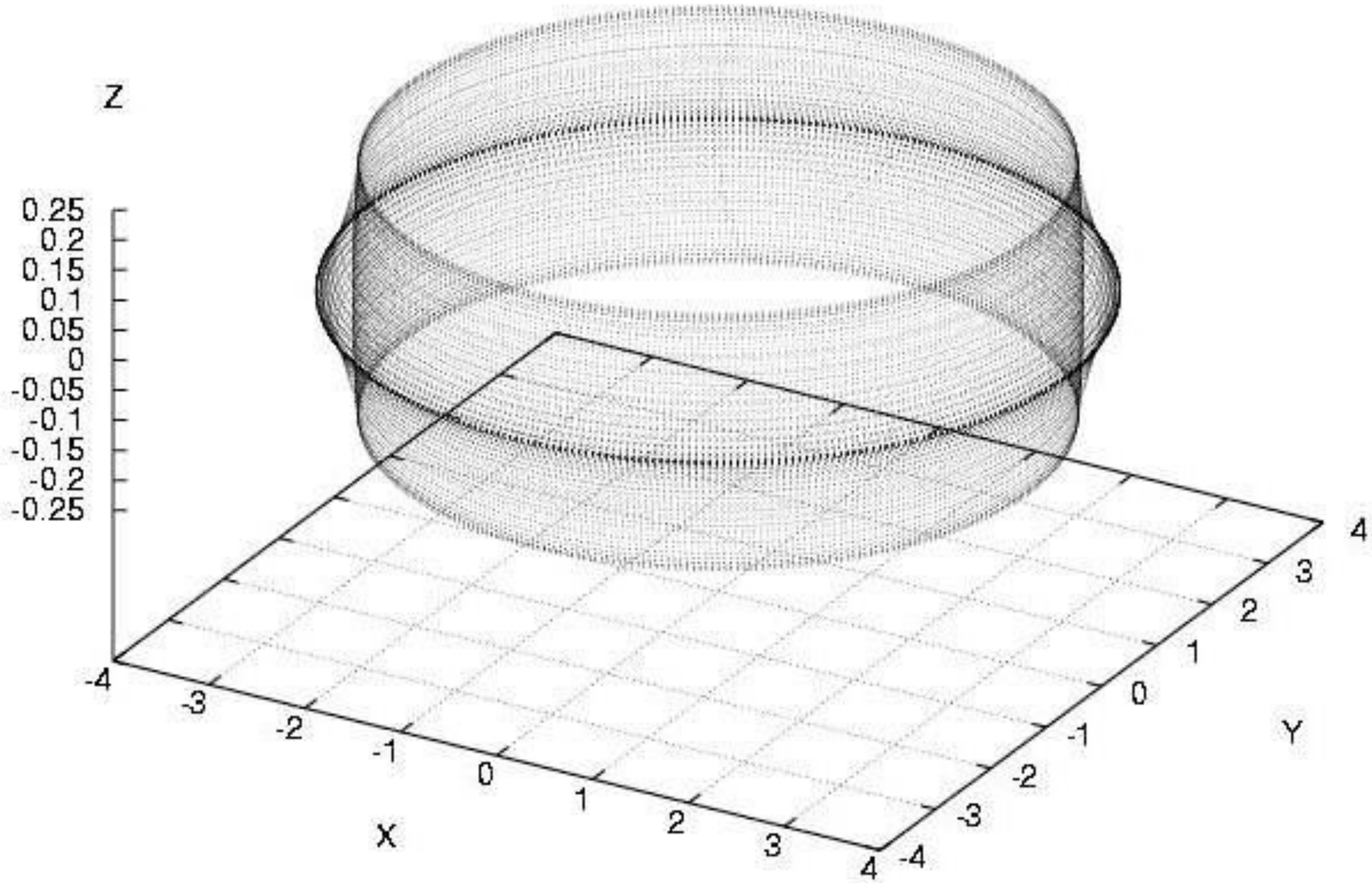
Galactic halos have inner caustics as
well as outer caustics.

If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

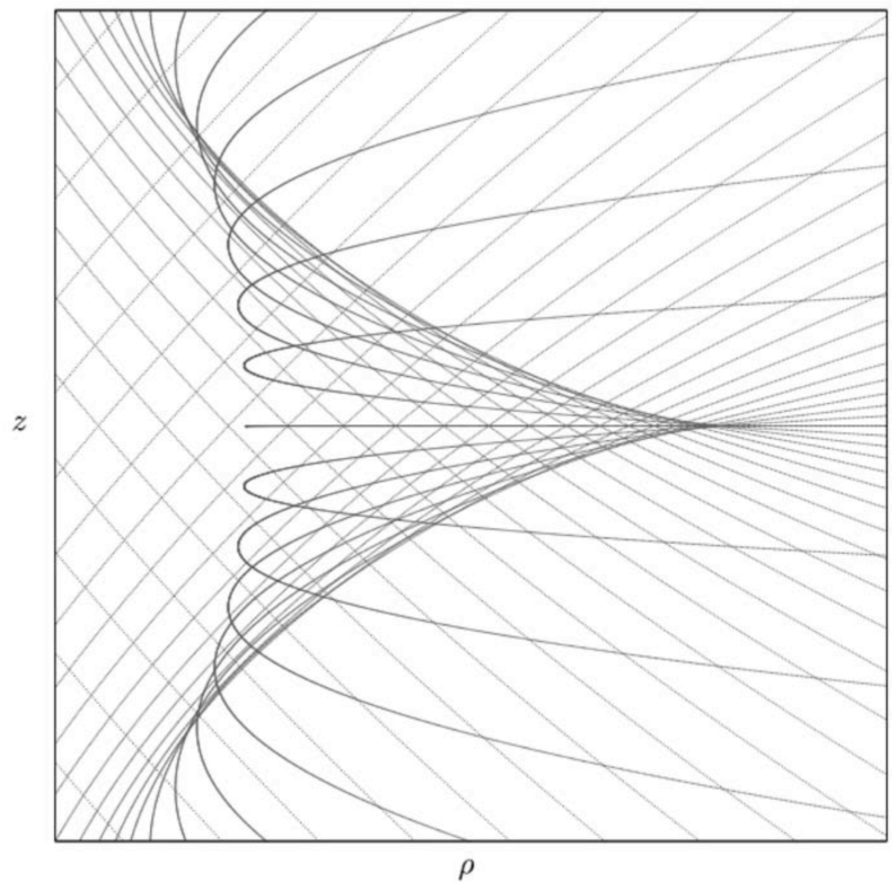
If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

