Fast Proton

Dimitri V. Nanopoulos

George P. and Cynthia W. Mitchell Institute for Fundamental Physics at Texas A&M University Houston Advanced Research Center (HARC) Academy of Athens

\mathcal{F} -ast Proton Decay

Dimitri Nanopoulos Texas A&M University

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I. INTRODUCTION

Grand Unification and Proton Decay



Fundamentals of Flipped SU(5)

Flipped SU(5)

In a flipped SU(5) x U(1)_x model, the electric charge generator Q is only partially embedded in SU(5):

$$Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$$

where Y' is the U(1) inside SU(5) and \tilde{Y} is U(1)_X

The photon is shared between SU(5) and $U(1)_X$

Flipped SU(5) Field Content

In a field-theoretic 'flipped' $SU(5) \otimes U(1)$ model the Standard Model states occupy $\overline{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ representations of the $\mathbf{16}$ of SO(10), with the quark and lepton assignments being 'flipped' $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional SU(5) GUT:

$$f_{\tilde{\mathbf{5}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L; \quad F_{\mathbf{10}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L d_L^c \quad \nu_L^c \right); \quad l_{\mathbf{1}} = e_L^c,$$

In particular, this results in the 10 containing a neutral component with the quantum numbers of ν_L^c . Spontaneous GUT symmetry breaking can be achieved by using a 10 and $\bar{10}$ of superheavy Higgs where the neutral components develop a large vacuum expectation value (*vev*), $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle$,

$$H_{10} = \{Q_H, d_H^c, \nu_H^c\} \quad ; \quad H_{\bar{10}} = \{Q_{\bar{H}}, d_{\bar{H}}^c, \nu_{\bar{H}}^c\},$$

while the electroweak spontaneous breaking occurs through the Higgs doublets H_2 and H_2 ,

$$h_5 = \{\mathbf{H}_2, \mathbf{H}_3\}$$
; $h_{\bar{5}} = \{\mathbf{H}_{\bar{2}}, \mathbf{H}_{\bar{3}}\}$.

The presence of a neutral component in the 10 and $\overline{10}$ of Higgs fields provides a very economical doublet-triplet splitting mechanism which gives a large mass to the Higgs triplets $(\mathbf{H}_3, \mathbf{H}_{\bar{3}})$ while keeping Higgs doublets $(\mathbf{H}_2, \mathbf{H}_{\bar{3}})$ light through trilinear superpotential couplings of the form,

$$FFh \rightarrow d_H^c \langle \nu_H^c \rangle H_3$$

 $\bar{F}\bar{F}\bar{h} \rightarrow \bar{d}_H^c \langle \bar{\nu}_H^c \rangle H_{\bar{3}}$

Flipped SU(5) Model Building Characteristics

Flipped SU(5) model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2) -triplet (H_3) splitting problem of the electroweak Higgs pentaplets h, \bar{h} through the trilinear coupling of the Higgs fields: $H_{10} \cdot H_{10} \cdot h_5 \rightarrow \langle \nu_H^c \rangle d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields $\phi, F_{10} \cdot \bar{H}_{\overline{10}} \cdot \phi \rightarrow \langle \nu_{\bar{H}}^c \rangle \nu^c \phi$.

The generic superpotential W for a flipped SU(5) model will be of the form :

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f}l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \dots \in W$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism. Consistency with Low Energy Phenomenology

Rudiments

SU(5): $\alpha_s(M_Z) = \frac{\frac{7}{3} \alpha_{\text{em}}}{5 \sin^2(\theta_w^{\text{eff}}) - 1}$ \mathcal{F} -SU(5): $\alpha_s(M_Z) = \frac{\frac{7}{3} \alpha_{\rm em}}{5 \sin^2(\theta_{\rm W}^{\rm eff}) - 1 + \frac{10}{\pi} \alpha_{\rm em} \ln \left(M_{32}^{\rm max} / M_{32} \right)}$ Particle Data Group (2008) $\alpha_{\rm em}(M_{\rm Z}) = \frac{1}{127.925 \pm 0.016}$; $\alpha_s(M_{\rm Z}) = .1176 \pm .0020$ $\sin^2 \theta_{\rm W}^{\rm \overline{MS}}(M_{\rm Z}) = .23119 \pm .00014$; $M_{\rm Z} = 91.1876 \pm .0021 \,{\rm GeV}$

Standard SU(5) unification predicts a value for the strong coupling at the Z-Boson mass which is much too large. Attempts to fix this with heavy thresholds only speed dimension 5 proton decay. We can lower α_s by Flipping SU(5).

