The growth rate index of large scale structure as a probe of the cause of cosmic acceleration

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Introduction and motivation

- Cosmic acceleration can be caused by a dark energy component in the universe
- Or a modification to gravity physics at cosmological scales
- An important question is to distinguish between the two possibilities
- The growth rate of large scale structure can be used to distinguish between the two competing alternatives
- Two methods have been proposed in literature so far:
  - Looking for inconsistencies in the dark energy parameter spaces
  - Constraining the growth index parameter (this talk)

Growth rate of large scale structure

- large scale matter density perturbation, $\delta = \delta\rho_m / \rho_m$, satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate $f = d\ln\delta / d\ln a$ as:

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

where the underlying gravity theory is expressed via the expression for G_{eff} .

A constant growth rate index parameter

- The growth function f *can be* approximated using the ansatz [Peebles, 1980; Fry, 1985; Lightman & Schechter, 1990]

$$f = \Omega_m^{\gamma}$$

where γ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad\qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.

- Probably, other anzats will be needed as we go to much higher redshifts

Growth rate index parameter and [General Relativity + Dark Energy] models

- [L. Wang and Steinhardt, 1998] considered Dark Energy models with slowly varying *w* and derived

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1-\Omega_m) \qquad f = \Omega_m^\gamma$$

- with the asymptotic early value $\gamma_\infty = \dfrac{3(1-w)}{(5-6w)}$

- This has been recently discussed by other authors [see for example Linder and Cahn, 2007; Mortonson, Hu, Huterer, 2009; Zhang et al. 2007; Gong, 2008; Polarski and Ganouji, 2008, Gong, Ishak, A. Wang 2009 …]

- The approximation provides a fit of about 1% to the growth function f as numerically integrated from the ODE

The growth index parameter as a discriminator for Gravity Theories

- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories

- Can be used to probe the underlying gravity theory and the cause of cosmic acceleration

- For example $\gamma=6/11=0.545$ for the Lambda-Cold-Dark-Matter model. (i.e. for $w=-1$)

- and $\gamma=11/16=0.687$ for the flat DGP model [e.g. Linder and Cahn, 2007; Gong 2008].

- As we will see, the dispersion of the values taken by γ for various values of w remains much smaller then the difference between two γ for two different theories

A redshift dependent parametrization for the growth rate index

- As seen from the previous expressions, while the asymptotic early value is a good approximation, γ does vary at late times.

- [Polarski and Ganouji, PLB 2008] proposed for small redshifts (z << 1) the following dependence.

$$\gamma(z) = \gamma_0 + \gamma' z \qquad \gamma' \equiv (d\gamma / dz)$$

- But we already have growth data for f from galaxy redshift distortions that go to much higher redshifts

An interpolated redshift parameterization for the growth rate index [Ishak and Dossett, PRD 2009]

- A parameterization that interpolates between a small redshift expression and a constant value at intermediate-higher redshifts:

$$\gamma(a) = \tilde{\gamma}(a)\,\frac{1}{1+(a_{ttc}/a)} + \gamma_{early}\,\frac{1}{1+(a/a_{ttc})}$$

or

$$\gamma(z) = \tilde{\gamma}(z)\,\frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_{early}\,\frac{1}{1+\frac{1+z_{ttc}}{1+z}}$$

where z_{ttc} is a transition resdhift from an early-time, almost constant value, to the following redshift dependent form

$$\gamma(a)_{late} = \tilde{\gamma}(a) = \gamma_0 + (1-a)\gamma_a$$

or

$$\gamma(z)_{late} = \tilde{\gamma}(z) = \gamma_0 + \left(\frac{z}{1+z}\right)\gamma_a$$

For the curved dark energy model with constant equation of state w, we have

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Omega_k - \frac{3}{2}[1 + w(1 - \Omega_m - \Omega_k)]. \tag{3}$$

The energy conservation equation tells us that

$$\Omega_m' = 3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2), we get

$$[3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k]\frac{df}{d\Omega_m} + f^2 + \left[\frac{1}{2} + \frac{1}{2}\Omega_k - \frac{3}{2}w(1 - \Omega_m - \Omega_k)\right] f = \frac{3}{2}\Omega_m. \tag{5}$$

Plugging $f = \Omega_m^\gamma$ into Eq. (5), we get

$$[3w(1-\Omega_m-\Omega_k)-\Omega_k]\Omega_m \ln\Omega_m \frac{d\gamma}{d\Omega_m} + \left(\gamma - \frac{1}{2}\right)[3w(1-\Omega_m-\Omega_k)-\Omega_k] + \Omega_m^\gamma - \frac{3}{2}\Omega_m^{1-\gamma} + \frac{1}{2} = 0. \tag{6}$$

