Gravitational Lensing and Dark Energy: A procedure to distinguish between dark energy models and modified gravity models

$$G^a_b + \Lambda \delta^a_b = \kappa T^a_b$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi \rho_{DE} (\frac{1}{3} + w)$$

$$S_{(5)} = \frac{1}{2} M^3_{(5)} \int d^4x\, dy \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{2} M^2_{(4)} \int d^4x \sqrt{-g_{(4)}} R_{(4)} + S_{matter}$$

$$P_\kappa(l) = \frac{9}{4} H^4_o \Omega^2_m \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(l/\sin_K(\chi), \chi\right) d\chi$$

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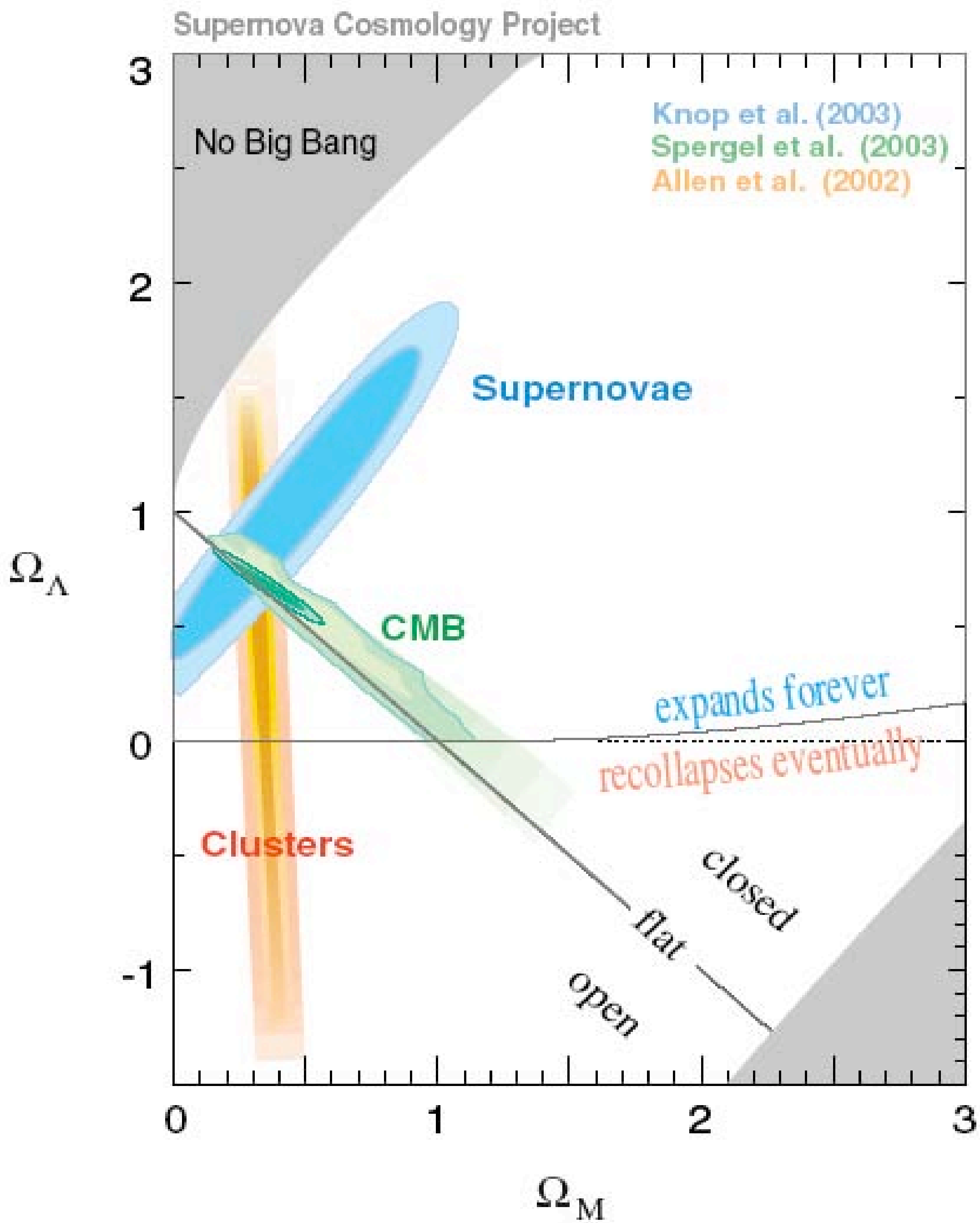
Evidence for the acceleration / dark energy from Complementary Cosmological Probes:

Supernovae,
CMB,
Clusters,
Galaxy Clustering,
Weak Gravitational lensing



In a golden era of cosmology, “We know a lot about our universe but we understand very little”

Contents of the universe:

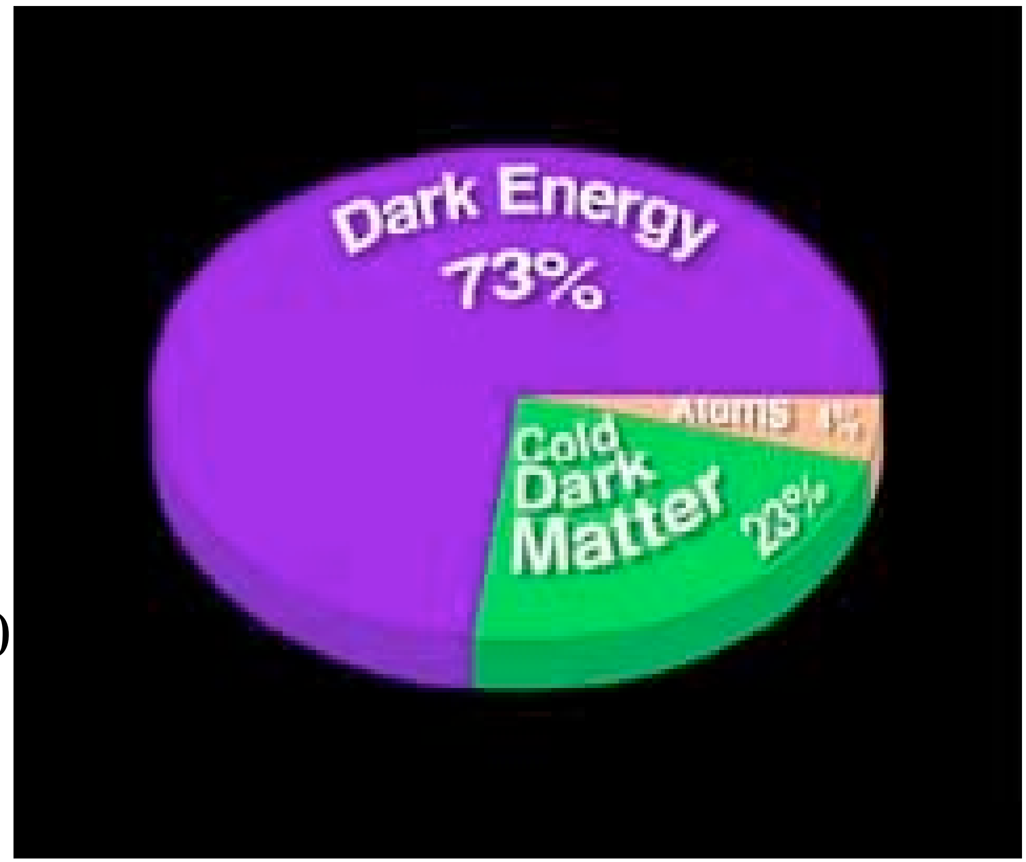
Baryons: 4%

Dark matter: 23%

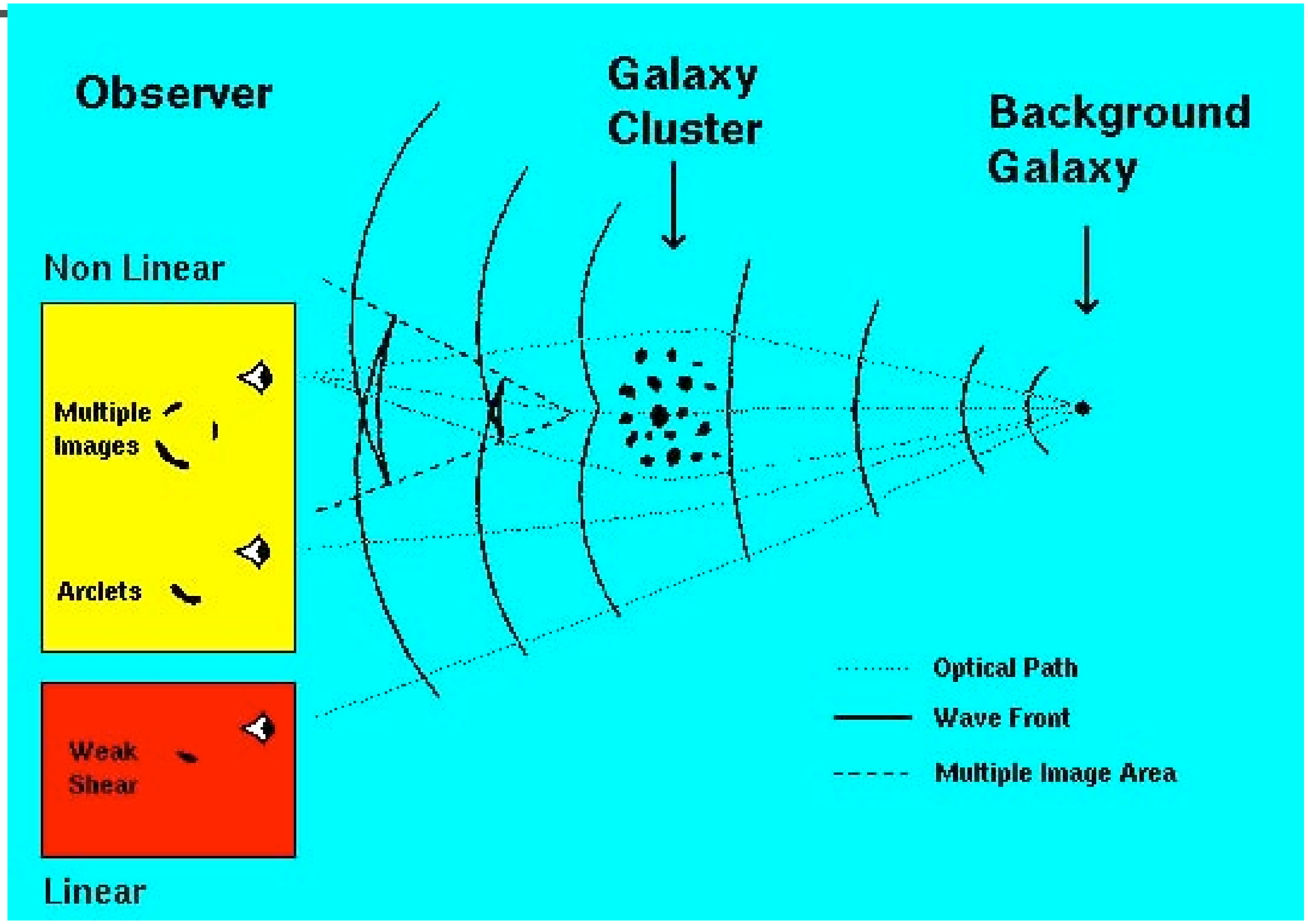
Dark energy: 73%

Massive neutrinos: 0.1%

Spatial curvature: very close to 0

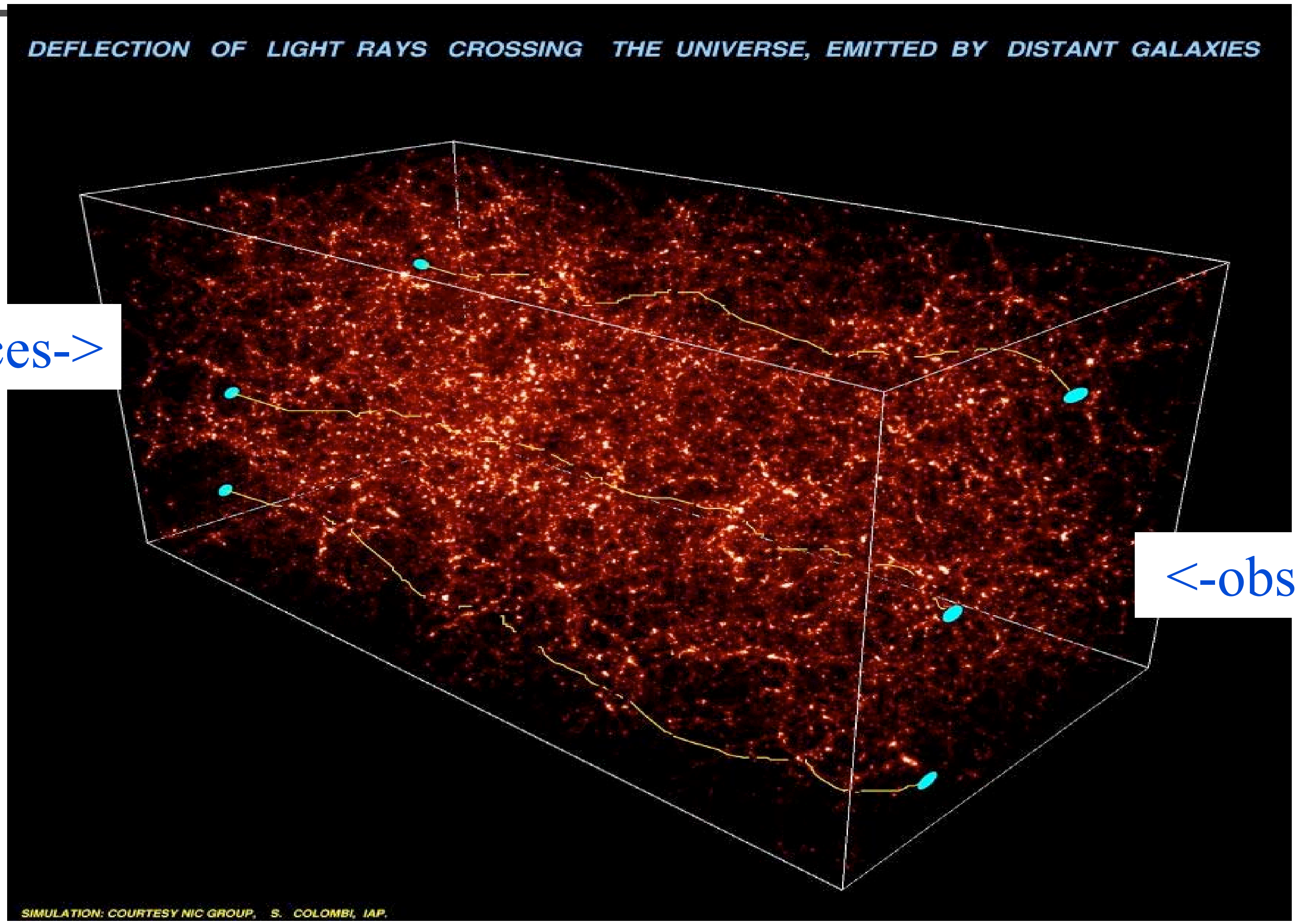


Gravitational Lensing is a powerful cosmological probe. The prediction of the bending of light trajectories by massive objects is a great triumph of General Relativity. Several techniques have been developed