Renormalization Group Equations With Thresholds

$$\frac{3}{5} \frac{(\Xi_{\rm Y} - \Theta_{\rm W})}{\alpha_{\rm em}} - \frac{1}{\alpha_1} = \frac{b_{\rm Y}}{2\pi} \ln \frac{M_{32}}{M_{\rm Z}}$$
$$\frac{\Theta_{\rm W}}{\alpha_{\rm em}} - \frac{1}{\alpha_5} = \frac{b_2}{2\pi} \ln \frac{M_{32}}{M_{\rm Z}}$$
$$\frac{\Xi_3}{\alpha_3} - \frac{1}{\alpha_5} = \frac{b_3}{2\pi} \ln \frac{M_{32}}{M_{\rm Z}}$$

Threshold and Second Loop Corrections are absorbed into an Effective Sine-Squared Weinberg angle Plus a corresponding shift term for the Hypercharge and Strong Coupling. The Second Loop

$$\frac{d\alpha_i}{dt} = \frac{b_i \alpha_i^2}{2\pi} + \frac{\alpha_i^2}{8\pi^2} \left[\sum_{j=1}^3 B_{ij} \alpha_j - \sum_{f=u,d,e} d_i^f \operatorname{Tr} (\lambda_f) \right]$$
$$\frac{d\lambda_u}{dt} = \frac{\lambda_u}{2\pi} \left[3\lambda_u + \lambda_d + 3\operatorname{Tr} (\lambda_u) - \sum_{i=1}^3 c_i^u \alpha_i \right]$$
$$\frac{d\lambda_d}{dt} = \frac{\lambda_d}{2\pi} \left[\lambda_u + 3\lambda_d + \operatorname{Tr} (3\lambda_d + \lambda_e) - \sum_{i=1}^3 c_i^d \alpha_i \right]$$
$$\frac{d\lambda_e}{dt} = \frac{\lambda_e}{2\pi} \left[3\lambda_e + \operatorname{Tr} (3\lambda_d + \lambda_e) - \sum_{i=1}^3 c_i^e \alpha_i \right]$$

A fresh numerical evaluation of the Second Loop has been performed for each scenario under consideration, including each distinct selection of $Tan(\beta)$ and the MSSM mass spectrum.

The Standard SU(5) Limit

$$M_{32}^{\max} = M_{\rm Z} \times \exp\left\{\frac{2\pi}{\alpha_{\rm em}\alpha_3} \left(\frac{3\,\alpha_3\,\Xi_{\rm Y} - 8\,\alpha_{\rm em}\,\Xi_3}{5b_{\rm Y} + 3b_2 - 8b_3}\right)\right\}$$

$$\alpha_5^{\max} \equiv \alpha_5(M_{32}^{\max}) = \left[\frac{\alpha_{\rm em}\Xi_3(5b_{\rm Y}+3b_2) - 3\alpha_3\Xi_{\rm Y}b_3}{\alpha_{\rm em}\alpha_3(5b_{\rm Y}+3b_2-8b_3)}\right]^{-1}$$

$$\Theta_{\rm W}^{\rm max} \equiv \Theta_{\rm W} \left(M_{32}^{\rm max} \right)$$
$$= \frac{5\alpha_{\rm em} \Xi_3 (b_{\rm Y} - b_2) + 3\alpha_3 \Xi_{\rm Y} (b_2 - b_3)}{\alpha_3 (5b_{\rm Y} + 3b_2 - 8b_3)}$$

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The "max" limit corresponds to a strict triple unification of the SM couplings. It is essential to recognize that the "max" effective Weinberg angle is a dependent variable. Failure to match the expected value signals a failure of unification itself.

Flipped SU(5) Solutions



It is critically important to select a properly orthogonalized set of dependent functions. For Flipped SU(5) it is convenient to choose the SU(5) and U(1)_X couplings, and either the unification scale M_{32} OR the effective Weinberg angle.