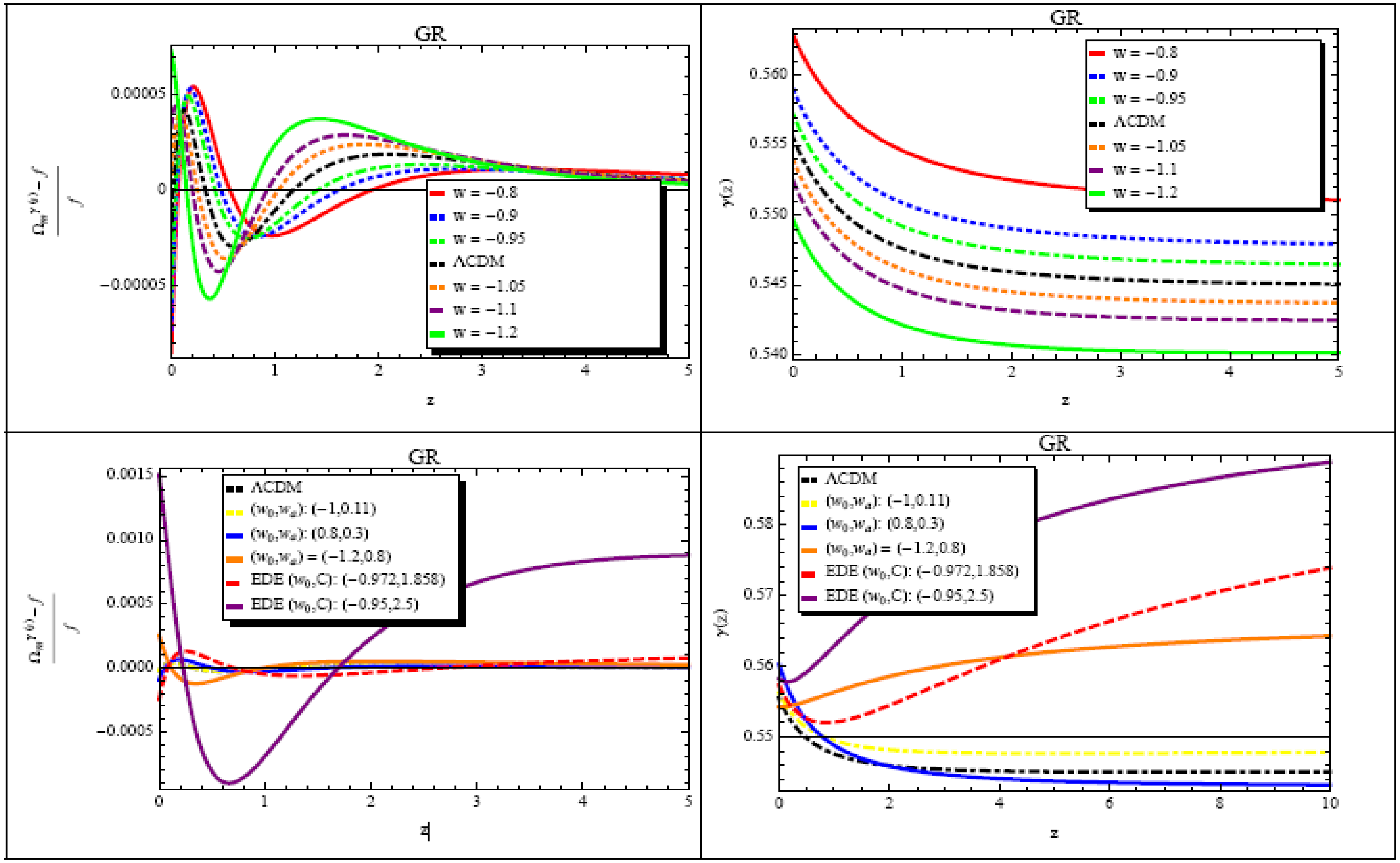
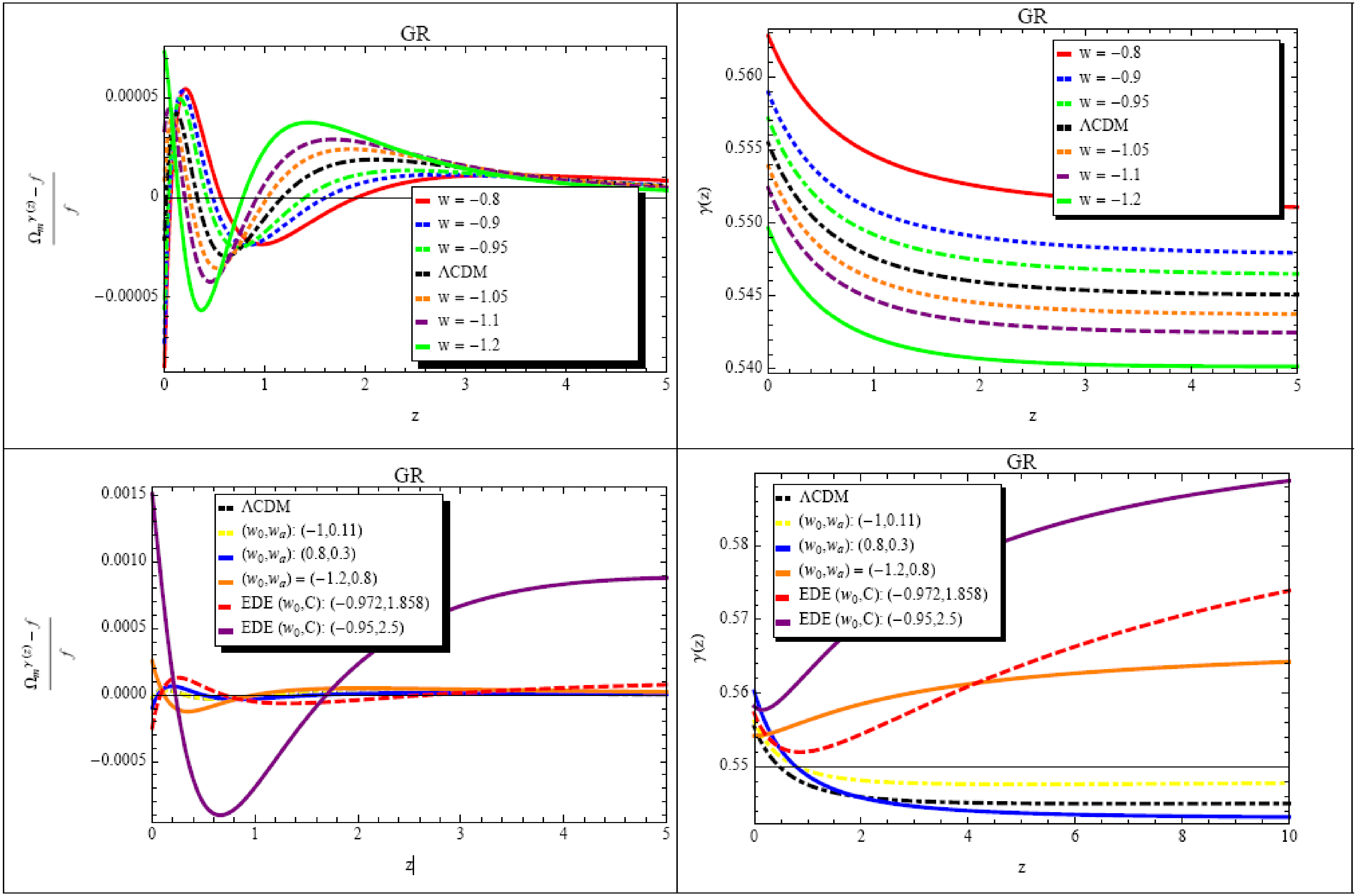


FIG. 1: GR - Dark Energy Models. TOP LEFT: We consider the QCDM models with a constant equation of state and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate, f, that is numerically integrated from the growth ODE. For the ΛCDM, we find the best fit parameters $\gamma_0 = 0.5655$ and $\gamma_a = -0.02710$ when $\gamma_\infty^{\Lambda CDM} = 6/11$. The fit approximate the growth function f to better than 0.004% while the best fit constant $\gamma_{const}^{\Lambda CDM} = 0.5509$ approximates the growth to 0.6%. Using our redshift dependent parameterizations of growth index provides an improvement to the fit of the growth of about a factor 150. TOP RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z) \frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty \frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various values of the constant equation of state w showing very little dispersion of the order of 0.015 at any given redshift. BOTTOM LEFT:We consider the QCDM models with a variable equation of state, as well as some Early Dark Energy models and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate, f, that is numerically integrated from the growth ODE. We find using our redshift dependent parameterizations of the growth index are able to approximate the growth to within 0.15%. BOTTOM RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z) \frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty \frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various dark dnergy models with a varying equation of state $w(a)$ including some early dark energy models.