(Arvind Natarajan and PS, 2005).

simulations by Arvind Natarajan



The caustic ring cross-section



D_{-4}

an elliptic umbilic catastrophe

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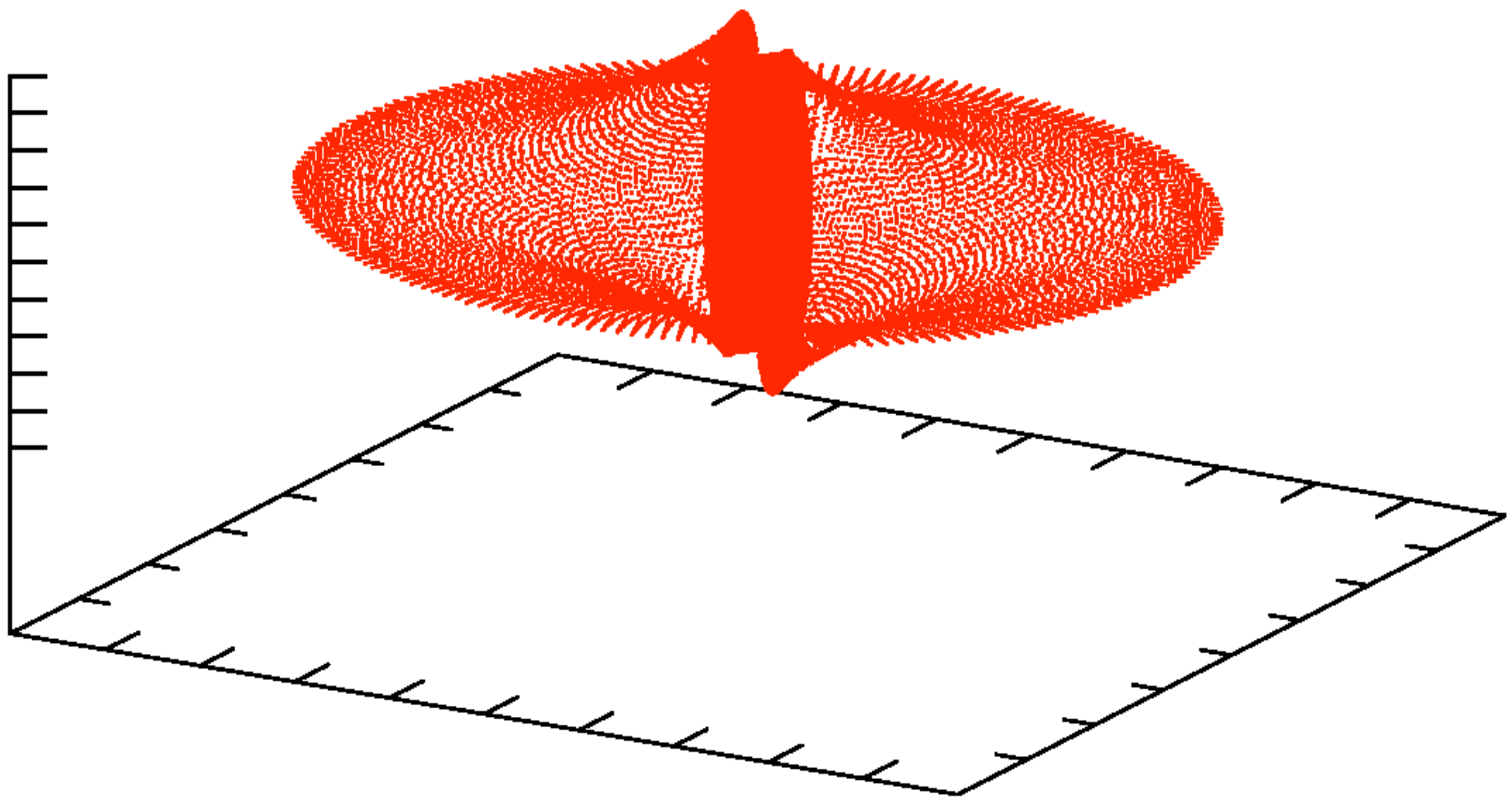