II. \mathcal{F} -Theory Model Building

*F***-Theory Model Building**

- The twelve-dimensional F-theory with seven-branes can be considered as the strongly coupled formulation of ten-dimensional Type IIB string theory with a varying axion (a)-dilaton (φ) field τ = a + ie^{-φ}.
- We compactify F-theory on a Calabi-Yau fourfold, which is elliptically fibered π : Y₄ → B₃ with a section σ : B₃ → Y₄. The base B₃ is the internal space dimensions in Type IIB string theory, and the complex structure of the T² fibre encodes τ at each point of B₃.
- The singularity types of the ellitically fibres fall into the familiar *ADE* classifications, and we identify the corresponding *ADE* gauge groups on the seven-brane world-volume.

GUT Models:

- The observable seven-branes with GUT models on its worldvolume wrap a complex codimension-one surface S in B₃.
- When h^{1,0}(S) ≠ 0, the low energy spectrum may contain the extra states obtained by reduction of the bulk supergravity modes of compactification. So we require that π₁(S) be a finite group.
- In order to decouple gravity and construct models locally, the extension of the local metric on S to a local Calabi-Yau fourfold must have a limit where the surface S can be shrunk to zero size. This implies that the anti-canonical bundle on S must be ample.
- The Hirzebruch surfaces with degree larger than 2 satisfy $h^{2,0}(S) = 0$ but do not define the fully consistent decoupled models.

S must be a del Pezzo n surface dP_n with $n \ge 2$ in which $h^{2,0}(S) = 0$.

GUT Models:

- The SU(5) and SO(10) gauge symmetries can be broken down to the SM and SU(5) × U(1) gauge symmetries, respectively by turning on U(1) fluxes.
- The SM fermions and Higgs can be obtained from the intersections between the observable seven-branes and the other seven-branes, where the singularity is enhanced.
- The Yukawa couplings can be realized at the triple intersection of three curves (two SM fermion curves and one Higgs curve), where the singularity is enhanced further.
- Unlike the perturbative D-brane model building, the exceptional gauge groups appear rather naturally at the triple intersections, and then all the SM fermion Yuakwa couplings can be generated.

Flipped $SU(5) \times U(1)_X$ Models:

- SO(10) models may be more interesting than SU(5) models since they have not only gauge interaction unification but also fermion unification.
- In SO(10) models, to eliminate the zero modes of the chiral exotic particles, we must break the SO(10) gauge symmetry down to the flipped SU(5) × U(1)_X gauge symmetry.
- Interestingly, in flipped $SU(5) \times U(1)_X$ models, we can solve the doublet-triplet splitting problem via the missing partner mechanism.
- In flipped SU(5) × U(1)_X models with SO(10) origin, there are two unification scales: the SU(2)_L × SU(3)_C unification scale M₂₃ and the SU(5) × U(1)_X unfication scale M_U.

Flipped $SU(5) \times U(1)_X$ Models:

- To separate the mass scales M₂₃ and M_U and realize the decoupling scenario, we introduce sets of vector-like particles in complete SU(5) × U(1)_X multiplets, whose contributions to the one-loop beta functions of the U(1)_Y, SU(2)_L and SU(3)_C gauge symmetries, Δb₁, Δb₂ and Δb₃ respectively, satisfy Δb₁ < Δb₂ = Δb₃.
- To avoid the Landau pole problem for the gauge couplings, we can only introduce the following two sets of vector-like particles around the TeV scale, which could be observed at the LHC

 $Z1 : XF = (\mathbf{10}, \mathbf{1}), \ \overline{XF} = (\overline{\mathbf{10}}, -\mathbf{1});$ $Z2 : XF, \ \overline{XF}, \ Xl = (\mathbf{1}, -\mathbf{5}), \ \overline{Xl} = (\mathbf{1}, \mathbf{5}).$

• We define the flipped $SU(5) \times U(1)_X$ models with Z1 and Z2 sets of vector-like particles as Type I and Type II models, respectively.

Flipped Unification with/without Vector Multiplets



Inclusion of TeV scale Vector Multiplets levels out the renormalization of the strong coupling, driving up the SU(5) coupling, and speeding proton decay.

Super Unification with Vector Multiplets



Continuing the two-loop renormalization group running beyond the 3-2 partial unification results in a super unification located near the reduced Planck scale.