GR
w = −0.8
w = −0.9
w = −0.95
ΛCDM
w = −1.05
w = −1.1
w = −1.2
z
γ(z)
(w0,wa): (−1,0.11)
(w0,wa): (0.8,0.3)
(w0,wa) = (−1.2,0.8)
EDE (w0,C): (−0.972,1.858)
EDE (w0,C): (−0.95,2.5)

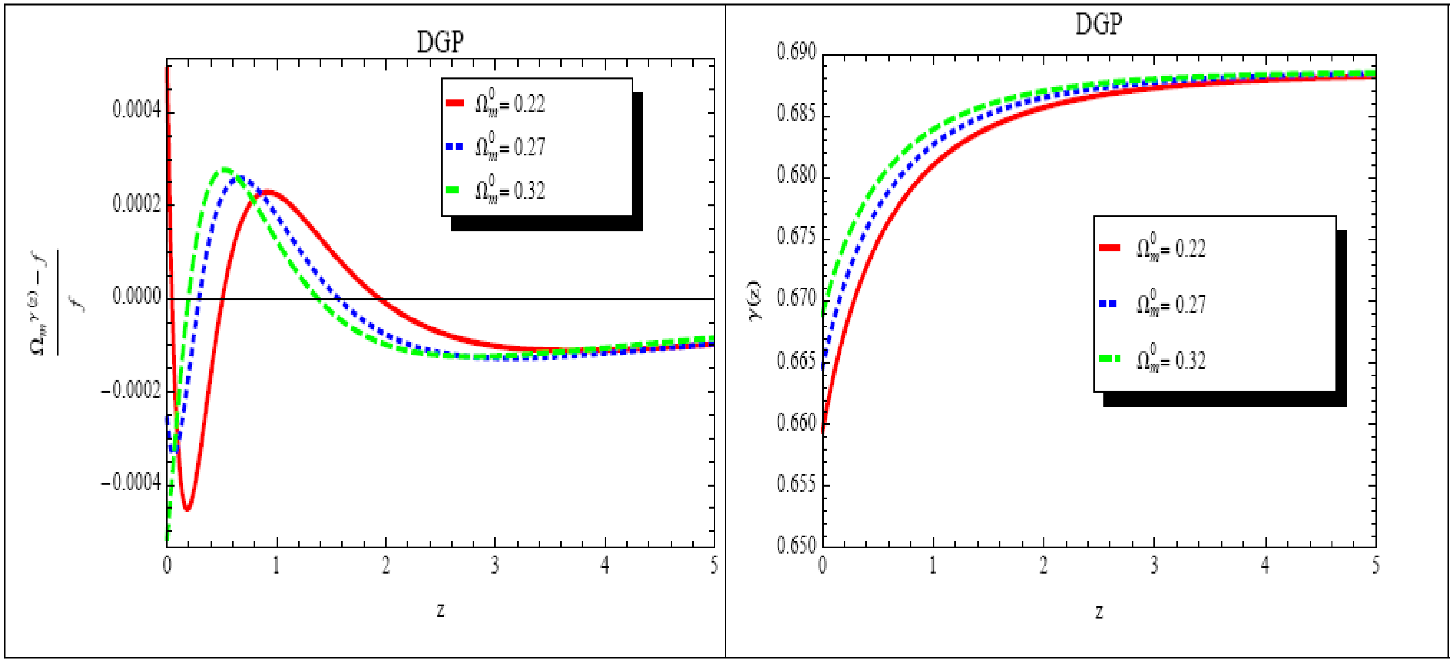
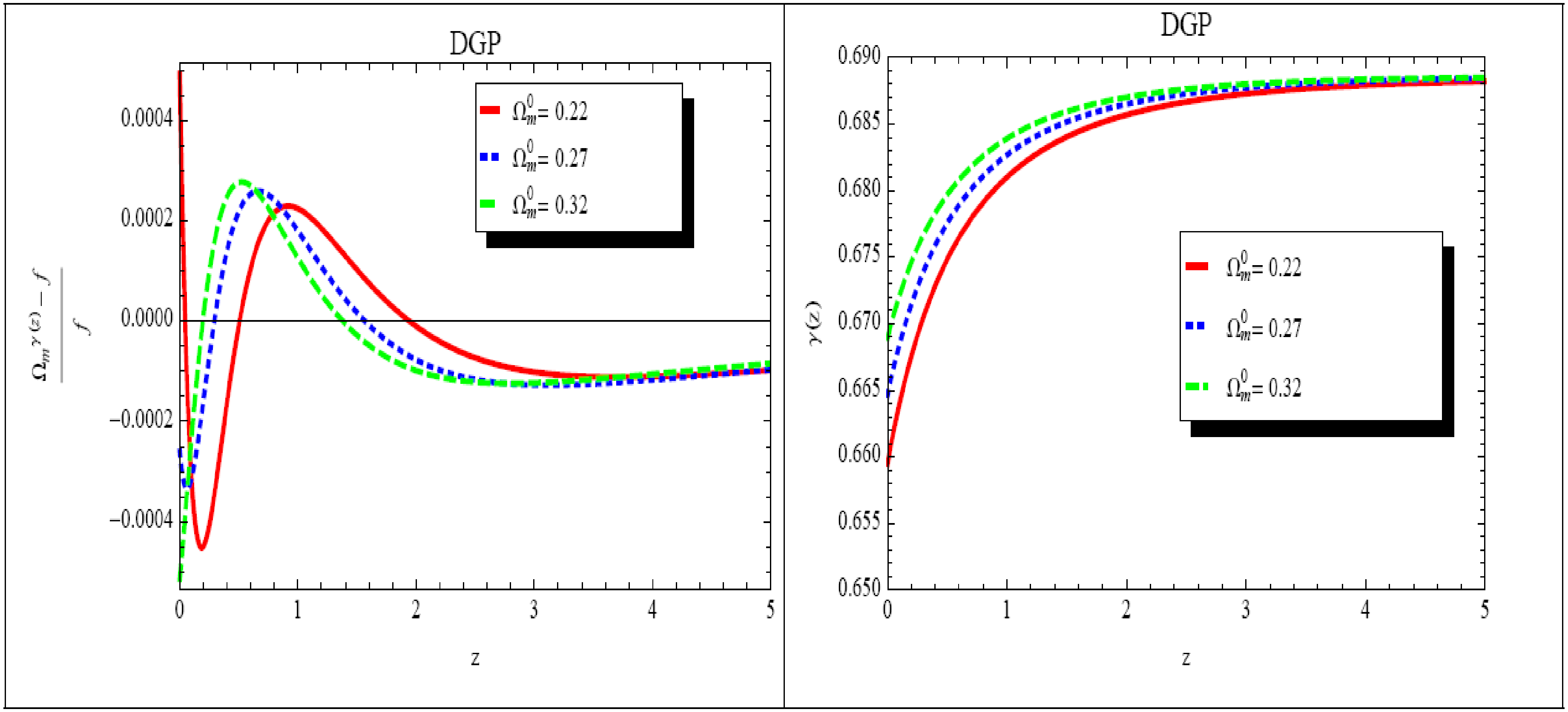