On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

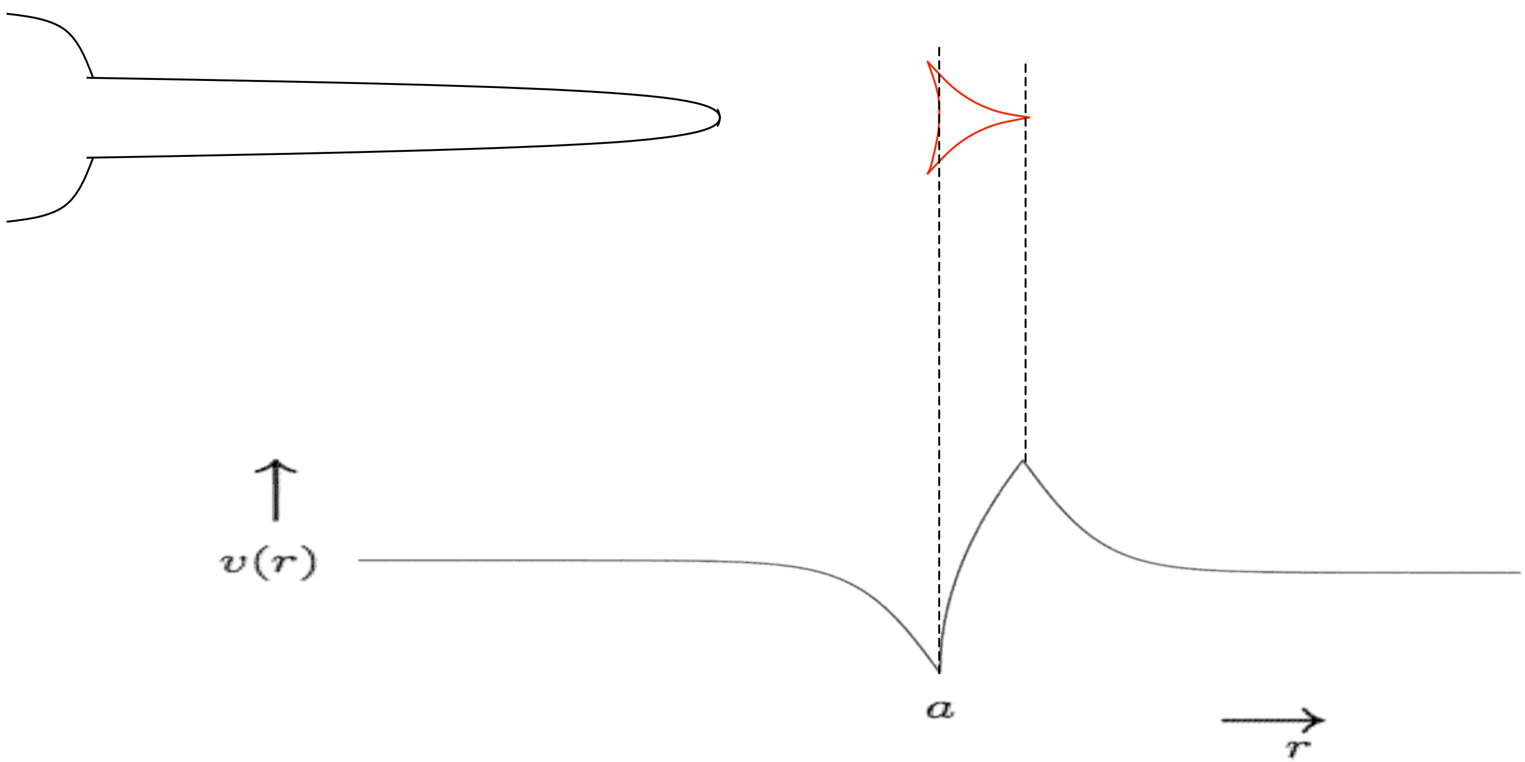
in the galactic plane

with radii $(n = 1, 2, 3 \ldots)$

$$a_n = \frac{40\,\mathrm{kpc}}{n}\left(\frac{v_{rot}}{220\,\mathrm{km/s}}\right)\left(\frac{j_{max}}{0.18}\right)$$

$j_{max} \cong 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

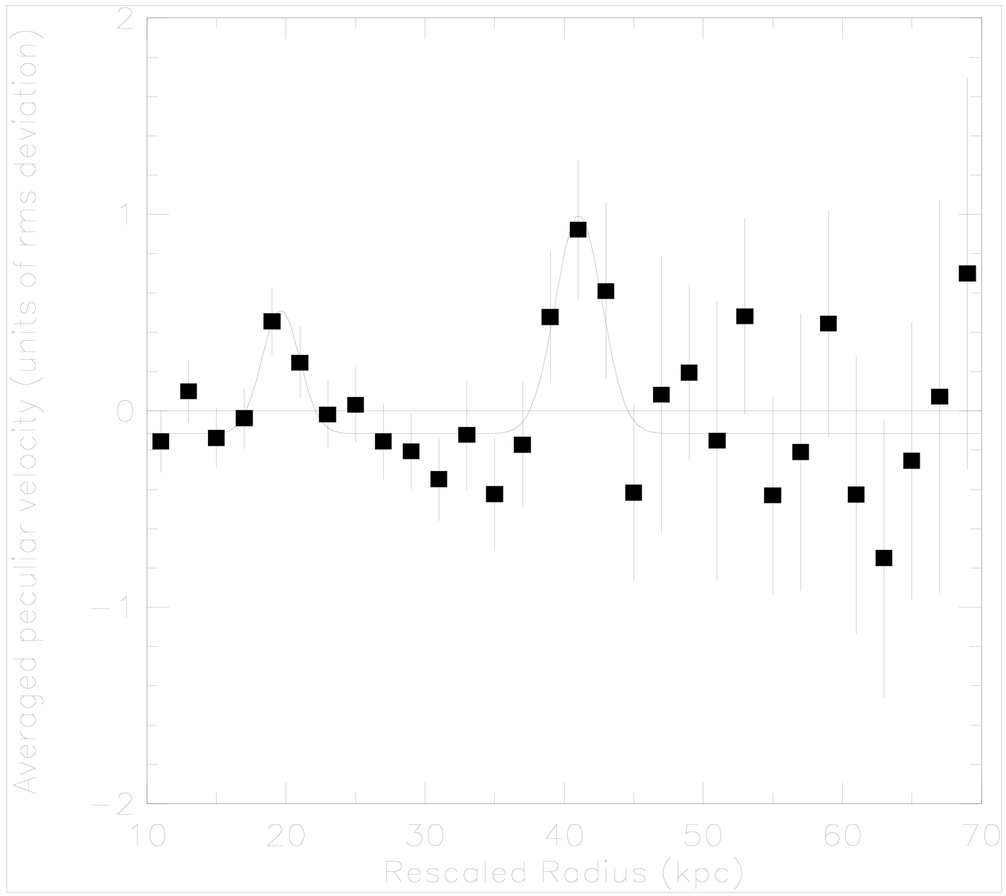
Effect of a caustic ring of dark matter upon the galactic rotation curve