III. \mathcal{F} -AST PROTON DECAY

Proton decay experiments:

- Super-Kamiokande, a 50-kiloton (kt) water Cherenkov detector, has set the current lower bounds of 8.2 × 10³³ and 6.6 × 10³³ years at the 90% confidence level for the partial lifetimes in the p → e⁺π⁰ and p → μ⁺π⁰ modes.
- Hyper-Kamiokande is a proposed 1-Megaton detector, about 20 times larger volumetrically than Super-Kamiokande, which we can expect to explore partial lifetimes up to a level near 2 × 10³⁵ years for p → e⁺π⁰ across a decade long run.
- The proposal for the DUSEL experiment features both 500 kt water Cherenkov and 100 kt liquid Argon detectors, with the stated goal of probing partial lifetimes into the order of 10³⁵ years for both the p→ e⁺π⁰ and p→ K⁺ν_µ channels.

Proton Decay Experiment Vs. Theory



Objective is to improve by at least 1 order of magnitude. Desirable to probe both $e^+\pi^0$ and $K^+\nu$ (plus many many other modes)

Detection Prospects for the $e^+\pi^0$ Mode



Detection Prospects for the K^+v Mode



Proton decay in flipped $SU(5) \times U(1)$:

• After integrating out the heavy gauge boson fields, we obtain the effective dimension six operator for proton decay

$$\mathcal{L} = \frac{g_{23}^2 \epsilon^{ijk}}{2M_{32}^2} \left[\left((\bar{d}_k^c \cos \theta_c + \bar{s}_k^c \sin \theta_c) \gamma^\mu P_L u_j \right) \times (u_i \gamma_\mu P_L e_L) + h.c. \right] \,.$$

- The decay amplitude is proportional to the overall normalization of the proton wave function at the origin. Relevant matrix elements have been calculated in a lattice approach with quoted errors below 10%, corresponding to an uncertainty of less than 20% in the proton partial lifetime. Thus, this uncertainty is negligible.
- Proton lifetime is sensitive to M_{23} and g_{23}

$$\tau(p \to e^+ \pi^0) \sim \left(\frac{M_{23}}{g_{23}}\right)^4$$

Proton decay:

- We consider two-loop RGE running for gauge couplings and one-loop RGE running for top and bottom Yukawa couplings.
- For the light M_Z -scale threshold corrections from the supersymmetric particles, we consider the CMSSM benchmark scenarios, which respect all the available experimental constraints.
- For the M_{23} scale threshold corrections from the triplet Higgs fields and heavy gauge fields of SU(5), from naturalness we assume

$$\frac{\sqrt{\lambda_1 \lambda_2}}{3} \le g_{23} \le 3\sqrt{\lambda_1 \lambda_2} \ .$$

Because XF and XF form complete SU(5) × U(1)_X multiplets and Xl and Xl are SU(5) singlets, we can assume degeneracy of these vector-like particles' masses at a central value of 1 TeV.

Model	g_1	g_{23}	M_{23} (GeV)	τ_p (Years)
Minimal	0.70	0.72	5.8×10^{15}	4.3×10^{34}
Type I	0.75	1.21	6.8×10^{15}	1.0×10^{34}
Type II	0.87	1.20	6.8×10^{15}	1.0×10^{34}

Table 1: Proton decay for benchmark scenario B'.

The central prediction of the proton partial lifetime for the minimal, Type I and Type II models is well below 10^{35} years, within the reach of the future Hyper-Kamiokande and DUSEL experiments. However, the uncertainty from heavy threshold corrections ever threatens to undo this promising result.

Proton Lifetime Within the $M_{1/2}$ - M_0 Plane



Proton Lifetime in the Flipped SU(5) $e^+\pi^0$ channel is depicted against the background of phenomenological constraints on the MSSM. Background figure courtesy of Keith Olive.

The $M_{1/2}$ - M_0 Plane with Vector Multiplets



Proton Lifetime in the Flipped SU(5) $e^+\pi^0$ channel, including TeV vector multiplets, is depicted against phenomenological constraints on the MSSM. Background figure courtesy of Keith Olive.

Variation of Vector Multiplet Scale



Proton lifetime (solid blue) is measured in 10^{35} [Y] on the left numeric scale. The dimensionless SU(5) coupling (dash red) is also measured on the left-hand scale. The unification mass (dot green) is measured in 10^{16} [GeV] on the right-hand scale.

