Bose-Einstein Condensation of Dark Matter Axions

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I’ll argue:

The Dark Matter is Axions

based on arXiv: 0901.1106
with Qiaoli Yang
How so?

• axions differ from ordinary CDM because they form a BEC, and the axion BEC continues to rethermalize, tracking the lowest energy state.

• axion BEC appears to solve two puzzles with CDM

1) evidence for caustic rings of dark matter implies that the dark matter falls onto galactic halos with "net overall rotation", implying \( \vec{\nabla} \neq 0 \)
whereas CDM predicts \( \vec{\nabla} = 0 \).

2) unexpected alignments among CMBR multipoles.
Outline

• Review of axion properties

• Cold dark matter axions form a rethermalizing Bose-Einstein condensate

• Axion BEC differs from ordinary CDM

• Compare CDM and axion BEC density perturbations with observations in three arenas

  1. upon entering the horizon
  2. in the linear regime within the horizon
  3. in the non-linear regime
\[ m_a \quad 6 \text{ eV} \quad \frac{10^6 \text{GeV}}{f_a} \]

\[ L_{a\bar{f}f} = i g_f \frac{\varphi}{f_a} \bar{f} \gamma_5 f \]

\[ L_{a\gamma\gamma} = g_\gamma \frac{\alpha}{\pi} \frac{\varphi}{f_a} \overrightarrow{E} \cdot \overrightarrow{B} \]

\[ g_\gamma = 0.97 \quad \text{in KSVZ model} \]
\[ 0.36 \quad \text{in DFSZ model} \]
The remaining axion window

- $m_a$ (eV)
- $f_a$ (GeV)

laboratory searches

stellar evolution

cosmology
There are two axion populations: hot and cold.

When the axion mass turns on, at QCD time,

\[ T_1 = 1 \text{ GeV} \quad t_1 = 2 \times 10^{-7} \text{ sec} \]

\[ p_a(t_1) = \frac{1}{t_1} = 3 \times 10^{-9} \text{ eV} \]
Axion production by vacuum realignment

\[ T \geq 1 \text{GeV} \]

\[ n_a(t_1) = \frac{1}{2} m_a(t_1) \varphi(t_1)^2 \]

\[ \rho_a(t_0) = m_a n_a(t_1) \left( \frac{a_1}{a_0} \right)^3 \propto m_a \]

\[ T \leq 1 \text{GeV} \]

\[ \frac{1}{2t_1} f_a^2 \theta(t_1)^2 \]
Cold axion properties

- **number density**
  \[ n(t) = \frac{4 \cdot 10^{47}}{\text{cm}^3} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{\frac{5}{3}} \left( \frac{a(t_1)}{a(t)} \right)^3 \]

- **velocity dispersion**
  \[ \delta v(t) = \frac{1}{m_a t_1} \frac{a(t_1)}{a(t)} \] if decoupled

- **phase space density**
  \[ N n(t) = \frac{(2\pi)^3}{4\pi} \frac{1}{(m_a \delta v)^3} \]
  \[ 10^{61} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{\frac{8}{3}} \]
The cold axions thermalize and therefore form a BEC

\[ L_a = \ldots + \frac{1}{4!} \frac{m^2}{f^2} \varphi^4 + \ldots \]

\[ \sigma_0 = \frac{1}{64\pi} \frac{m^2}{f^4} \quad 1.5 \times 10^{-105} \text{ cm}^2 \left( \frac{m}{10^{-5} \text{ eV}} \right)^6 \]

\[ n(t) \sigma_0 \delta v(t) \ll H(t) \quad \text{but \ldots} \]
At high phase space density \((N \approx 10^{61})\)

scattering rate

\[ \Gamma_{\text{scatt}} \approx n \sigma_0 \delta v N^2 \]

thermalization rate

\[ \Gamma_{\text{relax}} \approx n \sigma_0 \delta v N \]

\[ \Gamma_{\text{relax}}(t_1) \approx H(t_1) \]

A critical aspect of axion BEC phenomenology is whether the BEC continues to thermalize after it has formed.

Axion BEC means that almost all axions go to one state.