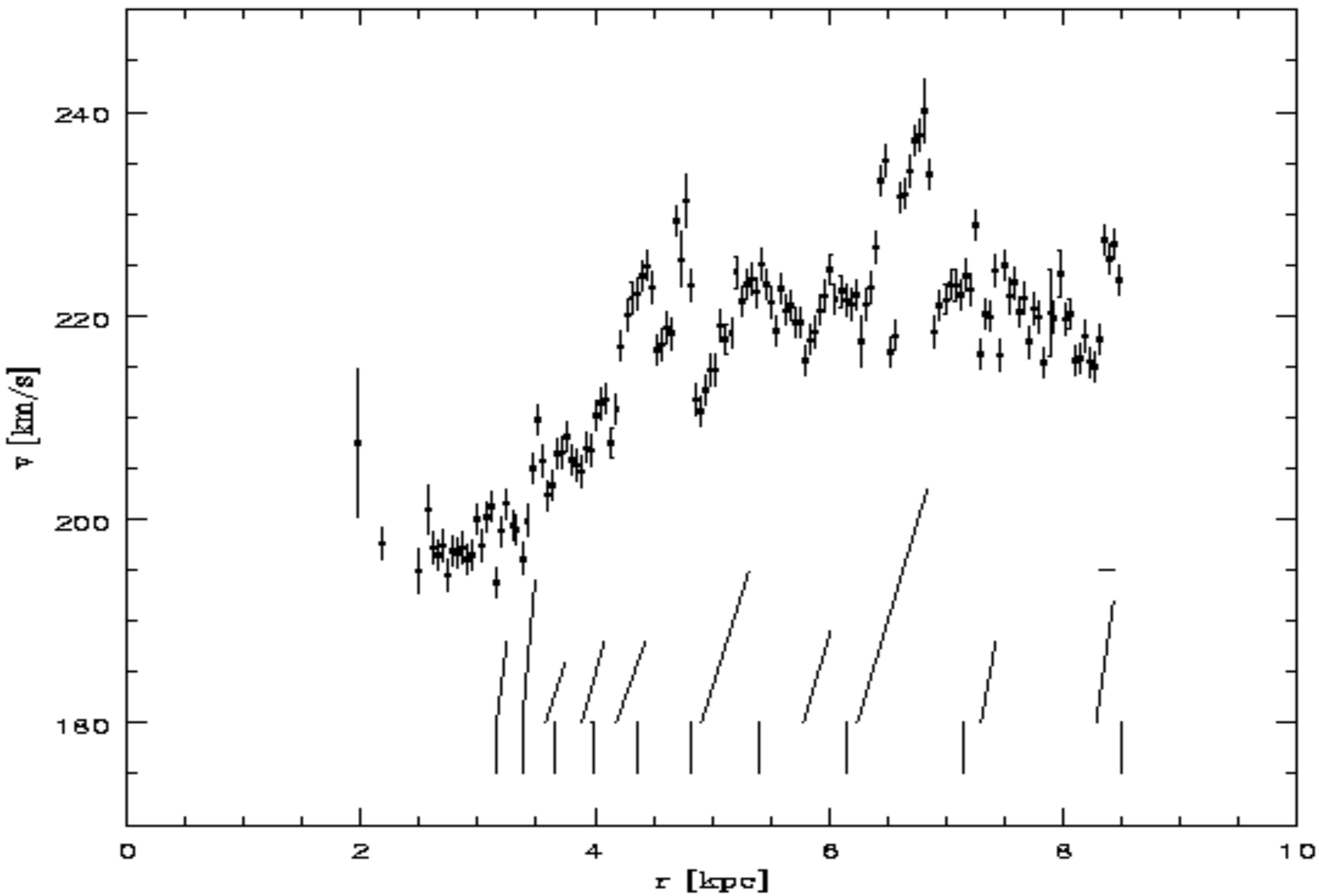
Composite rotation curve

(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy

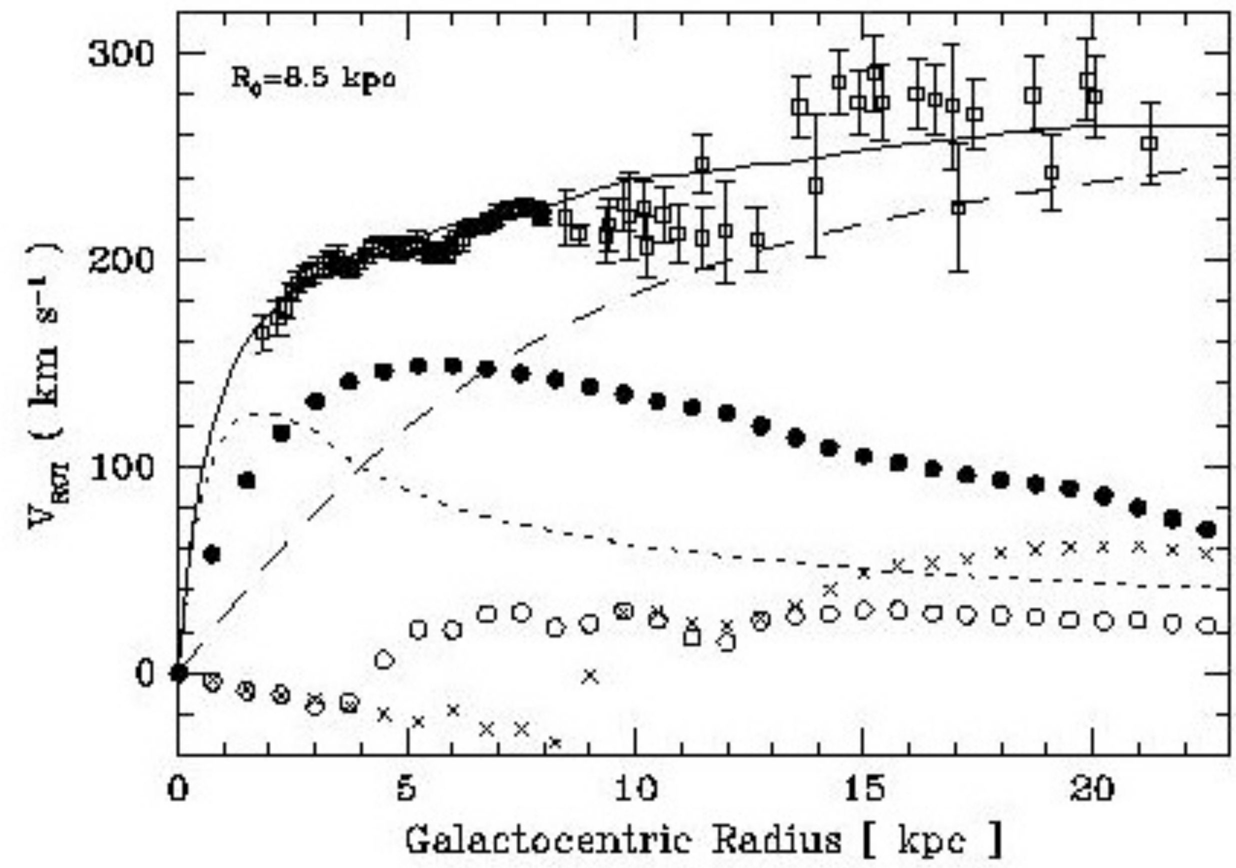


Inner Galactic rotation curve



from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)

Outer Galactic rotation curve



R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Monoceros Ring of stars

H. Newberg et al. 2002; B. Yanny et al., 2003; R.A. Ibata et al., 2003;