Variation of $Tan(\beta)$



Proton lifetime (solid blue) is measured in 10^{35} [Y] on the left numeric scale. The dimensionless SU(5) coupling (dash red) is also measured on the left-hand scale. The unification mass (dot green) is measured in 10^{16} [GeV] on the right-hand scale.

Heavy Thresholds

$$\begin{cases} \frac{2\pi}{\alpha_0} - \frac{2\pi}{\alpha_N} \} + \sum_{\text{light}} \delta b \ln \left(\frac{M}{M_0}\right) - \sum_{\text{heavy}} \delta b \ln \left(\frac{M_N}{M}\right) = b_{\text{light}} \ln \frac{M_N}{M_0} \\ \hline \\ & \\ \hline \\ \hline \\ M_{H_3} & M_{\Sigma} & M_V & M_{H_3} & M_{\overline{H}_3} & M_V \\ \hline \\ \hline \\ \delta b_Y & \frac{2}{5} & 0 & -10 & \frac{2}{5} & \frac{2}{5} & -10 \\ \hline \\ \delta b_2 & 0 & 2 & -6 & 0 & 0 & -6 \\ \hline \\ \delta b_3 & 1 & 3 & -4 & 1 & 1 & -4 \end{cases}$$

Light thresholds are treated additively, heavy thresholds subtractively. The heavy thresholds are compared to the upper mass scale rather than the lower. Heavy fields are not included in the main β -function coefficients.

Error Analysis and Uncertainty From Heavy Thresholds



Central values for proton lifetime at the MSSM Benchmarks lie on the white boundary. The dark red / blue bands show uncertainty of low energy measurements. The large light blue error bars depict plausible variation of the heavy thresholds.

Heavy Thresholds with Vector Multiplets



Central values for proton lifetime at the MSSM Benchmark Points lie on the white boundary. The dark red / blue bands show uncertainty of low energy measurements. The large light blue bars depict plausible variation of the heavy thresholds.

Reasons for the Decline in Proton Life Expectancy



Comparisons are relative to the 2002 report on Flipped SU(5). [Ellis, Nanopoulos, Walker]

Summary on Proton Decay:

• In the minimal model, the central partial lifetime is in the range of $4 - 7 \times 10^{34}$ years for benchmark scenarios from A' to I', and about $1 - 2 \times 10^{35}$ years for benchmark scenarios J' and K'. However, the uncertainties from the heavy threshold corrections at M_{23} are indeed quite large. Proton decay appears to be within the reach of the future Hyper-Kamiokande and DUSEL experiments if the heavy threshold corrections are more modest.

For Type II flipped SU(5) × U(1)_X model, the central values for the partial lifetime are about 1 – 2 × 10³⁴ years for benchmark scenarios from A' to I', and about 2 – 3 × 10³⁴ years for benchmark scenarios J' and K'. Even including uncertainties from the light and heavy threshold corrections, the lifetime is still less than 2 – 3 × 10³⁵ years for all scenarios considered. A strong majority of the parameter space for proton decay does indeed appear to be within the reach of the future Hyper-Kamiokande and DUSEL experiments for the Type II flipped SU(5) × U(1)_X model. This basic conclusion holds also for the Type I flipped SU(5) × U(1)_X model.



The 2009 central predictions for proton decay narrowly evades current detection bounds from Super-Kamiokande. Even with the possibility of substantial heavy thresholds, the majority of the predicted range is testable in the next generation.

CONCLUSION

Testable flipped $SU(5) \times U(1)_X$ **Models**

- These models can be realized in free fermionic string constructions and F-theory model building.
- The proton decay p → e⁺π⁰ from the heavy gauge boson exchange is within the reach of the future Hyper-Kamiokande and DUSEL experiments for a majority of the most plausible parameter space.
- Because the TeV-scale vector-like particles can be produced at the LHC, we predict a strong correlation between the most exciting particle physics experiments of the coming decade.
- The minimal flipped $SU(5) \times U(1)_X$ model is also testable if the heavy threshold corrections are small.

A Convergence of Large Experiments

Hyper-Kamiokande and DUSEL expect Sensitivity up to 10³⁵ [Y]

The Large Hadron Collider is Uniquely Poised to Detect Novel TeV Scale Fields

Proton decay, hastened by the inclusion of TeV scale Vector Multiplets, represents an imminently testable convergence of the most exciting particle physics experiments of the next decade.

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