Q - COSMOLOGY

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Motivation

- COBE/BOOMERANG/MAXIMA-1/DASI/WMAP/CBI ....

Cosmic Microwave Background Anisotropies

+ 

Type Ia Supernova Projects

\[ \Omega_{\text{DarkEnergy}} \approx 0.73 \quad \Omega_{\text{Tot}} = 0.73 \]

Cosmic Coincidence?

Are Inflation and Acceleration Related?

Brings Us Close To the Quantum Gravity Regime

M-Theory (includes "old Superstring Theory")
A Nano-Course in Non-Critical (Liouville) Strings

$\sigma$-Model: $S_{2d} \sim \int_{\Sigma} d^2 \sigma \sqrt{h} h^{\alpha\beta} G_{\mu\nu}(X) \partial_\alpha X^\mu \partial_\beta X^\nu + \cdots$

$\downarrow$

"Coupling Constant" $(g_i)$

$\rightarrow$ Conformal Invariance $\rightarrow$ $\beta_{G_{\mu\nu}}(X) = 0 \rightarrow$

$$\frac{\partial S^{D}_{\text{eff}}}{\partial G_{\mu\nu}} = 0$$

Where

$$S^{D}_{\text{eff}} \sim \int d^D X \sqrt{G} e^{-\Phi} (R^{(D)} + \cdots)$$

$\rightarrow$ Non-Criticality $\rightarrow$ $\beta_i \neq 0$ (At Least For Some $i$)

$\rightarrow$ Add to $S^{D}_{\text{eff}} \supset \int d^D X \sqrt{G} e^{-\Phi} [\cdots + Q^2(t) + \cdots]$

With $Q^2(t) = \frac{1}{3} (C - c^*)$ (Central Charge Deficit).

This leads to

$$\ddot{g}^i + Q \dot{g}^i = -\beta^i(g) = -G^{ij} \partial C[g] / \partial g^j$$

where $\dot{\cdot} = \frac{d}{d \ln \mu} = \frac{d}{d \phi} = -\frac{d}{dt}$
Whereas the Liouville zero-mode $\varphi$ is "dormant" (decouples) at criticality ($\beta_i = 0$), it gets very active in non-criticality ($\beta_i \neq 0$)

Such that, for (super-critical) $Q^2 > 0$:

$$ S_{2\text{-dim}}^{\text{quantum}} \supset \int d^2 \sigma \sqrt{h} \left( \cdots - h^{\alpha \beta} \partial_\alpha \varphi \partial_\beta \varphi + Q \varphi R^{(2)} + \cdots \right) $$

\[ \therefore \text{Extra time-like d.o.f. !} \]

\[ \Rightarrow \text{The tale of two times} \cdots \]

\[ \Rightarrow_{\text{dynamics}} \varphi = -t \]

\[ \cdots \text{"Quantum Origin" of Time} \cdots \]

i.e. from the would-be ($D+1$) spacetime, we live in the $(\varphi + t) = 0$ D-spacetime \cdots

$$ V_{\text{effective}} \propto \cosh(t + \varphi) \to \frac{dV_{\text{eff}}}{d\varphi} = 0 \quad @ \varphi + t = 0 $$
\[ S_\sigma = S^* + \int_\Sigma g^i V_i, \quad \beta^i \equiv \frac{dg^i}{d\ln \mu} \neq 0 \]

\( S^* \) fixed point (conformal) action, \( \mu \) RG scale on world-sheet \( \Sigma \), \( \{g^i\} = \) space-time fields \( = \{G_{\mu\nu}, \Phi, A_\mu\ldots\} \).

\[ C[g] = 2z^4 < T(z)T(0) > -3z^3 \bar{z} < T(z)\Theta(0) > -6z^2 \bar{z}^2 < \Theta(z)\Theta(0) > \]

Zamolodchikov's running central charge (counts d.o.f.).

NB: Critical strings: \( 0 = \beta^i = \frac{\delta F}{\delta g^i} \) eqs. of motion, \( F \propto C[g] \) effective target-space action.
Irreversibility of RG flow (c-theorem):

\[ \frac{\partial C}{\partial \ln \mu} = -12 \beta^i < V_i(z) V_j(0) > \beta^j \leq 0 \]

for unitary \( \sigma \)-models.