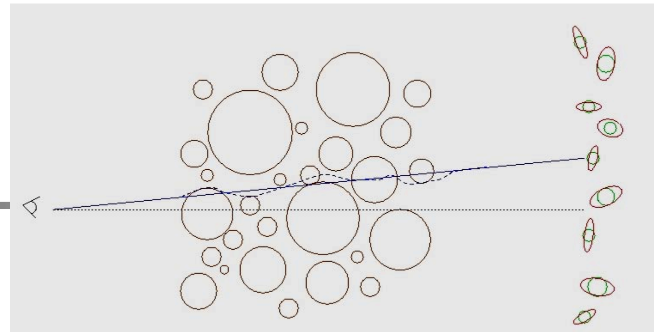


From MPA lensing group

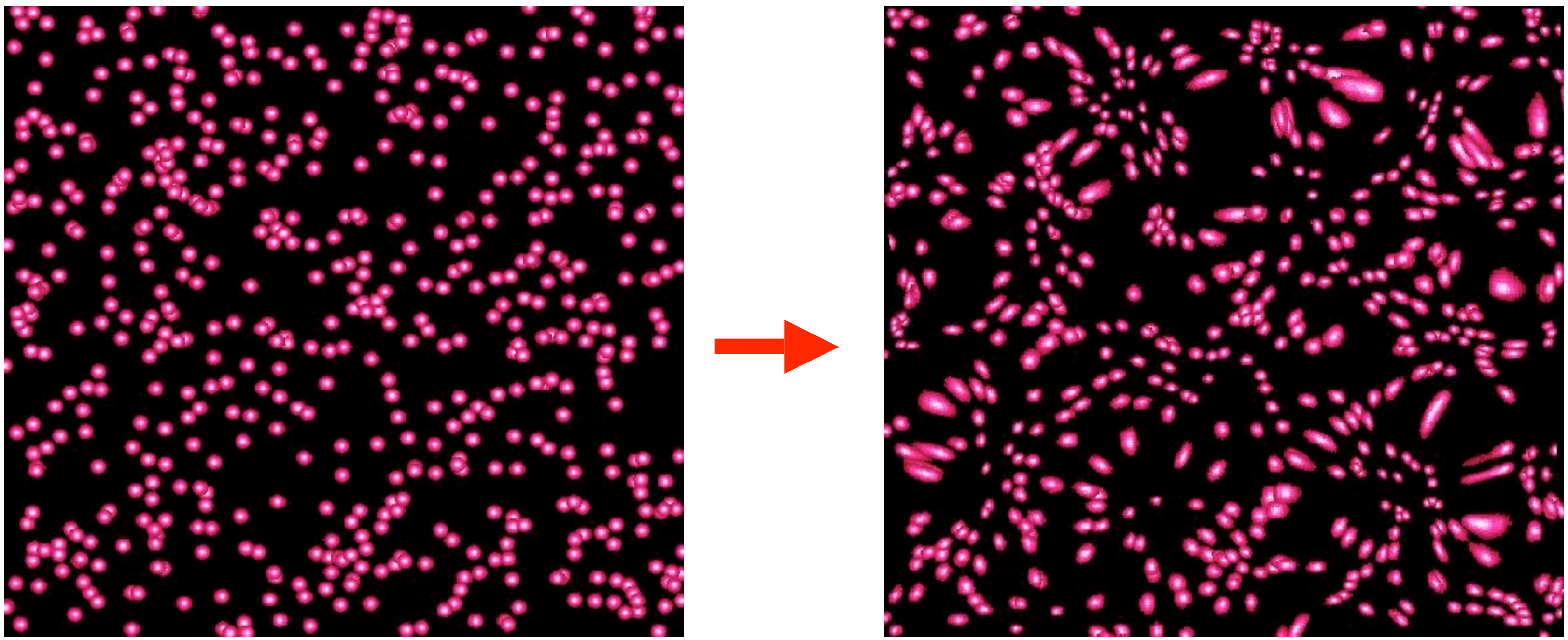
Weak gravitational lensing (cosmic shear)



Weak Gravitational Lensing



Distortion of background images by foreground matter



Unlensed **Lensed**

Credit: SNAP WL group

The information on the magnification and distortion of images is contained in the convergence power spectrum

$$P_\kappa(l) = \frac{9}{4} H_o^4 \Omega_m^2 \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(l / \sin_K(\chi), \chi\right) d\chi$$

$$g(\chi) = \int_\chi^{\chi_H} n(\chi') \frac{\sin_K(\chi' - \chi)}{\sin_K(\chi')} d\chi'$$