H.J. Rocha-Pinto et al, 2003; J.D. Crane et al., 2003; N.F. Martin et al., 2005

in the Galactic plane

at galactocentric distance $r \quad 20\,\mathrm{kpc}$

appears circular, actually seen for $100^0 < l < 270^0$

scale height of order 1 kpc

velocity dispersion of order 20 km/s

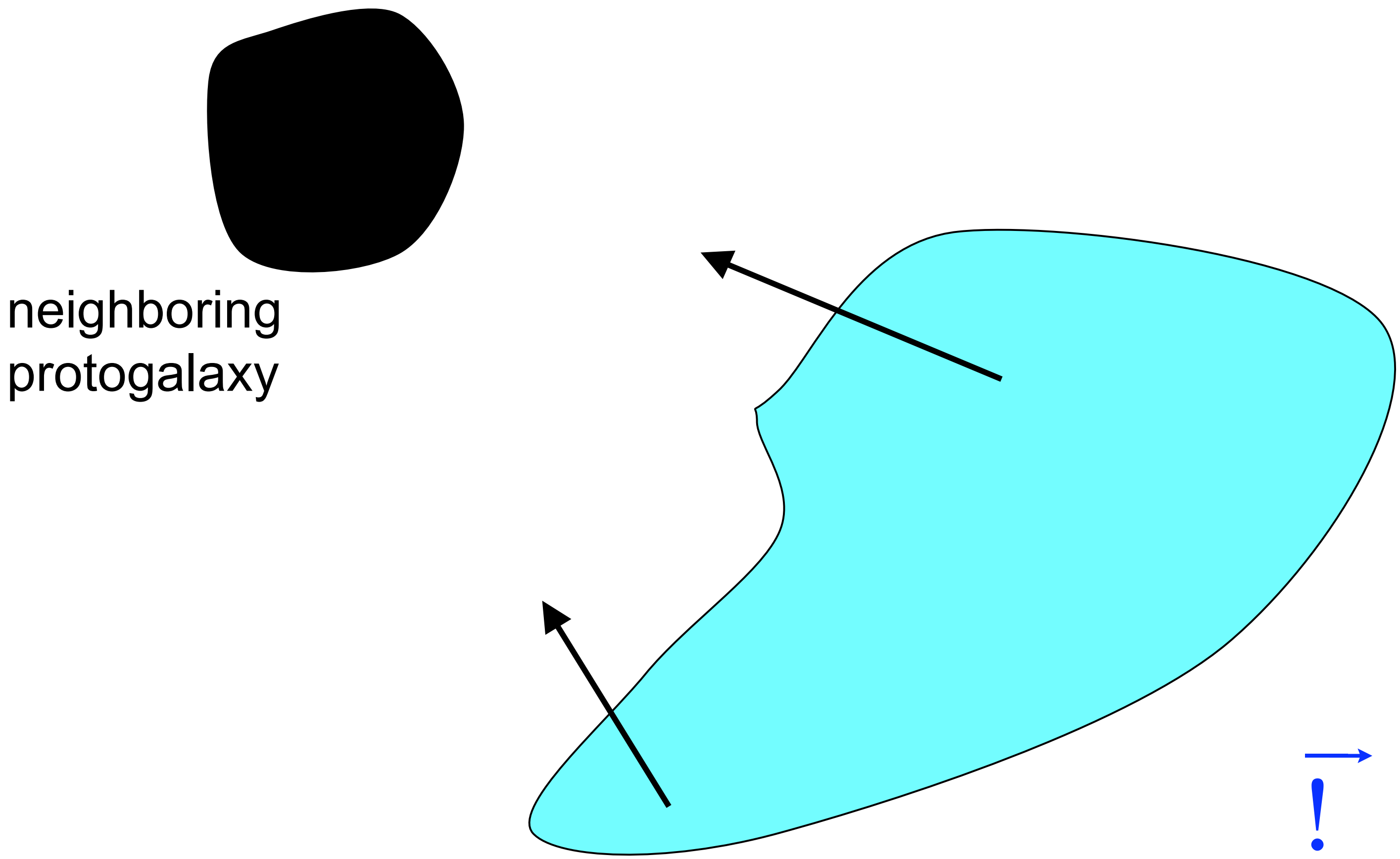
may be caused by the n = 2 caustic ring of dark matter (A. Natarajan and P.S. '07)

