CONTRIBUTION OF Λ TO THE BENDING OF LIGHT

(HAS RELEVANCE TO GRAVITATIONAL LENSING IN COSMOLOGY)

W.R. + M. ISHAK, Phys. Rev. D 76 (2007) 043006
M. ISHAK, W.R. et al, arXiv 0710.4726v2 [astro-ph] 6 Dec 2007 MNRAS 388 (2008) 1279
M. ISHAK, W.R., J. DOSSETT, arXiv 2009

WIDELY ACCEPTED "RESULT":

N. J. ISLAM, Phys Lett. A 97 (1983) 239
etc, etc

"Λ DOES NOT AFFECT BENDING"

STRANGE NEGLECT:

EDDINGTON 1923 ("Math. Th. of Rel")
ALREADY HAS CONTRIBUTION
OF Λ TO ADVANCE OF PERIHELION (*)

WHY NOT TO BENDING OF LIGHT?

[(*) 1" per century $\leftrightarrow$ $\Lambda \approx 5 \times 10^{-42}\ cm^{-2}$]

FIELD OF MASS-POINT:

WITHOUT Λ - FIELD EQS $\Rightarrow$

SCHWARZSCHILD: $\boxed{c=1}$ $\boxed{G=1}$

$$\underline{ds}^2 = \left(1-\frac{2m}{r}\right)dt^2 - \left(\ \right)^{-1}dr^2 - r^2\{d\theta^2 + \sin^2\theta\, d\phi^2\}$$

<u>WITH</u> Λ [OR SCHWARZSCHILD-DE SITTER]

KOTTLER:

$$ds^2 = \left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)dt^2 - \left(\ \right)^{-1}dr^2 - r^2\{\ \}$$

STATIC METRICS:

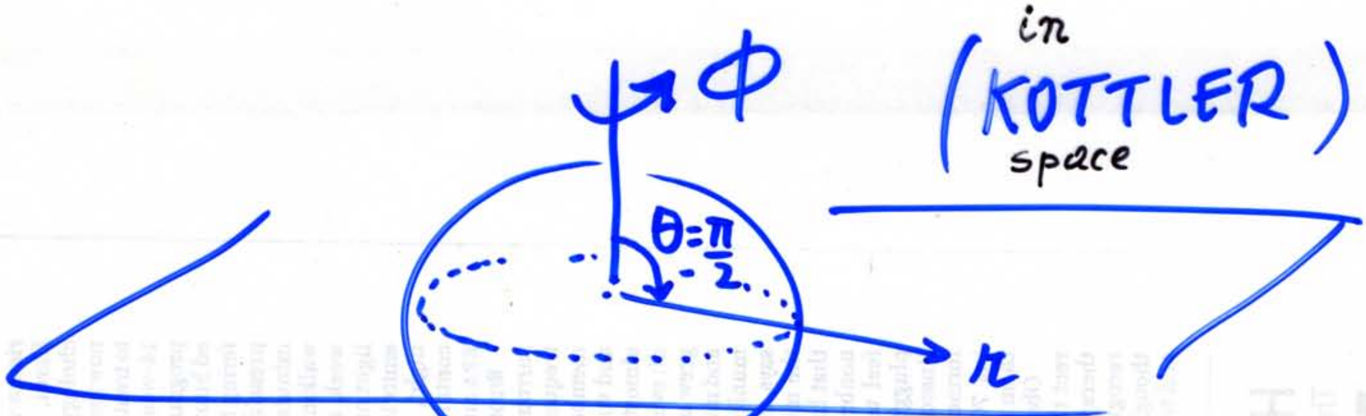
$$\underbrace{\left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)}_{\therefore\ -2\Phi} = e^{2\Phi} \approx 1 + 2\Phi$$

$$\boxed{-\nabla\Phi = -\frac{Gm}{r^2} + \frac{c^2\Lambda}{3}r}$$

RADIAL FORCE FIELD

Expect less bending!