The power spectrum is sensitive to several cosmological parameters

Weak lensing captures the effect of Dark Energy on the expansion history and its effect on the growth factor of large-scale structure

$$W(H_o, \Omega_m, \chi(z), H(z))$$

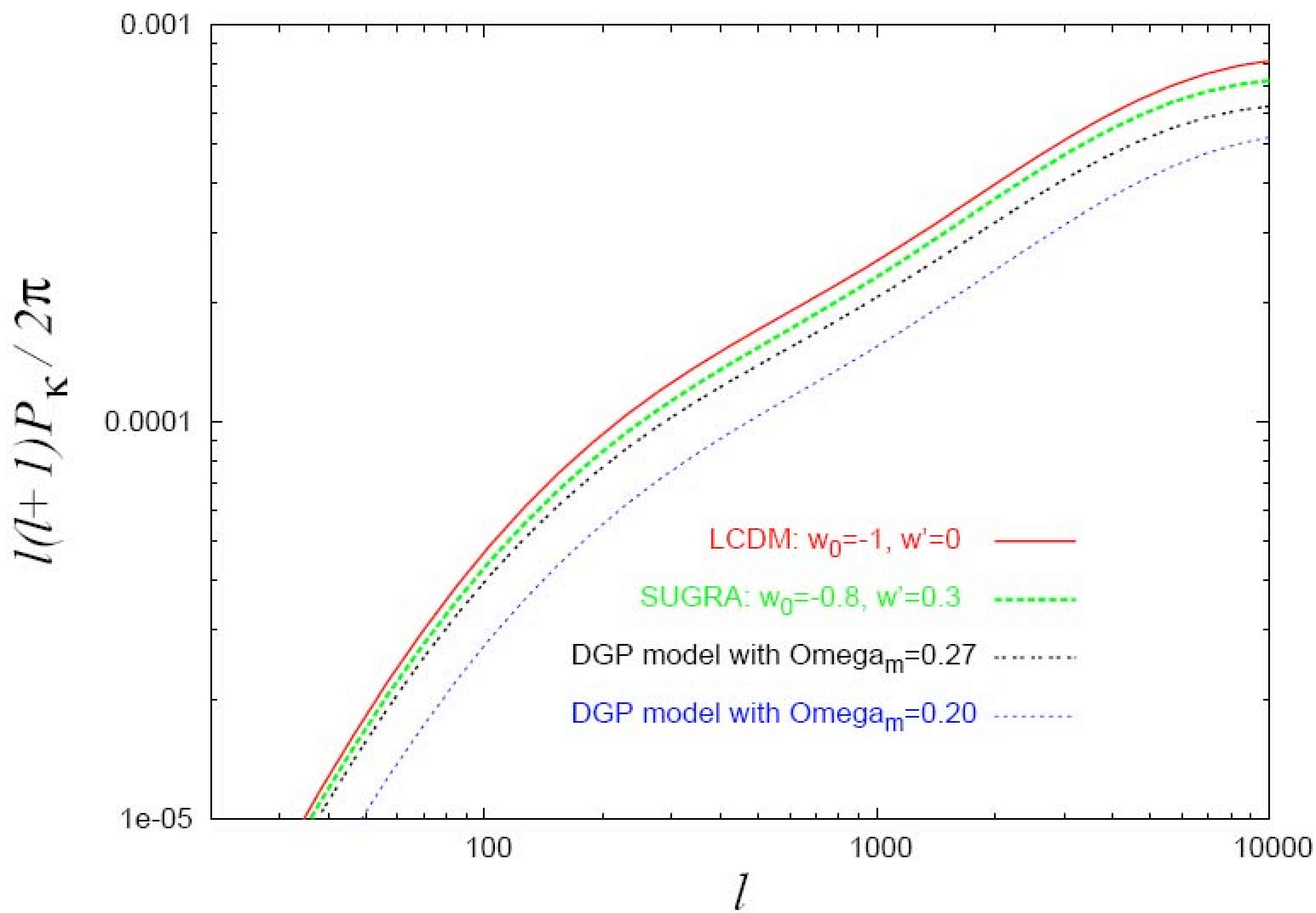
$$P_{3D}(k,z) \propto P_{3D}^{prim}(k) T^2(k,z) \left[\frac{G(z)}{G(0)}\right]^2 NLM(k,z)$$