FIG. 2: DGP models. LEFT: We consider the DGP model and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate f_{num} that is numerically integrated from the growth ODE. We find the best fit parameters $\gamma_0 = 0.6418$ and $\gamma_a = 0.06261$ for $\Omega_m^0 = 0.27$ when $\gamma_\infty^{DGP} = 11/16$. The fit approximates the growth function f to better than 0.04% while the best fit constant $\gamma_{const}^{DGP} = 0.6795$ approximates the growth to 1.95%. So using our redshift dependent parameterization of the growth index provides an improvement to the fit of about a factor 50 for the DGP model. RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z)\ \frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty^{DGP}\ \frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various values of Ω_m^0 showing very little dispersion of the order 0.01 or less at any redshift.


DGP
0.0004
0.0002
0.0000
−0.0002
−0.0004
0
1
2
3
4
5
z
$\frac{\Omega_m^{\gamma(z)} - f}{f}$
$\Omega_m^0 = 0.22$
$\Omega_m^0 = 0.27$
$\Omega_m^0 = 0.32$
DGP
0.690
0.685
0.680
0.675
0.670
0.665
0.660
0.655
0.650
$\gamma(z)$
z

Parameters for various QCDM models.		
$(\mathrm{w_0}, \mathrm{w_a})$	γ_0	γ_a
$(-0.8, 0)$	0.5690	-0.02131
$(-0.9, 0)$	0.5683	-0.022525
$(-0.95, 0)$	0.5676	-0.02699
$(-1, 0)$	0.5655	-0.02718
$(-1.05, 0)$	0.5635	-0.02735
$(-1.1, 0)$	0.5617	-0.02749
$(-1.2, 0)$	0.5583	-0.02771
$(-1, 0.11)$	0.5641	-0.02464
$(-0.8, -0.3)$	0.5720	-0.03074
$(-1.2, 0.8)$	0.5409	-0.01417
Parameters for some EDE models.		
$(\mathrm{w_0}, \mathrm{C})$	γ_0	γ_a
$(-0.972, 1.858)$	0.5498	-0.02915
$(-0.95, 2.5)$	0.5165	-0.05578
Parameters for various DGP models.		
Ω_m^0	γ_0	γ_a
0.22	0.6314	0.07324
0.27	0.6418	0.06261
0.32	0.6504	0.05279

TABLE I: We list the parameter values for in our interpolation parameterization for various QCDM, EDE, and DGP models. These values were found by fitting our parameterization to the numerically integrated solution of ODE for the growth function, f (e.g. we use for $\gamma(z)$, Eqs.(18) with(9) for dark energy models, and Eqs. (25) with (9) for DGP models). We see that the QCDM and EDE models have a negative values for the parameter γ_a, while the DGP models have a positive value for γ_a, thus providing parameter that observational data can constrain to distinguish between the two gravity theories, additionally γ_0 takes on distinct values for each theory.

Introducing spatial curvature
[Gong, Ishak, Wang, PRD 2008]

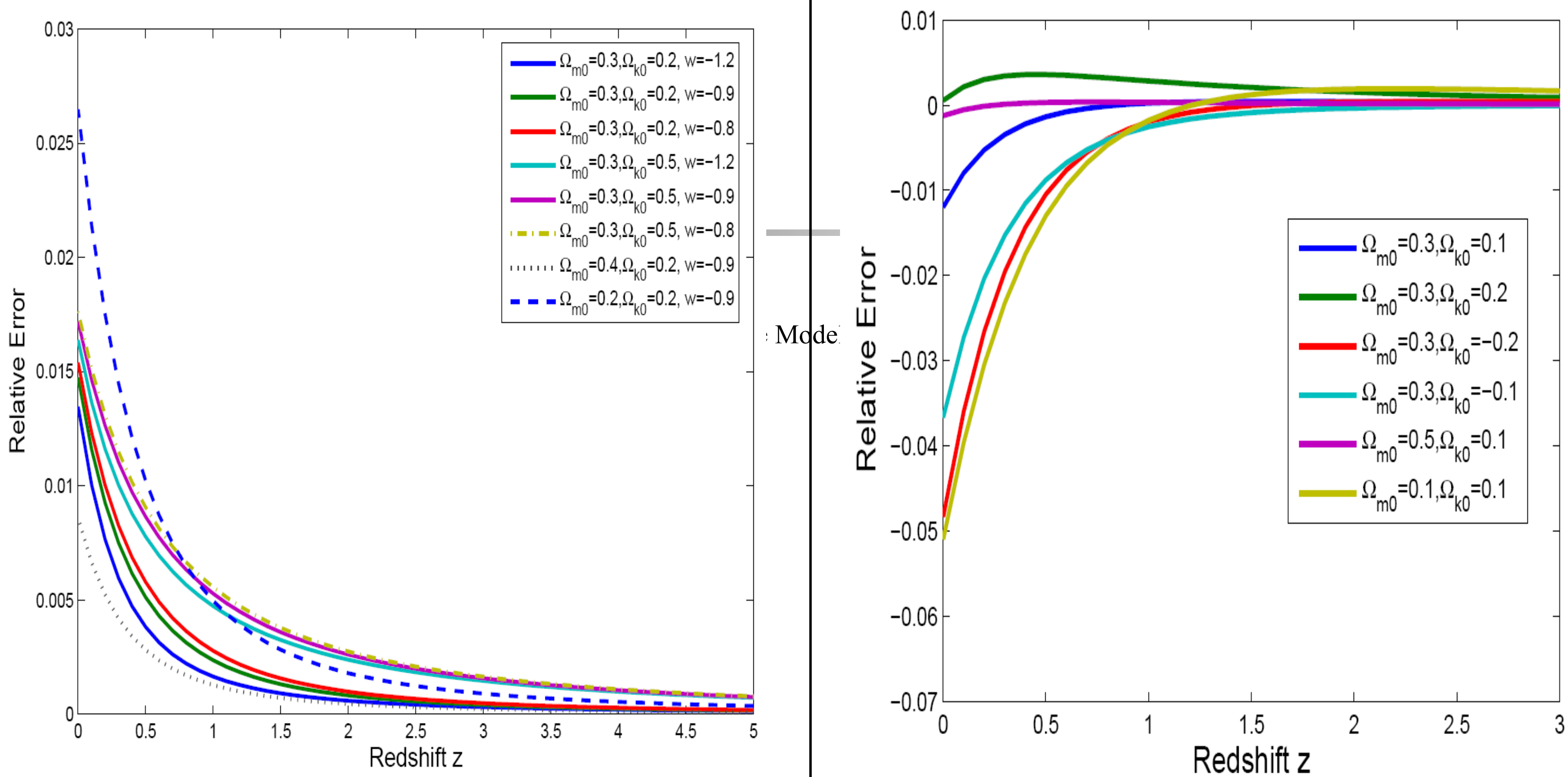
- Introducing small contributions from spatial curvature gives