SIMPLE ARGUMENT FOR NO Λ-BENDING:



ORBIT EQ (GEODESIC) IN $\theta = \frac{\pi}{2}$ (EQUATORIAL) "PLANE"

$$\frac{d^2u}{d\phi^2} + u = \frac{m}{h^2} + 3mu^2 - \frac{\Lambda}{3h^2u^3}$$

EXACT!

$$h = r^2 \frac{d\phi}{ds} = \text{const}, \quad u = 1/r$$

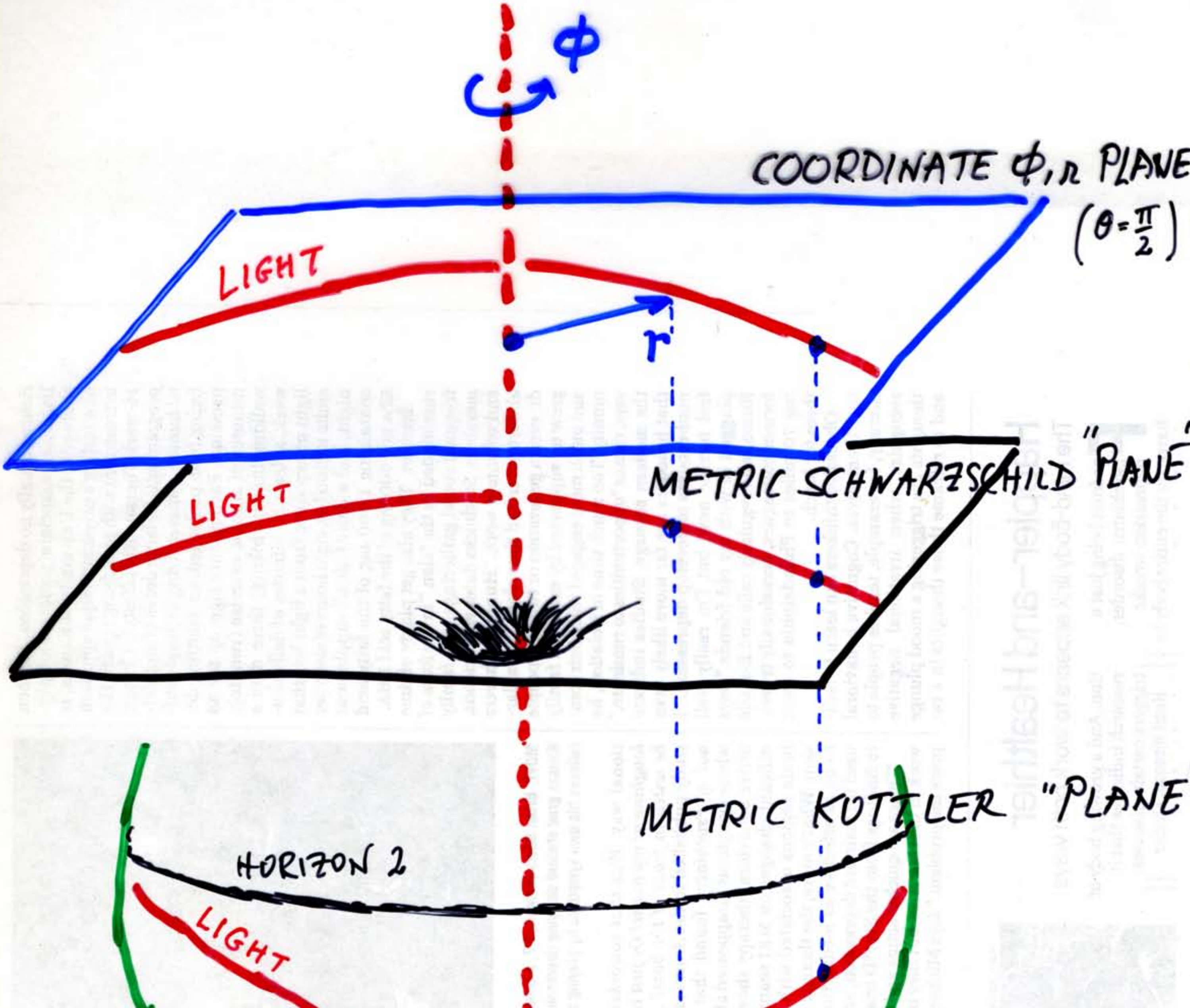
FOR $ds \neq 0$ Λ contributes

FOR $ds = 0$ Λ drops out!