$$D(a) = \frac{\delta(a)}{\delta(1)}$$

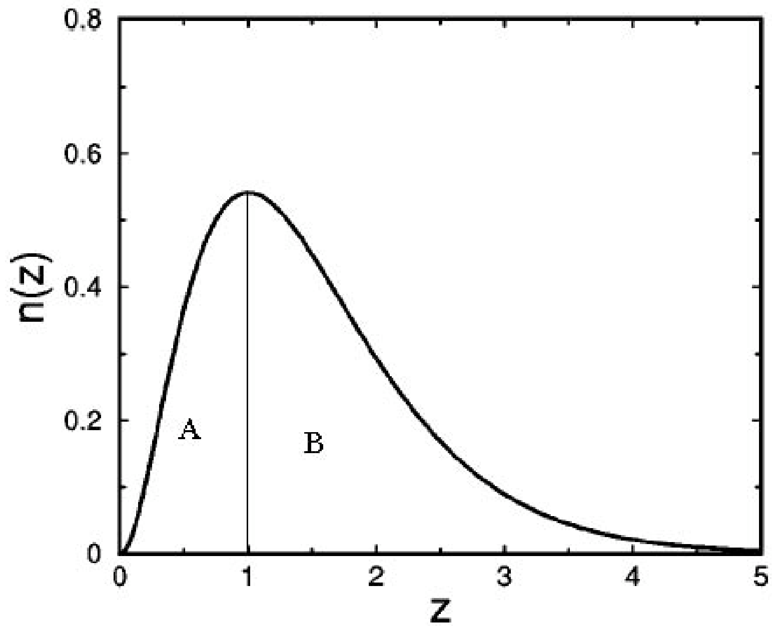
$$G(a) = \frac{D(a)}{a}$$

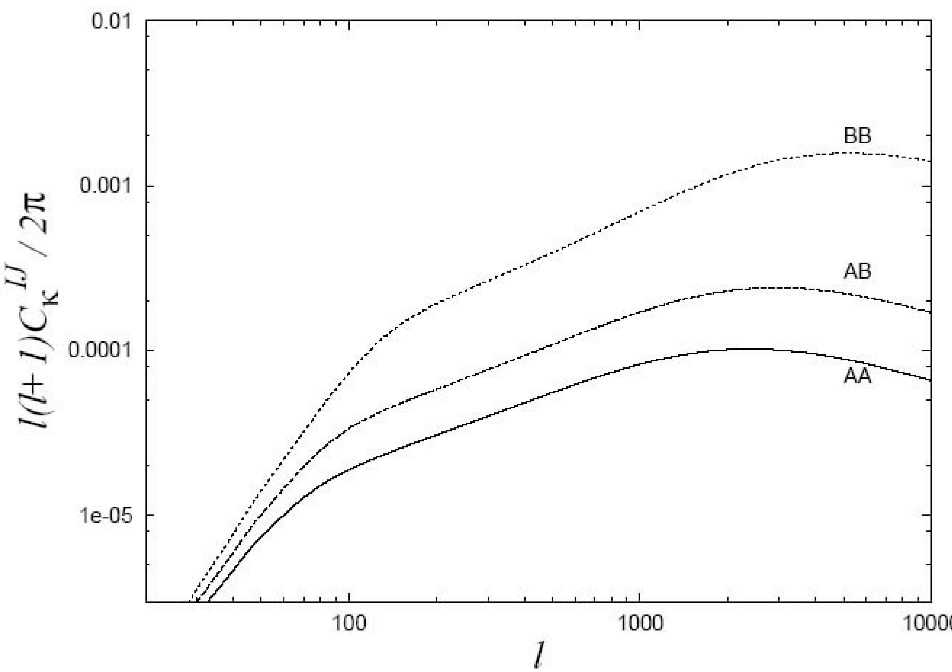
$$G'' + \left[\frac{7}{2} - \frac{3}{2} \frac{w(a)}{1 + X(a)}\right] \frac{G'}{a} + \frac{3}{2} \frac{1 - w(a)}{1 + X(a)} \frac{G}{a^2} = 0$$

Convergence Power Spectrum



Cosmic shear tomography





A promising technique of weak lensing is called tomography: it requires dividing the source galaxies into intervals of red shift called tomography bins

Tomography auto power spectra and the cross-correlation for 2 red shift bins (it should be possible to do about 10 bins with the best proposed future experiments)

Current major approach to the cosmic acceleration problem: Putting tight constraints on the equation of state of dark energy, w

The equation of state of a "cosmic fluid": $$p = w\rho$$

Negative w < -1/3 gives an accelerating expansion

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi}{3}(\rho_{DE} + 3p_{DE}) \qquad \frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE}(\frac{1}{3} + w)$$

Two levels of difficulty:
1) A constant EOS w.
2) A variable EOS w(z)

$$w(a) = w_0 + w_a \frac{z}{1+z} = w_0 + w_a(1-a)$$

$$w(z) = \begin{cases} w_0 + w_1 z & \text{if } z < 1 \\ w_0 + w_1 & \text{if } z \geqslant 1. \end{cases}$$

$$w(a) = w_0 + w_a(1-a) + w_b(1-a)^2$$

What kind of answers are we going to be able to provide from the Dark Energy equation of state approach? (e.g. Ishak 2005, MNRAS, V363, 2, 469 (2005); Upadhye, Ishak and Steinhardt, PRD, 72, 063501 (2005))

- The most decisive answer will be if the data can show conclusively that Dark Energy is not a cosmological constant

- It will be possible from several combinations of experiments to exclude some proposed models, trackers or some SUGRA inspired models with for w_0=-0.8 and w_1=0.3

- A very suggestive but less decisive answer will be to show that the Dark Energy parameters are those of a cosmological constant to a very high level of precision (a few percent?)