$$\gamma_b = \frac{3(w-1)(1-\Omega_m) - (3w+1)\Omega_k}{(6w-5)(1-\Omega_m) - 2(3w+1)\Omega_k} \qquad f = \Omega_m^{\gamma}$$

- Or using another anzats provides:

$$f_a = \Omega_m^{\gamma} + (\gamma - 4/7)\Omega_k$$

- However, introducing curvature diminished the goodness of the approximations



GR + Dark Energy + Spatial Curvature. We plot again the relative error in order to compare the fit of the growth parameterization to that of the growth rate, *f*, that is numerically integrated from the growth ODE. The fit is now good to only 5%. RIGHT: DGP + Curvature where the fit is now only goof to 10%. Introducing curvature has affected the goodness of the significantly the fits compared to the flat cases where the errors are well below 1% for all cases.

Comparisons to observations. Current publicly available growth data [Dossett, Ishak, Yungui, Wang, PRD submitted 2009]

z	f_{obs}	References
0.15	0.49 ± 0.1	[19, 20]
0.35	0.7 ± 0.18	[21]
0.55	0.75 ± 0.18	[22]
0.77	0.91 ± 0.36	[19]
1.4	0.9 ± 0.24	[23]
3.0	1.46 ± 0.29	[24]
$2.125 - 2.72$	0.74 ± 0.24	[25]
$2.2 - 3$	0.99 ± 1.16	[26]
$2.4 - 3.2$	1.13 ± 1.07	[26]
$2.6 - 3.4$	1.66 ± 1.35	[26]
$2.8 - 3.6$	1.43 ± 1.34	[26]
$3 - 3.8$	1.3 ± 1.5	[26]

No significant constraints from current data

Constraints on interpolated parameterization for the growth index, Eq. (6), using current growth data		
Model	Best fit parameters	
	γ_0	γ_a
LCDM	$1.07^{+1.12}_{-0.98}$	$-1.71^{+5.19}_{-4.75}$
DGP	$0.978^{+0.879}_{-0.780}$	$-2.68^{+3.28}_{-3.24}$

Constraints on exponential parameterization for the growth index, Eq. (11), using current growth data		
Model	Best fit parameters	
	γ_∞	γ_b
LCDM	$0.26^{+1.19}_{-0.98}$	$0.59^{+1.50}_{-1.65}$
DGP	$0.040^{+0.682}_{-0.579}$	$0.748^{+0.982}_{-0.979}$

Constraints on a constant growth index from current growth data, γ	
Model	γ
LCDM	$0.670^{+0.179}_{-0.201}$
DGP	$0.542^{+0.143}_{-0.171}$

TABLE III: Constraints from current observational data on the growth index from the data list of TABLE II. We see on the last line that the DGP theoretical value (i.e. $\gamma = 0.68$) for a constant γ lay on the 1σ but current data is unable to constrain any of the parameters well enough to draw any conclusions.

Simulated future growth data

- Using discussions from current papers on data, we extrapolated two scenarios for uncertainties on future data.
- A pessimistic scenario where we assume that we will have more data but the uncertainties will get only slightly better then the ones of the current data: 20% to 30%
- A moderate scenario with: 10% to 20% (an improvement of a factor of 2 or 1.5).
- We generated 80 points (or bins) for the growth rate that are almost equally spaced