BUT THE ϕ, u EQ. IS NOT EVERYTHING!

ϕ, u (r) ARE MERE COORDINATES

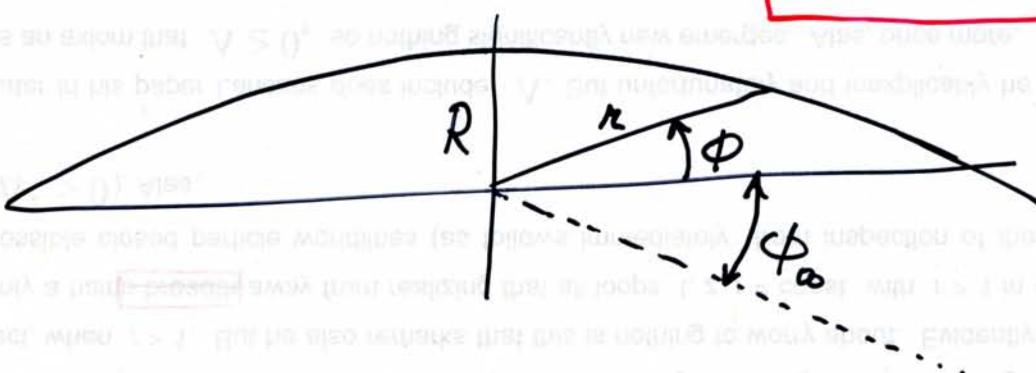
ONLY THE METRIC TELLS US WHAT IS MEASURED!


φ
COORDINATE φ, r PLANE
(θ = π/2)
LIGHT
r
METRIC SCHWARZSCHILD "PLANE"
LIGHT
METRIC KOTTLER "PLANE"
HORIZON 2
LIGHT
HORIZON 1

Usual 1st approximation to solution of photon orbit D.E.:

$$\frac{1}{r} = \frac{\sin\phi}{R} + \frac{3m}{2R^2}\left(1+\frac{1}{3}\cos 2\phi\right)$$

$\boxed{\text{Schw+Ko}}$ (I)



Schwarzschild bending angle:

$$r \to \infty \qquad \phi \to -\phi_\infty \quad (\text{SMALL!})$$

$$(\mathrm{I}): \quad \cancelto{0}{\frac{1}{r}} = \frac{\cancelto{\phi_\infty}{\sin\phi}}{R} + \frac{3m}{2R^2}\left(1+\frac{1}{3}\cancelto{1}{\cos 2\phi}\right)$$

$$\phi_\infty = \frac{3m}{2R}\cdot\frac{4}{3} = \boxed{\frac{2m}{R}}$$

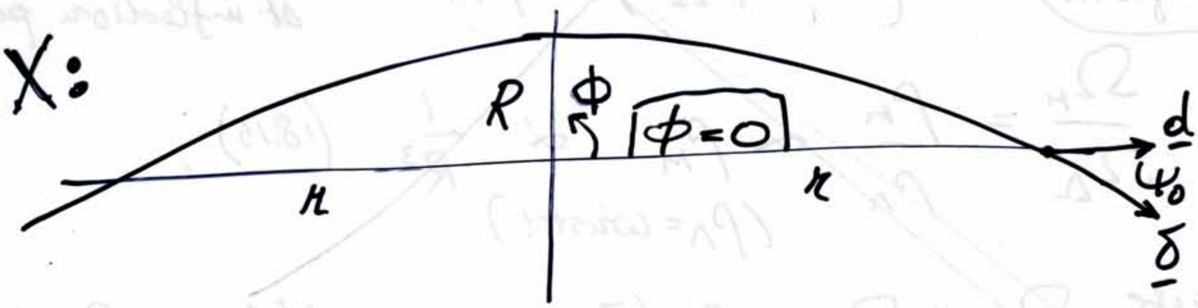
Total bending : $2\phi_\infty = \dfrac{4m}{R}$ (Einstein)

WITH Λ :

SCHWARZSCHILD $\rightarrow$ KOTTLER

cannot let $r \rightarrow \infty$!