- In all cases, a burning question is to know if what is obtained from the data is the equation of state of some dark energy cosmic fluid or a just a result obtained because we tried to force a dark energy model on the top of a modified gravity model?

Probing cosmic acceleration beyond the equation of state: A new procedure.

- Cosmic acceleration can be caused by:
  - 1) An energy component in universe (e.g. dark energy, vacuum energy)
  - 2) A modification to gravity at cosmological scales

- An important step will be to distinguish between the 2 causes

- We proposed a procedure to distinguish between cosmic acceleration due to dark energy and cosmic acceleration due to modified gravity models at cosmological scales (Ishak, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513, astro-ph/0507184)

- The procedure also answers the paramount question of true or forced equation of state

Basic idea of the approach

- The cosmic acceleration affects cosmology in two ways:
  - 1) It effects the expansion history of the universe
  - 2) It effects the growth rate of large scale structure in the universe (the rate at which clusters and super clusters of galaxies forms over the history of the universe)

- The idea explored is that, for dark energy models, these two effects must be consistent one with another because they are mathematically related by General Relativity equations

- The idea has been discussed by our group and others groups as well but the challenge was to implement it using cosmological probes

- The presence of significant inconsistencies between the expansion history and the growth rate could be the signature of some modified gravity at cosmological scales.

- The proposed procedure detects such inconsistencies when they are present.

- The key step is to compare constraints on the expansion and the growth using different and specific pairs of cosmological probes in order to detect inconsistencies (Ishak, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513 , astro-ph/0507184)

An example of modified gravity at cosmological scales: The Dvali, Gabadadze and Poratti model (DGP model, 2000)

- The action for this 5-dimentional theory is given by

$$S_{(5)} = \frac{1}{2} M_{(5)}^3 \int d^4x\, dy \sqrt{-g_{(5)}}\, R_{(5)} + \frac{1}{2} M_{(4)}^2 \int d^4x \sqrt{-g_{(4)}}\, R_{(4)} + S_{matter}$$

- The idea is that our universe is a "brane" embedded in a 5D bulk
- The first term describes the bulk while the 2 others are the usual 4D ones
- The 2 different pre-factors in front of the bulk and the brane actions give rise to a characteristic length scale r_c
- At distance scales much smaller than this characteristic distance, we have the usual gravitational physics; On scales larger then r_c, the full 5D physics is recovered
- The result is that gravity is weakened at scales comparable to r_c and that will cause the cosmic expansion to accelerate
- This model is consistent with current cosmological data and is a good example to use to test the proposed procedure (note: we are not particularly interested in the viability of this model but only in using it as an example to illustrate the procedure)

The consistency relation between the expansion history and the growth rate of large scale structure

- For the standard FLRW model with k=0 and a Dark Energy component, the expansion history is expressed by the Hubble function and is given by

$$H(z) = Ho\sqrt{(1-\Omega_{de})(1+z)^3 + \Omega_{de}\varepsilon(z)} \qquad (1)$$

- And the growth rate G(a=1/(1+z)) is given by integrating the ODE:

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1+X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1-w(a)}{1+X(a)}\frac{G}{a^2} = 0; \quad G(a) = \frac{D(a)}{a}; \quad D(a) = \frac{\delta(a)}{\delta(1)} \qquad (2)$$

- For Modified Gravity DGP models and k=0, the expansion history is given by

$$H(z) = Ho\left[\frac{1}{2}(1-\Omega_m) + \sqrt{\frac{1}{4}(1-\Omega_m)^2 + \Omega_m(1+z)^3}\right] \qquad (3)$$

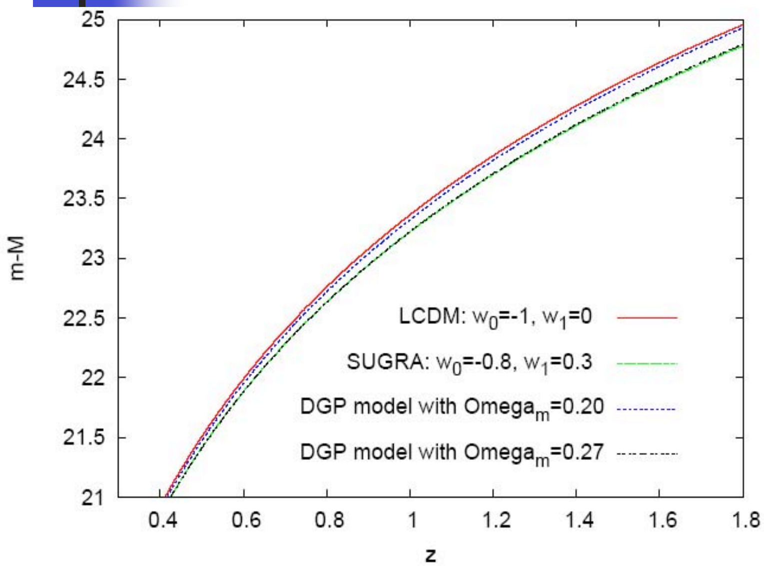
- And the growth rate of function is given by