Extension of irreversibility for Non-Unitary \( \sigma \)-models

\[ c_{\text{eff}} \equiv c - 24 \Delta_m \quad \Delta_m = \text{scaling dim of lowest energy state} < 0 \text{ for non unitary theories.} \]

For stringy \( \sigma \)-models time-like dilatons induce non-unitarity, hence central charge may oscillate before settling to non-trivial infrared fixed point.
Non-Critical (Liouville) Strings and Cosmic Inflation

The Origins: (ABEN '87 – '89)

\[ \ddot{g}^i + Q\dot{g}^i = -\beta^i(g) = -G^{ij} \partial C[g] / \partial g^j \]

Applying this to scalar, inflaton-like string modes

\[ \Rightarrow \ddot{\phi}_i + 3H\dot{\phi}_i = -\frac{\partial V}{\partial \phi_i} \]

\[ \dddot{\phi}_i \cdot \text{Standard field equations for scalar fields in de Sitter (inflationary) spacetime} \]

Provided \( Q = -3H \)

Example: \( \{g_i\} = \{G_{\mu\nu}, B_{\mu\nu}, \Phi\} \)

\[ \beta^G_{\mu\nu} = \alpha'(R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H^{\rho\sigma}) \]

\[ \beta^B_{\mu\nu} = \alpha'(-\frac{1}{2} \nabla_\rho H^\rho_{\mu\nu} + H^\rho_{\mu\nu} \partial_\rho \Phi) \]

\[ \tilde{\beta}^\Phi = \beta^\Phi - \frac{1}{4} G^{\rho\sigma} \beta^G_{\rho\sigma} = \frac{1}{6}(C - 26) \]

where we take \( H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b \) with \( b = b(t) = \beta t \), \( \beta = \text{constant} \) and \( \Phi(t, x) = c_1 + c_2 t \), \( c_1, c_2 = \text{constant} \).
This "linear in time dilaton" field makes no contribution to $\beta^G_{\mu\nu}, \beta^B_{\mu\nu}$

Turn on $Q(\neq 0) = \text{constant}$
(Initial quantum fluctuation; D-brane collision)

Liouville Dressing $\implies \Phi(t, x, \varphi) = Q\varphi + c_1 + c_2 t$
with $Q = c_2$; thus: $\Phi(t, x, \varphi) = c_1 = \text{constant}$, and:

$$\ddot{G}_{ij} + Q\ddot{G}_{ij} = -R_{ij} + \frac{1}{2} \beta^2 G_{ij}$$
$$\ddot{G}_{00} + Q\ddot{G}_{00} = -R_{00}$$

which accepts as solution the RWF Inflationary (de Sitter) metric

$$G_{00} = -1, \ G_{ij} = e^{2Ht} \eta_{ij}$$

$$H = -\frac{Q}{3}, \ \beta^2 = \frac{5}{9} Q^2 \quad \text{Q.E.D.}$$
On the Origin of the Big Bang

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DAWN OF TIME

inflation

tiny fraction of a second

380,000 years

13.7 billion years
SPACE-TIME FOAM

AN ARTIST'S IMPRESSION OF SPACE-TIME FOAM

(AFTER WEINBERG 99)
Brany Liouville (Non-critical) Inflation/Acceleration

- Big Bang is a Catastrophic Cosmic Event Due to the Collision of D-Branes (or Brane Worlds)

Departure from Conformal Invariance of Underlying String Theory, i.e.

Non-Criticality $\beta_i \neq 0$ of the 2-d world-sheet $\sigma$ model

Collision is viewed as a non-equilibrium stringy process formulated within Non-critical (Liouville) String Theory
(ii) Catastrophic events (e.g. collision) in brane worlds, take branes out of equilibrium $\rightarrow$ relaxation to critical state, which is reached asymptotically. Collision $\rightarrow$ recoil problem in brane worlds. Break SUSY at TeV by compactifying higher branes to three branes in magnetized tori Relaxing to zero quintessence like cosmological constant, $1/t^2$, $t =$ age of Universe$=10^{60}$ (Planck units), quintessence fields: dilaton, instabilities (tachyonic condensates from breaking of SUSY) etc.

Formalism: Logarithmic CFT deformations of brane-world recoil crucial
Figure 2: A model for supersymmetric D-particle foam consisting of two stacks each of sixteen parallel coincident D8-branes, with orientifold planes (thick dashed lines) attached to them. The space does not extend beyond the orientifold planes. The bulk region of ten-dimensional space in which the D8-branes are embedded is punctured by D0-particles (dark blobs). The two parallel stacks are sufficiently far from each other that any Casimir contribution to the vacuum energy is negligible. Open-string interactions between D0-particles and D8-branes are also depicted (wavy lines). If the D0-particles are stationary, there is zero vacuum energy on the D8-branes, and the configuration is a consistent supersymmetric string vacuum.
The String Coupling Accelerates the Expansion of the Universe

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DAWN OF TIME

tiny fraction of a second

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**An Example**

10-0 Type-0 string theory

Compactlyified on a space with five-flat dimensions and a non-trivial flux parallel to the remaining dimension.