Ex:



$\rightarrow$ AS A SIMPLE EXAMPLE, CALCULATE ψ_0

$\rightarrow$ MEASURED ANGLE IN 2 or 3-D RIEM. SPACE:

$$\cos\psi_0 = \frac{\underline{d}\cdot\underline{\delta}}{|\underline{d}|\,|\underline{\delta}|} = \frac{g_{ij}\, d^i \delta^j}{\left(g_{ij}\, d^i d^j\right)^{1/2}\left(g_{ij}\, \delta^i \delta^j\right)^{1/2}} \qquad \text{II} \quad !$$

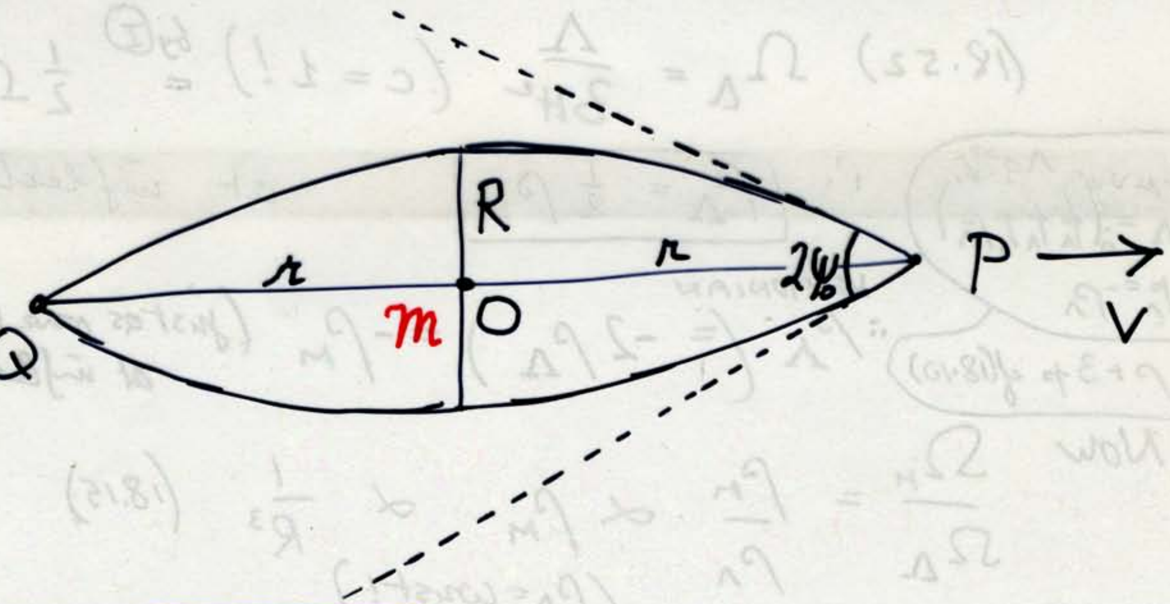
PUT $\boxed{\Phi = 0}$ IN I $\uparrow$ to find $\boxed{r = R^2/2m}$

then (to lowest order) find from II :

$$\boxed{\psi_0 = \frac{2m}{R}\left(1 - \frac{\Lambda R^4}{24 m^2}\right)} \qquad \text{III}$$

EINSTEIN

A SPECIFIC APPLICATION TO LENSING : EINSTEIN RING



$$\boxed{\psi_0 = \frac{2m}{R}\left(1 - \frac{\Lambda R^4}{24 m^2}\right)}$$

m, R <u>same</u> in Schw. and Kottler

→ Hence formulae comparable !