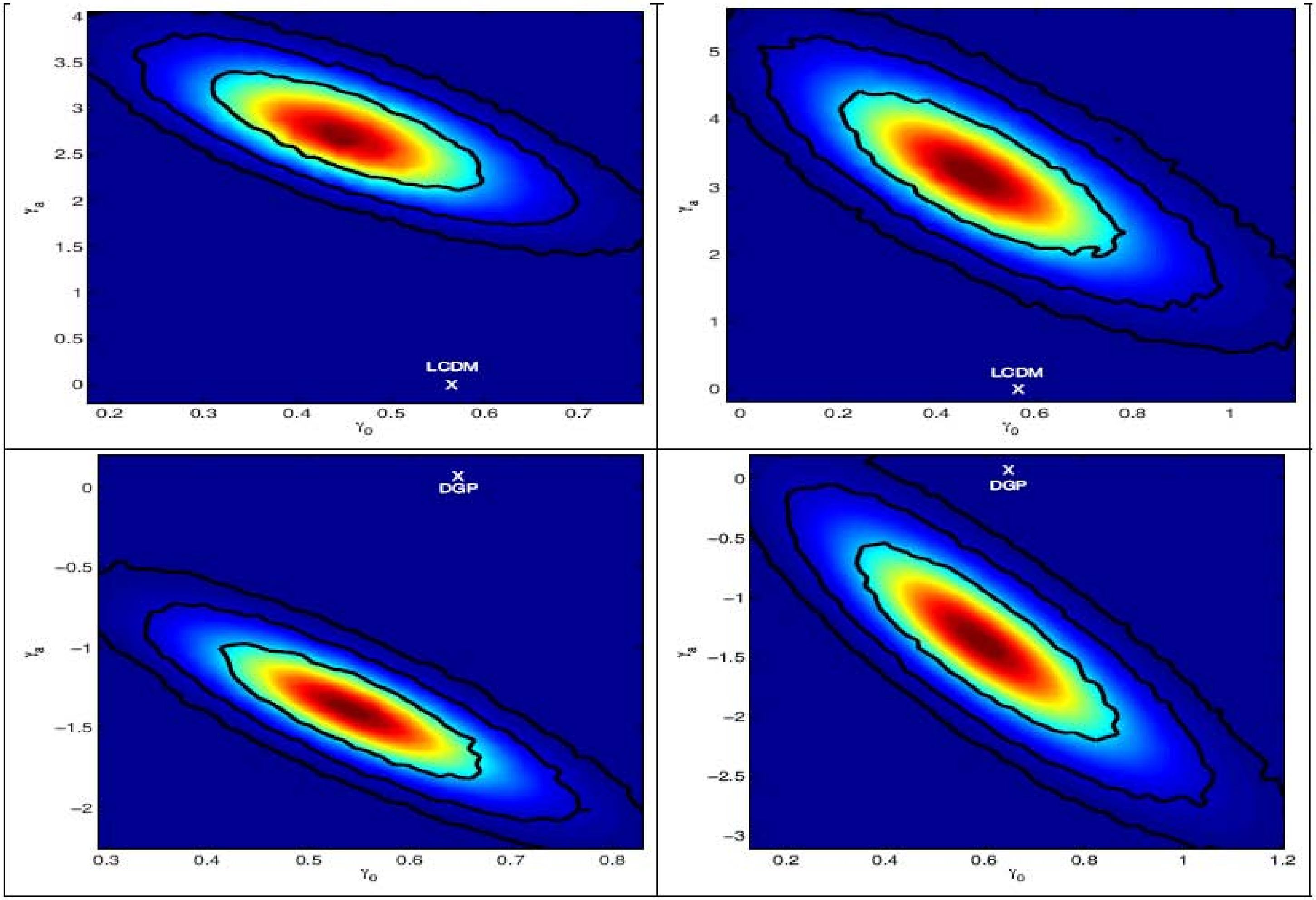


FIG. 3: Interpolated parameterization. TOP LEFT: Moderate scenario fitting fiducial DGP data on an assumed LCDM background. TOP RIGHT: Pessimistic scenario fitting fiducial DGP data on an assumed LCDM background. BOTTOM LEFT: Moderate scenario fitting fiducial LCDM data on an assumed DGP background. BOTTOM RIGHT: Pessimistic scenario fitting fiducial LCDM data on an assumed DGP background. As shown on the figures, in each case the incorrect assumed background model is ruled out to 99.7%.

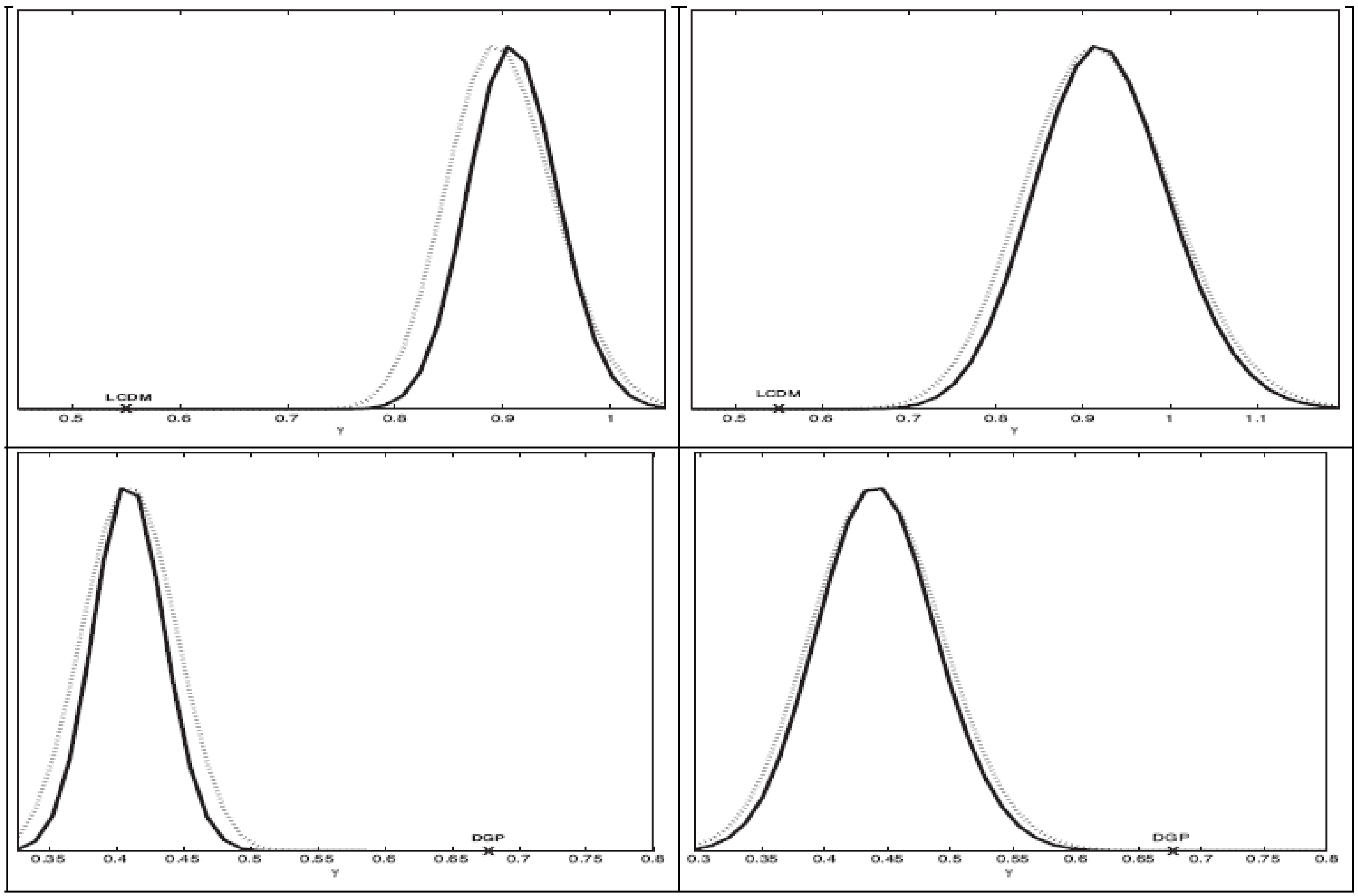


FIG. 5: Constant Growth Index γ. TOP LEFT: Moderate scenario fitting fiducial DGP data on an assumed LCDM background. TOP RIGHT: Pessimistic scenario fitting fiducial DGP data on an assumed LCDM background. BOTTOM LEFT: Moderate scenario fitting fiducial LCDM data on an assumed DGP background. BOTTOM RIGHT: Pessimistic scenario fitting fiducial LCDM data on an assumed DGP background. As shown on the figures, in each case the incorrect assumed background model is ruled out to 99.7%.