However, only if the BEC continually rethermalizes does the axion state track the lowest energy state.
After $t_1$ the thermalization rate due to self-interactions is

$$\Gamma_\lambda \propto \lambda n m^{-2}$$

$$\Gamma_\lambda/H \sim O(1) \quad \text{at time } t_1$$

$$\Gamma_\lambda(t)/H(t) \propto t a(t)^{-3} \propto a(t)^{-1}$$

Self-interactions are insufficient to rethermalize axion BEC after $t_1$ even if they cause axion BEC at $t_1$. 
However, the thermalization rate due to gravitational interactions

\[ \Gamma_g = G n m^2 l^2 \]

with \( l = (m \delta v)^{-1} \)

\[ \Gamma_g / H = 4 \cdot 10^{-8} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{2/3} \]

at time \( t_1 \)

\[ \Gamma_g(t) / H(t) \propto t a(t)^{-1} \propto a(t) \]
Gravitational interactions thermalize the axions and cause them to form a BEC when the photon temperature

\[ T_\gamma \sim 100 \text{ eV} \left( \frac{f}{10^{12} \text{ GeV}} \right)^{\frac{1}{2}} \]

After that

\[ \delta V \sim \frac{1}{m t} \]

\[ \Gamma_g(t)/H(t) \propto t^3 a(t)^{-3} \]
\[ D_\mu D^\mu \varphi(x) = g^{\mu\nu} [\partial_\mu \partial_\nu - \Gamma^\lambda_{\mu\nu} \partial_\lambda] \varphi(x) = m^2 \varphi(x) \]

\[ \varphi(x) = \sum_\alpha [a_\alpha \Phi_\alpha(x) + a_\alpha^\dagger \Phi_\alpha^*(x)] \]

Except for a tiny fraction, all axions are in the same state \((\alpha = 0)\)

\[ |N > = \frac{1}{\sqrt{N!}} (a_0^\dagger)^N |0 > \]

\(N\) is the number of axions
\[
\langle N | T_{\mu\nu} | N \rangle = N \left[ \partial_\mu \Phi_0^* \partial_\nu \Phi_0 + \partial_\nu \Phi_0^* \partial_\mu \Phi_0 \\
+ g_{\mu\nu} (-\partial_\lambda \Phi_0^* \partial^\lambda \Phi_0 - m^2 \Phi_0^* \Phi_0) \right]
\]

\[\Phi_0 = \frac{A}{3} e^{-imt} \quad \frac{1}{a(t)^2}\]

in a homogeneous and isotropic space-time.
In Minkowski space-time

Let

$$\Phi_0(x) = e^{-imt} \Psi(x)$$

then

$$i \partial_t \Psi = -\frac{1}{2m} \nabla^2 \Psi$$

for non-relativistic motion

Let

$$\Psi(\vec{x},t) = \frac{1}{\sqrt{2mN}} B(\vec{x},t) e^{i\beta(\vec{x},t)}$$

then

$$T_{00}(\vec{x},t) \equiv \rho(\vec{x},t) = m (B(\vec{x},t))^2$$
\[ T_{0j} \equiv -\rho \mathbf{v}_j = -B^2 \partial_j \beta \]

hence
\[ \mathbf{v}(\bar{x}, t) = \frac{1}{m} \mathbf{\nabla} \beta(\bar{x}, t) \]

\[ T_{jk} = \rho \mathbf{v}_j \mathbf{v}_k + \frac{1}{4m^2} \left( \frac{1}{\rho} \partial_j \rho \partial_k \rho - \delta_{jk} \nabla^2 \rho \right) \]

stresses related to the Heisenberg uncertainty principle tend to homogenize the axion BEC
We have

\[ \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \]

and

\[ \partial_t \vec{v} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla q \]

with

\[ q = -\frac{1}{2m^2} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \]

To recover CDM, let \( m \) go to infinity.
In the linear regime, within the horizon, axion BEC density perturbations obey

\[ \partial_t^2 \delta(\vec{k},t) + 2H \partial_t \delta(\vec{k},t) - \left( 4\pi G \rho_0 - \frac{k^4}{4m^2 a^4} \right) \delta(\vec{k},t) = 0 \]

**Jeans’ length**

\[ \ell_J = \left(16\pi G \rho m^2 \right)^{\frac{1}{4}} = 1.02 \cdot 10^{14} \text{ cm} \left( \frac{10^{-5} \text{ eV}}{m} \right)^{\frac{1}{2}} \left( \frac{10^{-29} \text{ g/cc}}{\rho} \right)^{\frac{1}{4}} \]
In the linear regime within the horizon, axion BEC and CDM are indistinguishable on all scales of observational interest,

but

axion BEC differs from CDM in the non-linear regime & upon entering the horizon
quadrupole:

CMB multipoles are aligned

octupole:

C. Copi D. Huterer D. Schwarz G. Starkman, 2006
Upon entering the horizon

CDM density perturbations evolve linearly

the density perturbations in the axion BEC evolve non-linearly because the axion BEC rethermalizes upon entering the horizon.

axion BEC provides a possible mechanism for the alignment of CMBR multipoles through the ISW effect.
DM forms caustics in the non-linear regime
Phase space structure of spherically symmetric halos
Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine’s book)
Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)
Phase space structure of spherically symmetric halos
Galactic halos have inner caustics as well as outer caustics.

If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

(Arvind Natarajan and PS, 2005).
simulations by Arvind Natarajan
The caustic ring cross-section

an elliptic umbilic catastrophe

$D_4$
Galactic halos have inner caustics as well as outer caustics.

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(Arvind Natarajan and PS, 2005).
On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be in the galactic plane with radii $(n = 1, 2, 3...)$

$$
a_n = \frac{40\text{kpc}}{n} \left( \frac{v_{\text{rot}}}{220\text{km/s}} \right) \left( \frac{j_{\text{max}}}{0.18} \right)$$

$j_{\text{max}} \approx 0.18$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.
Effect of a caustic ring of dark matter upon the galactic rotation curve
Composite rotation curve
(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy
Inner Galactic rotation curve

from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)
Outer Galactic rotation curve

Monoceros Ring of stars

in the Galactic plane
at galactocentric distance \( r = 20 \text{ kpc} \)
appears circular, actually seen for \( 100^0 < l < 270^0 \)
scale height of order \( 1 \text{ kpc} \)
velocity dispersion of order \( 20 \text{ km/s} \)

may be caused by the \( n = 2 \) caustic ring of
dark matter \( \text{(A. Natarajan and P.S. '07)} \)
TABLE I. Velocity vectors $\vec{v}^{n \pm}$ and densities $d_n^{\pm}$ of the first 40 flows in the caustic ring halo model, in galactic coordinates. The flow of velocity vector $\vec{v}^{n \pm}$ has density $d_n^{\pm}$ or $d_n^\mp$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$v_G^{n \pm}$ (km/s)</th>
<th>$v_yG^{n \pm}$ (km/s)</th>
<th>$v_zG^{n \pm}$ (km/s)</th>
<th>$v_xG^{n \pm}$ (km/s)</th>
<th>$d_n^\perp$ ($10^{-26}$ gr/cm$^3$)</th>
<th>$d_n^-$ ($10^{-26}$ gr/cm$^3$)</th>
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</table>
Tidal torque theory with CDM

The velocity field remains irrotational

$\vec{v} = 0$
For collisionless particles

\[
\frac{d \vec{v}}{dt}(\vec{r}, t) = \frac{\partial \vec{v}}{\partial t}(\vec{r}, t) + \left( \vec{v}(\vec{r}, t) \cdot \vec{\nabla} \right) \vec{v}(\vec{r}, t) \\
= -\vec{\nabla} \Phi(\vec{r}, t)
\]

If \( \vec{\nabla} \times \vec{v} = 0 \) initially,

then \( \vec{\nabla} \times \vec{v} = 0 \) for ever after.
Tidal torque theory with axion BEC

Net overall rotation is produced because, in the lowest energy state, all axions fall with the same angular momentum

$\vec{\nabla} \times \vec{v} \neq 0$
For axion BEC

\[ E = \sum_{i=1}^{N} \frac{L_i^2}{2I} \]

is minimized for given

\[ L = \sum_{i=1}^{N} L_i \]

when \( L_1 = L_2 = L_3 = \ldots = L_N \).
\( \nabla \times \vec{v} \neq 0 \) is allowed through the appearance of vortices; see discussion by Tanja Rindler-Daller and Paul Shapiro at this meeting.
Summary

- axion BEC and CDM are indistinguishable in the linear regime inside the horizon on all scales of observational interest.

- axion BEC may provide a mechanism for net overall rotation in galactic halos.

- axion BEC may provide a mechanism for the alignment of CMBR multipoles.