Si'yer extra dimensions RANeY® fixed t© s© fixed val®

Positive central-charge defect $Q^2 \rightarrow Q_0^2$

\[ q_E^2 (t_e) = \frac{Q_0}{\delta} \sqrt{1 + \delta^2 t_e^2} \]

\[ H(t_e) = \frac{\delta^2 t_e}{1 + \delta^2 t_e^2} \]

\[ \Lambda_C (t_e) \approx \frac{\delta^2}{1 + \delta^2 t_e^2} ; \]

\[ \Phi_0 \sim \delta \delta(t_e) \]

\[ q (t_e) = \left( -\frac{d^2 \phi_E / dt_e^2}{(d \phi_E / dt_e)^2} \right) \phi_E \approx -\frac{1}{\delta^2 t_e^2} \]

\[ q(t_e) = -\delta^2 \phi_0 = -\frac{\delta^2}{\delta^2 t_e^2} \]

\[ \frac{g_5}{g_5} \sim \frac{4}{t_e} \rightarrow \frac{4}{t_0} \sim 10^{-60} \]
\[ q_v = -\frac{g^2}{\Lambda} \]

\[ q_v = \frac{4}{3} (\Omega_m)_0 - (\Omega_{\gamma})_0 \]

\[ (\Omega_m)_0 = 0.27 \]

\[ (\Omega_{\gamma})_0 = 0.73 \]

As observed!!!
Figure 3: The evolution of the energy densities of matter, radiation and of the quintessence field (dilaton) vs. the scale factor of the Universe in the Einstein frame. At early stages the energy density of the quintessence field decreases significantly, as compared with the rest, and the coincidence situation is lost. This is due to the behaviour of the central charge deficit of the model, shown in figure 1, which dives into zero for a short period immediately after inflation.
$$3 \dot{H}^2 - \ddot{e}_m - e_\phi = \frac{e^{2\phi}}{2} \ddot{\phi}$$
$$2 \dot{H} + \ddot{e}_m + e_\phi + \ddot{p}_m + p_\phi = \frac{\ddot{g}_{ii}}{a^2}$$
$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{4} \frac{\partial V}{\partial \phi} + \frac{1}{2} (\ddot{e}_m - 3\ddot{p}_m) = -\frac{3}{2} \frac{\ddot{g}_{ii}}{a^2} - \frac{e^{2\phi}}{2} \ddot{g}_\phi.$$

$$\ddot{g}_\phi = e^{-2\phi} (\ddot{\phi} + \dot{\phi}^2 + Qe^\phi \dot{\phi})$$
$$\ddot{g}_{ii} = 2a^2 (\ddot{\phi} + 3H\dot{\phi} + \dot{\phi}^2 + (1 - q)H^2 + Qe^\phi (\dot{\phi} + H)).$$

$$\frac{dn}{dt} = -3Hn - \langle \sigma v \rangle (n^2 - n_{eq}^2) + \dot{n} + "G/m_X".$$

$$\frac{d\ddot{e}_m}{dt_E} + 3H(\ddot{e}_m + \ddot{p}_m) + \frac{\dot{Q}}{2} \frac{\partial V}{\partial Q} - \dot{\phi} (\ddot{e}_m - 3\ddot{p}_m) = 6(H + \phi) \frac{\ddot{g}_{ii}}{a^2}.$$

$$\frac{d\ddot{g}_\phi}{dt_E} = -6e^{-2\phi} (H + \dot{\phi}) \frac{\ddot{g}_{ii}}{a^2}.$$
Figure 1: Left panel: The dilatons $\phi$, the (square root of the) central charge deficit $Q$ and the ratio $a/a_0$ of the cosmic scale factor as functions of the Einstein time $t_{\text{Einstein}}$. The present time is located where $a/a_0 = 1$ and in the figure shown corresponds to $t_{\text{today}} \simeq 1.07$. The input values for the densities are $\rho_b = 0.238, \rho_\gamma = 0.0$ and $w_\gamma$ is 0.5. The dilaton value today is taken $\phi = 0.0$. Right panel: The values of $\Omega_i \equiv \rho_i/\rho_c$ for the various species as functions of $t_{\text{Einstein}}$.