Interpolated parameterization for the growth index, Eq. (6), with 3% error on Ω_m					
Fiducial Model	Assumed Background	Moderate Scenario		Pessimistic Scenario	
		γ_0	γ_a	γ_0	γ_a
DGP	LCDM	$0.440^{+0.159}_{-0.153}$	$2.61^{+0.85}_{-0.64}$	$0.444^{+0.332}_{-0.249}$	$3.10^{+1.43}_{-1.24}$
LCDM	DGP	$0.556^{+0.142}_{-0.153}$	$-1.38^{+0.41}_{-0.44}$	$0.615^{+0.277}_{-0.280}$	$-1.38^{+0.83}_{-0.84}$

Exponential parameterization for the growth index, Eq. (11), with 3% error on Ω_m					
Fiducial Model	Assumed Background	Moderate Scenario		Pessimistic Scenario	
		γ_∞	γ_b	γ_∞	γ_b
DGP	LCDM	$1.36^{+0.23}_{-0.13}$	$-0.800^{+0.197}_{-0.259}$	$1.41^{+0.36}_{-0.25}$	$-0.886^{+0.373}_{-0.455}$
LCDM	DGP	$0.237^{+0.089}_{-0.070}$	$0.309^{+0.143}_{-0.125}$	$0.270^{+0.162}_{-0.135}$	$0.305^{+0.272}_{-0.244}$

Constant growth index, γ with 3% error on Ω_m			
Fiducial Model	Assumed Background	Moderate Scenario	Pessimistic Scenario
		γ	γ
DGP	LCDM	$0.889^{+0.062}_{-0.054}$	$0.893^{+0.110}_{-0.081}$
LCDM	DGP	$0.419^{+0.025}_{-0.041}$	$0.450^{+0.047}_{-0.063}$

TABLE IV: Here we summarize our best fits for multiple parameterizations of the growth index, when we fit fiducial data over the wrong model as described above. We see that all of the parameterizations are able to find inconsistencies in at least one parameter compared to their theoretical values. We would expect: $(0.565, -0.027)$ for LCDM and $(0.648, 0.054)$ for DGP in the interpolation parameterization; $(0.546, 0.010)$ for LCDM and $(0.687, -0.024)$ for DGP in the exponential parameterization; and (0.551) for LCDM and (0.678) for DGP with a constant γ.

Summary and conclusion

- We discussed parameterizations of the growth index that cover wide redshift ranges and interpolates to constant values at intermediate-high redshifts
- The parameterizations are found to fit the growth function to fractions of a percent for LCDM, various QCDM and DGP models
- We find that the best fit values for the growth index parameters take distinctive values for dark energy models versus modified gravity models. For example (γ_0, γ_a)=(0.5655,-0.02718) for the LCDM model and (γ_0, γ_a)= (0.6418,0.06261) for the flat DGP model
- γ_a is of a different sign for the two models. This distinction hold even when looking at more complex dark energy models.
- This provides a way for observational data to distinguish between dark energy models and modified gravity models as cause of cosmic acceleration.
- we find that current growth data is unable to put significant constraints on the growth parameters.
- Using a Monte-Carlo-Markov-Chain analysis, we find that either a pessimistic or moderate scenarios for future growth data uncertainties will be able to rule out an incorrect background model using any of the parameterizations discussed.
- We find that a constant growth index parameter will be able to rule out an incorrect as well
- But the slope of the redshift dependence provides a second significant test to rule out an incorrect model
- These parameterizations should be useful for ongoing and future high precision missions.

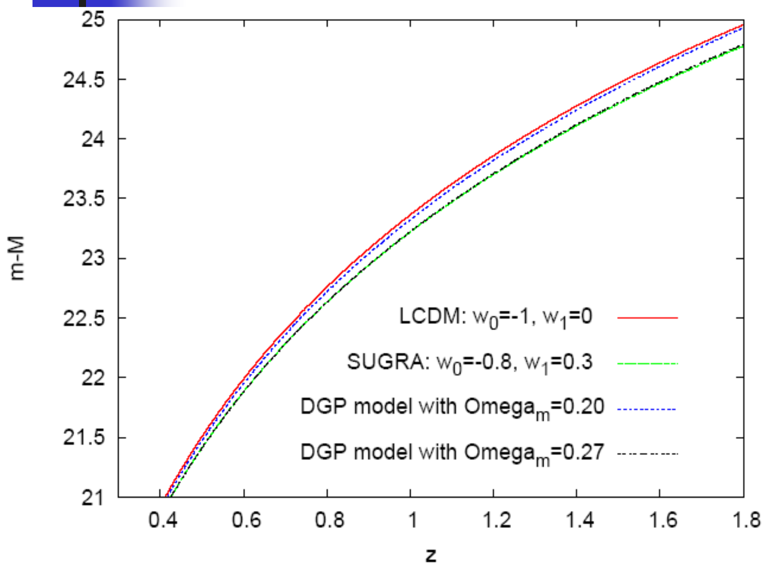
Early dark energy models

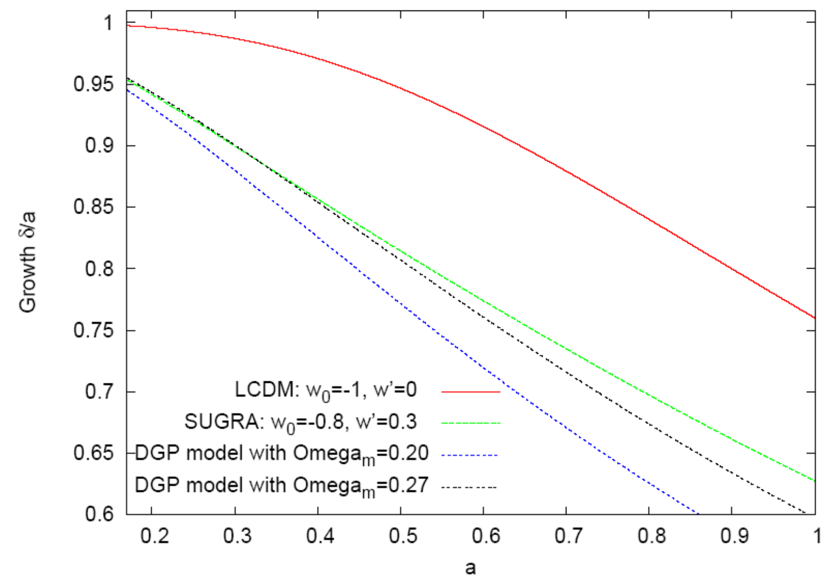
For early dark energy we use a Mocker model first introduced by [35]. The dark energy equation of state for these models is given by:

$$w(a) = -1 + \left[1 - \frac{w_0}{1+w_0} a^C\right]^{-1}. \qquad (26)$$

In these models the dark energy component behaves like nonreletavistic matter at high redshifts, having an equation of state $w = 0$, but assymptotes to a cosmological constant with $w = -1$. We use parameter values for w_0 and C given by [37] which are said to fit CMB and SN Ia constraints very well. See the aformentioned references as well as