TABLE I. Velocity vectors $\vec{v}^{n\pm}$ and densities $d_n^{\pm}$ of the first 40 flows in the caustic ring halo model, in galactic coordinates. The flow of velocity vector $\vec{v}^{n\pm}$ has density $d_n^{\pm}$ or $d_n^{\mp}$.

n	$v_G^{n\pm}$ (km/s)	$v_{yG}^{n\pm}$ (km/s)	$v_{zG}^{n\pm}$ (km/s)	$v_{xG}^{n\pm}$ (km/s)	d_n^+ (10^{-26}gr/cm^3)	d_n^- (10^{-26}gr/cm^3)
1	620	130	±605	/	0.3	0.3
2	560	230	±510	/	0.8	0.8
3	530	320	±420	/	1.4	1.4
4	500	405	±300	/	3.4	3.4
5	480	470	0	±100	170.	15.
6	465	400	0	±240	6.5	3.4
7	450	330	0	±305	4.1	1.3
8	430	295	0	±320	2.0	1.1
9	420	240	0	±340	1.5	0.7
10	410	200	0	±355	1.0	1.0
11	395	180	0	±350	0.9	0.9
12	385	160	0	±350	0.8	0.8
13	375	150	0	±345	0.7	0.7
14	365	135	0	±340	0.7	0.7
15	355	120	0	±335	0.6	0.6
16	350	110	0	±330	0.6	0.6
17	340	105	0	±320	0.5	0.5
18	330	95	0	±315	0.5	0.5
19	320	90	0	±310	0.5	0.5
20	310	80	0	±300	0.4	0.4