Figure 2: Left panel: Ratios of $\Omega$'s for the dilaton ($\phi$), exotic matter ($e$) and the non-critical terms ("noncrit") to the sum of "dust" and radiation $\Omega_b + \Omega_r$ densities. Right panel: The quantities $\rho_b \, a^3$, for "dust", $\rho_r \, a^4$ and $\rho_e \, a^2$ as functions of $t_{\text{Einstein}}$. 

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Figure 3: Left panel: The deceleration $q$ and the dimensionless Hubble expansion rate $\dot{H} \equiv \frac{H}{\sqrt{3}H_0}$ as functions of $t_{Einstein}$. Right panel: The derivative of the dilaton and its ratio to the dimensionless expansion rate.

Figure 4: Left panel: The dilaton $\phi$, the (square root of the) central charge deficit $Q$ and the ratio $a/a_0$ of the cosmic scale factor as functions of the Einstein time $t_{Einstein}$. The inputs are as in figure 1 with only changing the dilaton to $\phi_0 = -1.0$. Right panel: The deceleration and $\ddot{H}$ for the same inputs.
Figure 5: Left panel: The ratio $|q|/g_s^2$ as function of the redshift for $z$ ranging from $z = 0.2$ to future values $z = -0.6$, for the inputs discussed in the main text. The rapid change near $z \approx 0.16$ signals the passage from deceleration to the acceleration period. Right panel: The values of the string coupling constant plotted versus redshift value in the range $z = 0.0 - 1.0$.

Figure 6: The deceleration as function of redshift values in the range $z = 0.0 - 1.0$. The inputs are as in figure 5.
FIG. 7: Reconstruction of $q(z)$ using its 2 most well-constrained principle components (Eq. 13). The short-dashed line is the best 2 mode fit to SNeIa data; it has a 14% goodness-of-fit. The gray band shows the error corresponding to 1-cT uncertainties in both $a$ and $\langle 2 \rangle$. The solid line is $q(z)$ for $\Lambda$CDM with $\Omega M = 0.3$. As in Fig. 1, the kinematic model suggests a later transition to acceleration than $\Lambda$CDM does.
\[ H(z) = H_0 \left( \Omega_3 (1 + z)^3 + \Omega_4 (1 + z)^{4.1} + \Omega_2 (1 + z)^2 \right)^{1/2} \]

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<th>( \Omega_M )</th>
<th>( \chi^2 )</th>
<th>( \chi^2/\text{dof} )</th>
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\[
\frac{dn}{dt} + 3\frac{\dot{a}}{a} n = \Gamma(t)n + \int \frac{d^3p}{E} \mathcal{C}[f] .
\]
\[
\Gamma(t) \equiv \Phi + \frac{1}{2} n e^{-\Phi} g^{\mu\nu} \tilde{\gamma}_\mu \tilde{\gamma}_\nu
\]
\[
\frac{dn}{dt} = -3\frac{\dot{a}}{a} n - \langle v\sigma \rangle (n^2 - n_{eq}^2) + \Gamma n
\]
\[
n_{eq} a^3 = n_{eq}^{(0)} a^3(t_0) \exp \left( \int_{t_0}^t \Gamma dt \right)
\]
\[
\Omega_\chi h_0^2 = (\Omega_\chi h_0^2)_{no-source} \times \left( \frac{g_s}{g} \right)^{1/2} \left( 1 + \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{\psi(x)} dx \right)
\]
\[
(\Omega_\chi h_0^2)_{no-source} = \frac{1.066 \times 10^9 \text{ GeV}^{-1}}{M_{\text{Planck}} \sqrt{g_s} J}
\]
\[
\psi(x) \equiv x \exp \left( - \int_{x_0}^x \Gamma H^{-1} dx/x \right)
\]
mSUGRA Benchmark Points

$m_0$

$m_{1/2}$

$m_{h,b \rightarrow s\gamma}$

$g-2$
$A_0 = 0$, $\tan \beta = 40$, $\mu > 0$

$m_0$ (GeV)

$m_{\tilde{c}} = 104$ GeV

$M_{1/2}$ (GeV)

TH
Conclusions:

- Novel way to model the origin of the Big Bang/Universe and its Evolution
- Based upon Fundamental Principles of M-Theory (Non-Criticality, D-Brane Dynamics...)
- In Sharp Contrast to (Pseudo)-Effective String Theory approaches ...