Dark Energy versus DGP model Hubble diagrams and growth function

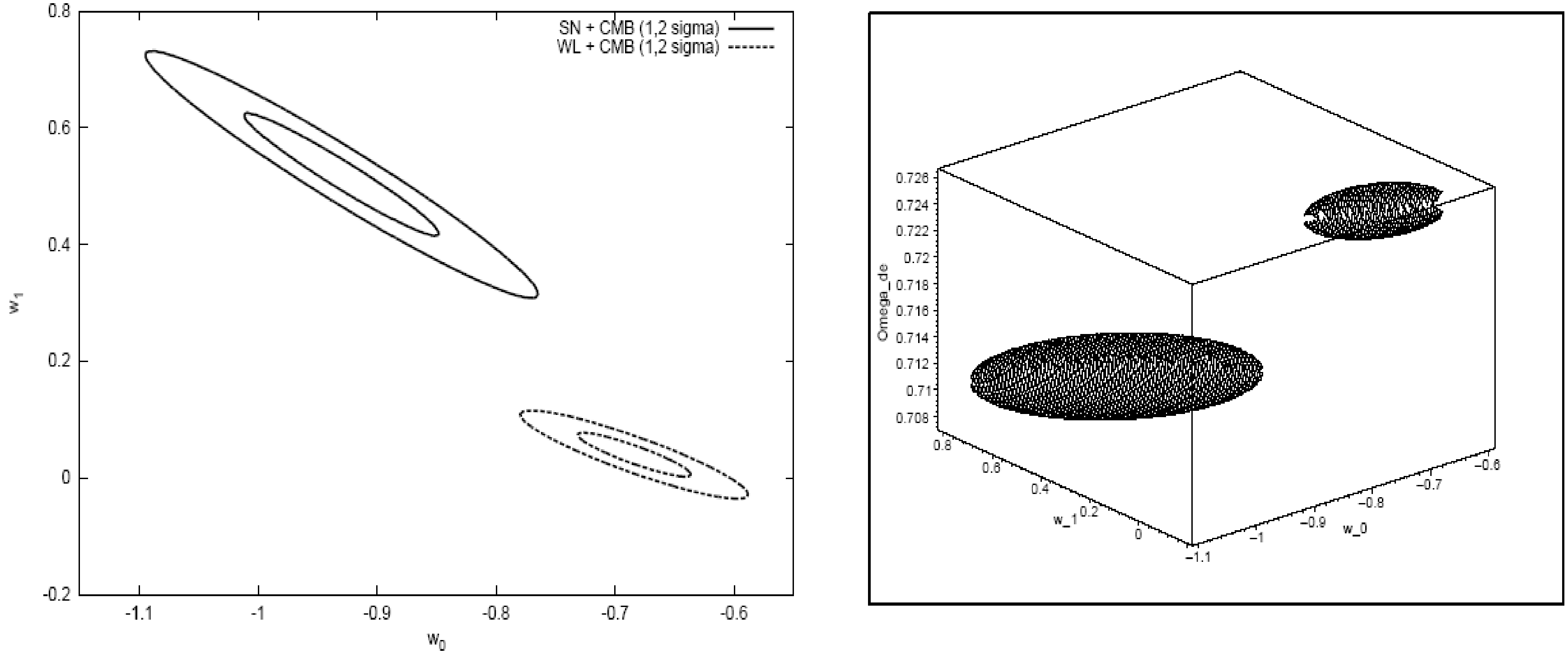




Supernova Hubble diagrams for several dark energy and DGP models. Note that the LCDM model (red solid line) and the Ωm=0.20 DGP model (blue dotted) have nearly identical Hubble diagrams, but different growth factors as shown in Fig. 1b. The same is true of the SUGRA (green dashed) and Ωm =0.27 DGP (black double dotted) models.

Growth factor of linear density perturbations for several dark energy and DGP models. Note that the growth factor in the Ωm=0.27 DGP model is suppressed with respect to that in the LCDM model, which has the same Ωm.

Results: Equations of state found using two different combinations of simulated data sets. Solid contours are for fits to the [Supernova + CMB] data combination, while dashed contours are for fits to [Weak Lensing + CMB] data combination.
(MI, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513 , astro-ph/0507184)



The significant difference (inconsistency) between the equations of state found using these two combinations is a due to the DGP model and should be detectable by future experiments.
In this simulated case, The inconsistency tells us that we are in presence of modified gravity rather than GR+dark energy.

Distinguishing dark energy and modified gravity has now generated a lot of discussion and work in the field:

[7] An incomplete list includes: L. Knox, Y.-S. Song, and J. A. Tyson, preprint, astro-ph/0503644; I. Sawicki and S. M. Carroll, preprint, astro-ph/0510364; K. Koyama and R. Maartens, JCAP **0601**, 016 (2006); M. Ishak, A. Upadhye, and D. N. Spergel, Phys. Rev. D **74**, 043513 (2006); P. Zhang, Phys. Rev. D **73**, 123504 (2006); K. Koyama, JCAP **0603**, 017 (2006); E. Bertschinger, Astrophys. J. **648**, 797 (2006); D. Huterer and E. V. Linder, Phys. Rev. D, **75**, 023519 (2007); Y.-S. Song, W. Hu, and I. Sawicki, Phys. Rev. D **75**, 044004 (2007); Y.-S. Song, I. Sawicki, and W. Hu, Phys. Rev. D **75**, 064003 (2007); R. Bean, D. Bernat, L. Pogosian, A. Silvestri, and M. Trodden, Phys. Rev. D **75**, 064020 (2007). P. Zhang, M. Liguori, R. Bean, and S. Dodelson, preprint, astro-ph/0704.1932; L. Amendola, M. Kunz, and D. Sapone, preprint, astro-ph/0704.2421.

- Sheng Wang, Lam Hui, Morgan May, Zoltan Haiman, May 2007) where they applied the same method we proposed but using some current data and Ω_Λ and w instead.

- Also Robert Caldwell, Asantha Cooray, Alessandro Melchiorri, Mars 2007; Kazuhiro Yamamoto, David Parkinson, Takashi Hamana, Robert C. Nichol, Yasushi Suto, April 2007; and others…