IF LENS IS HUGE CLUSTER OF GALAXIES, <u>COSMOLOGICAL EXPANSION MUST BE TAKEN INTO ACCOUNT</u>

OBSERVER RECEDES, SAY, AT SPEED V

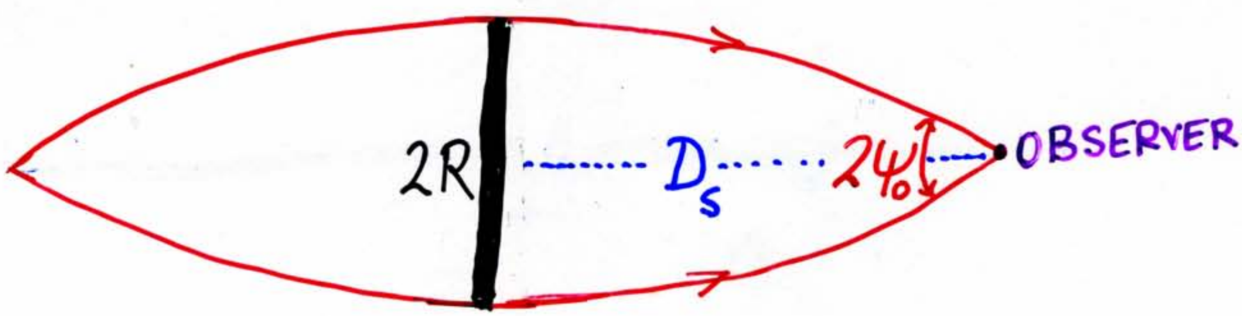
$$2\psi_0 \mapsto 2\psi_0 \sqrt{\frac{1+V}{1-V}}$$

ABERRATION FACTOR

→ **RATIO** of Einstein : Λ-bending <u>unchanged</u>

ANOTHER INTERPRETATION OF ψ_0 :

AFFECTS DISTANCE FROM APPARENT SIZE



$$2\psi_0 D_s = 2R$$

$$D_s = \frac{R}{\psi_0} = \frac{R^2}{2m}\left(1 + \frac{\Lambda R^4}{24\, m^2}\right)$$

PROBABLE BEST HOPE OF DETECTION:

LENSING BY LARGE! CLUSTERS OF GALAXIES

$$M \approx 10^{13} - 10^{14} M_\odot$$

BENDING:

$$\frac{\lambda\text{-term}}{m\text{-term}} \sim \frac{1}{200}$$

$$\frac{\lambda\text{-term}}{2^{nd} \text{ order } m\text{-term}} \sim 10$$

GENERAL CASE

Q: IMPORTANT QUESTION:

HOW FAR TO INTEGRATE TO GET TOTAL BENDING BY A GIVEN SOURCE

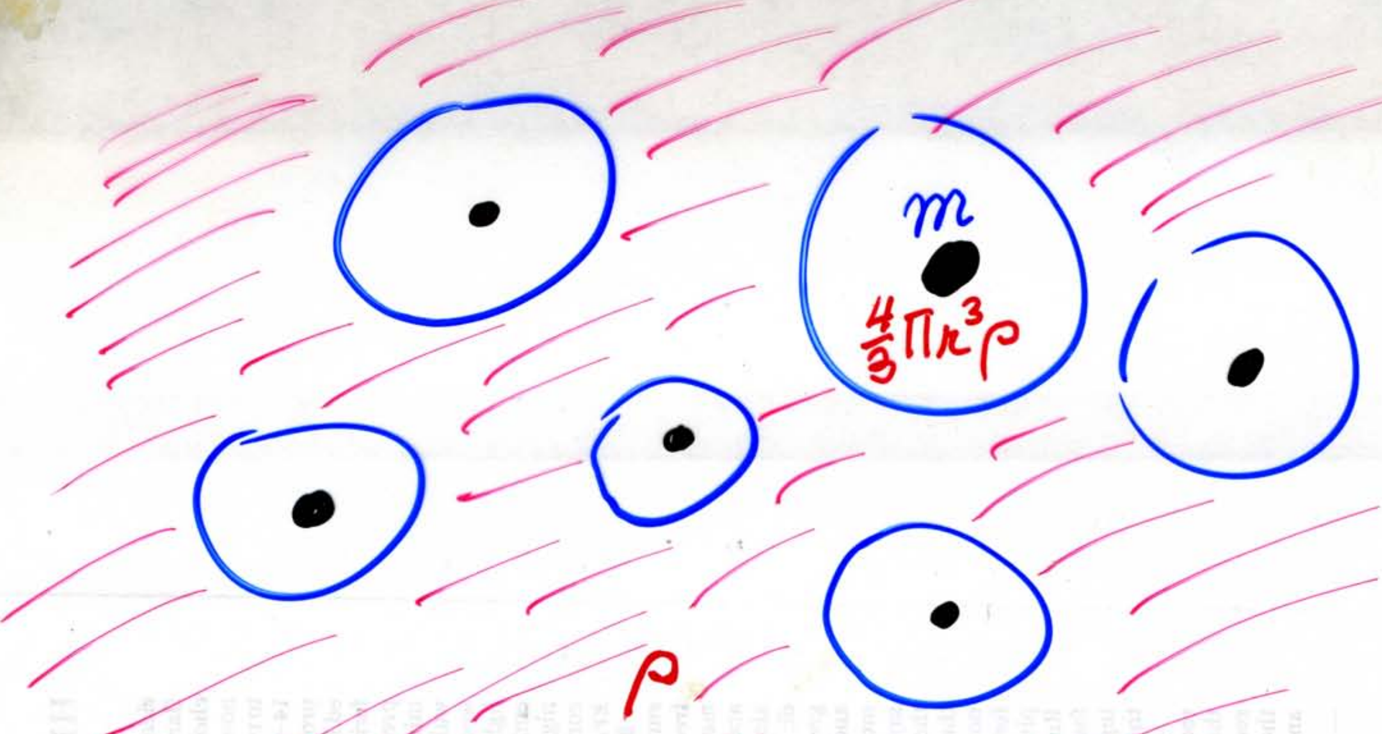
SCHWARZSCHILD:

a) space gets flat
b) m-attraction → 0 } as $r \to \infty$

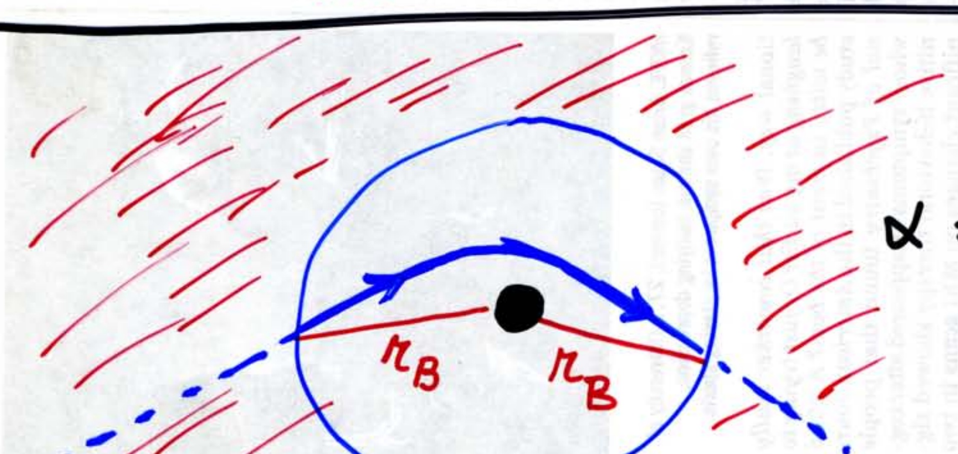
KOTTLER:

a) space never gets flat
b) Λ-repulsion increases } as $r \nearrow$

A: "SWISS-CHEESE" COSMOLOGY



FRIEDMAN UNIVERSE
+ SCHWARZSCHILD (KOTTLER) BUBBLES
(JUNCTION CONDITIONS SATISFIED)



$$\alpha = 2(\psi - \phi)\Big|_{r = r_B}$$

ALL BENDING TAKES PLACE IN BUBBLE
NO FURTHER BENDING OUTSIDE

$$\alpha = \frac{4mG}{Rc^2} - \frac{\Lambda R r_B}{3}$$ (R = radius of lens)