from L. Duffy and PS, Phys. Rev. D78 (2008) 063508

Tidal torque theory with CDM



$$\vec{\nabla} \times \vec{v} = 0$$

The velocity field remains irrotational

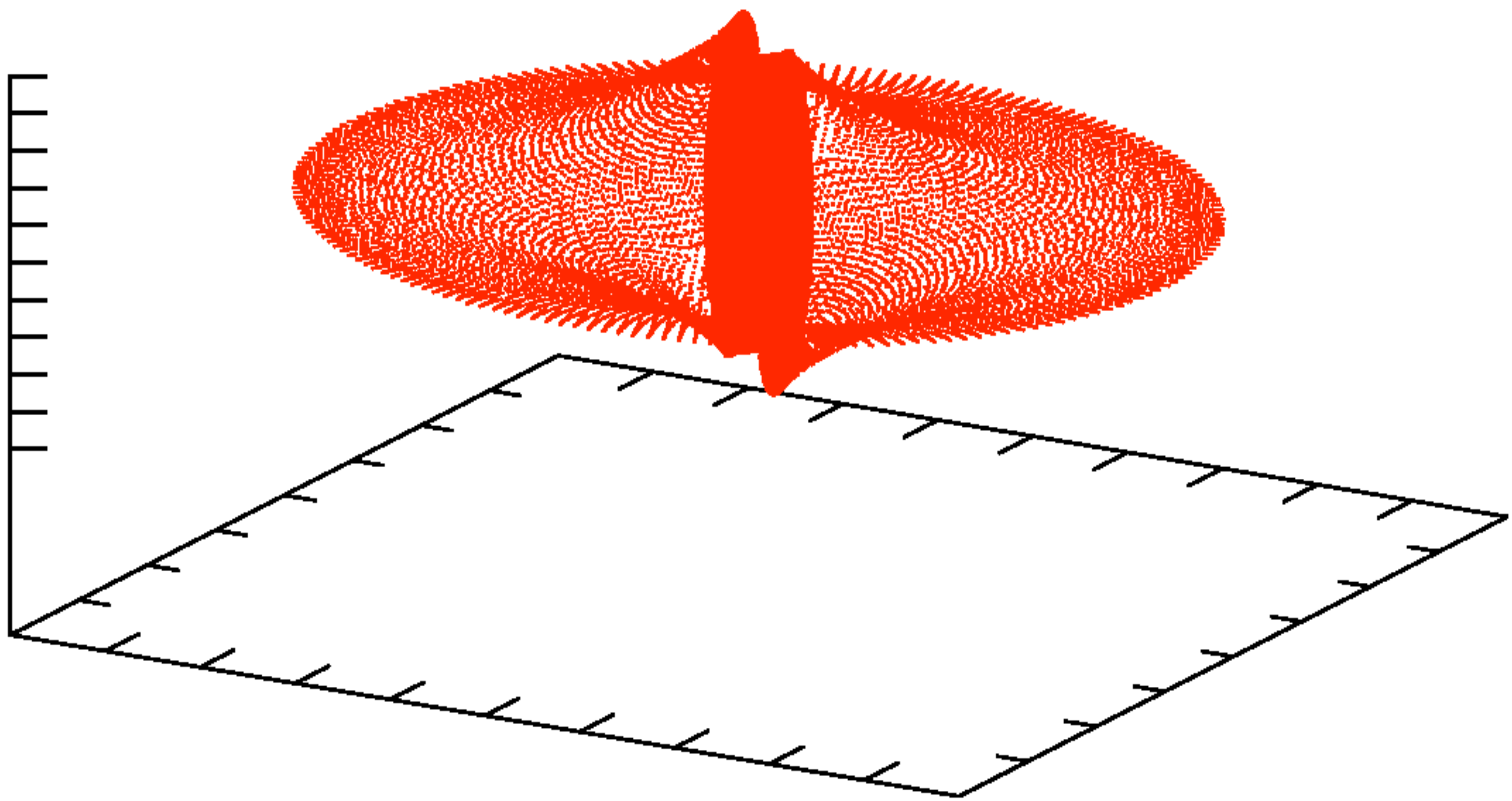
For collisionless particles

$$\frac{d\,\vec{\mathrm{v}}}{dt}(\vec{r},t) = \frac{\partial\,\vec{\mathrm{v}}}{\partial t}(\vec{r},t) + \left(\vec{\mathrm{v}}(\vec{r},t)\cdot\vec{\nabla}\right)\vec{\mathrm{v}}(\vec{r},t)$$

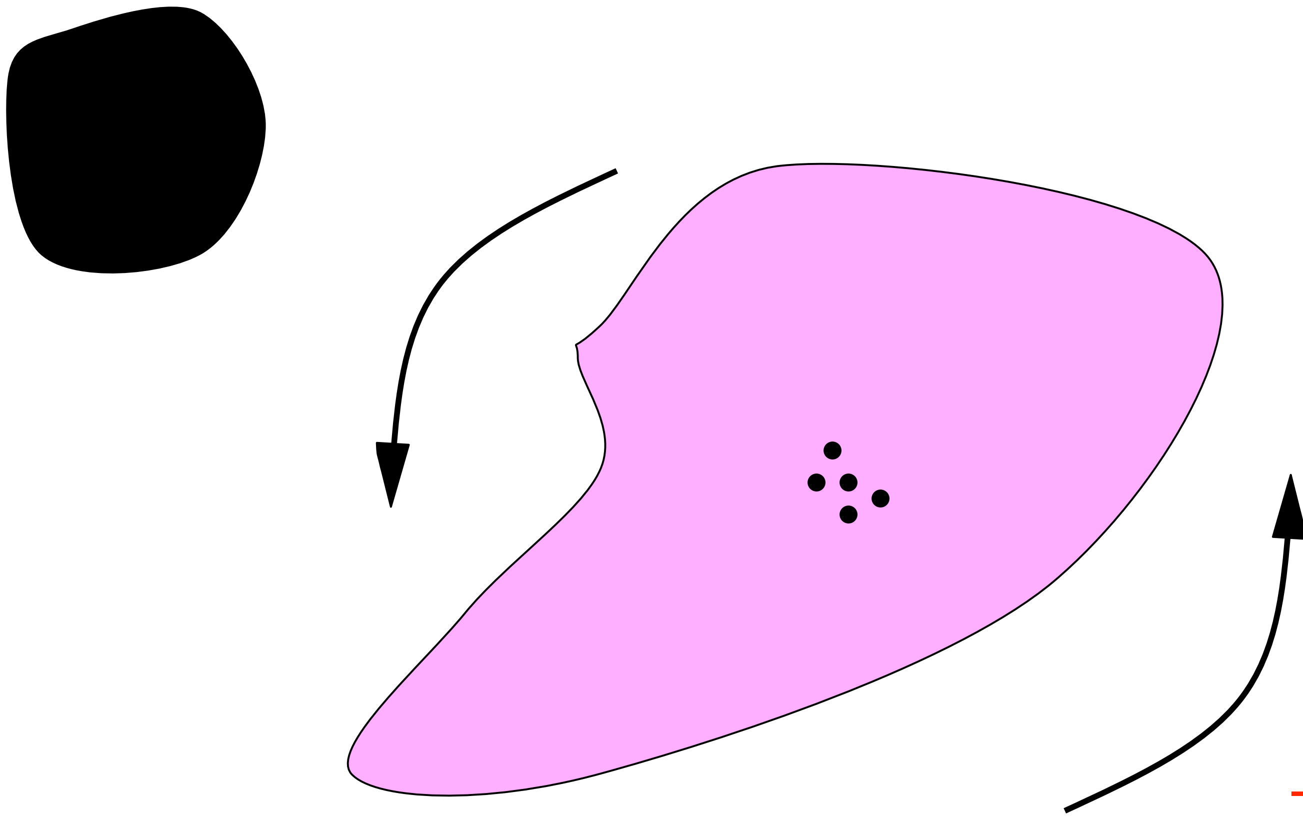
$$= -\vec{\nabla}\Phi(\vec{r},t)$$

If $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ initially,

then $\vec{\nabla}\times\vec{\mathrm{v}} = 0$ for ever after.



Tidal torque theory with axion BEC



$$\vec{\nabla} \times \vec{v} \neq 0$$

Net overall rotation is produced because, in the lowest energy state, all axions fall with the same angular momentum

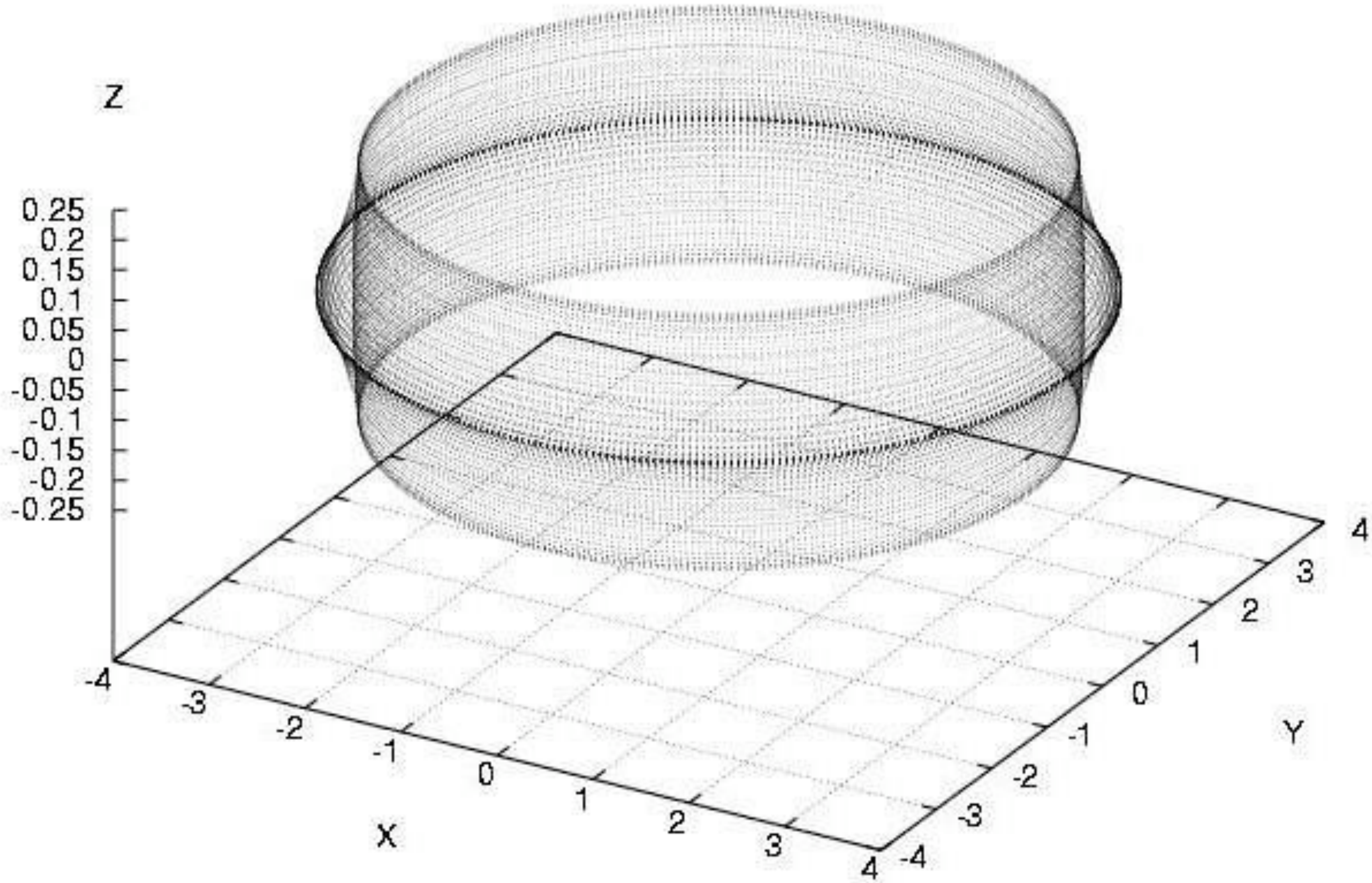
For axion BEC

$$E = \sum_{i=1}^{N} \frac{L_i^{\,2}}{2I}$$

is minimized for given $L = \sum_{i=1}^{N} L_i$

when $L_1 = L_2 = L_3 = \ \dots \ = L_N$.

$\vec{\nabla} \times \vec{v} \neq 0$ is allowed through the appearance of vortices; see discussion by Tanja Rindler-Daller and Paul Shapiro at this meeting.


Z
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-4
-3
-2
-1
0
1
2
3
4
X
-4
-3
-2
-1
0
1
2
3
4
Y

Summary

- axion BEC and CDM are indistinguishable in the linear regime inside the horizon on all scales of observational interest.

- axion BEC may provide a mechanism for net overall rotation in galactic halos.

- axion BEC may provide a mechanism for the alignment of CMBR multipoles.