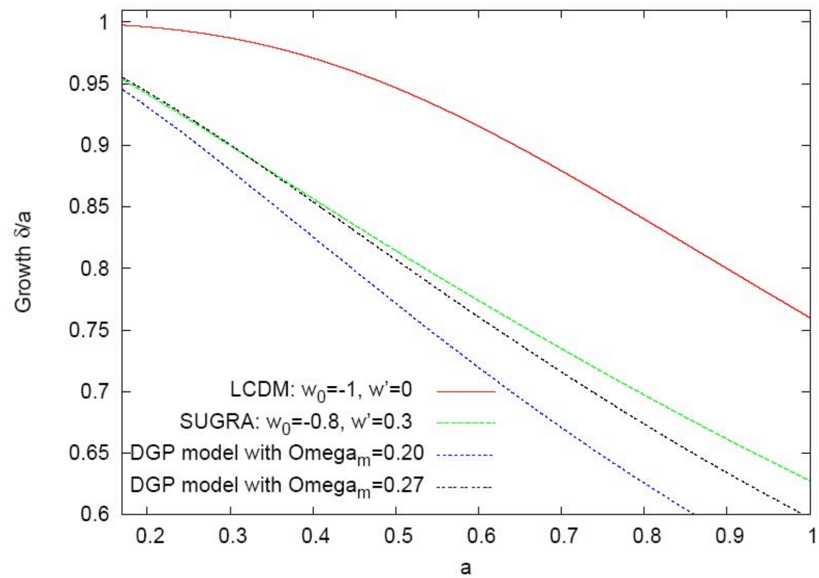
$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0 \qquad \beta = 1 - 2r_c H\left(1 + \frac{\dot{H}}{3H^2}\right) \qquad (4)$$

- Equation (1) and (2) must be mathematically consistent one with another via General Relativity. Similarly, equation (3) and (4) must be consistent one with another via DGP theory

- Our approach uses cosmological probes in order to detect inconsistencies between equations (1) and (2).

Dark Energy versus DGP model Hubble diagrams and growth function

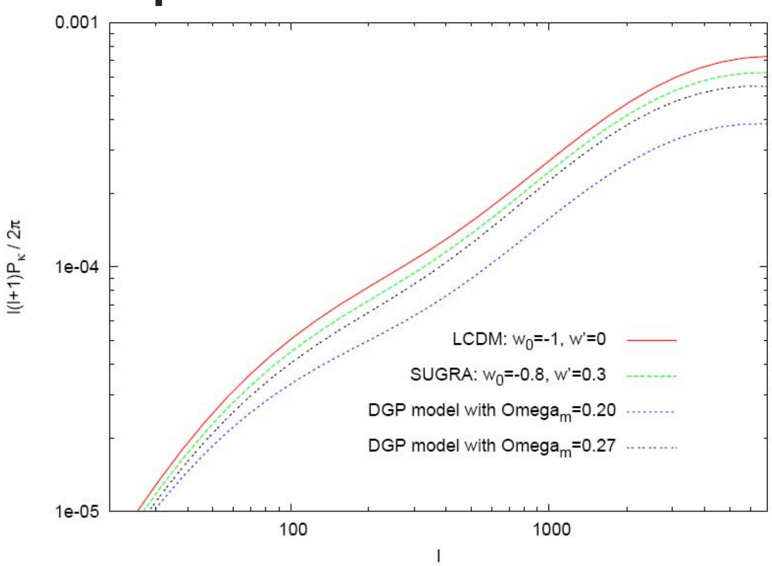


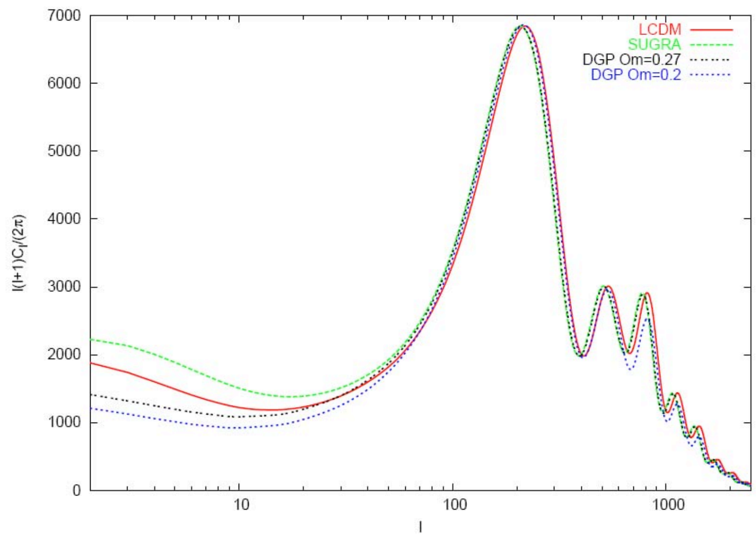


Supernova Hubble diagrams for several dark energy and DGP models. Note that the LCDM model (red solid line) and the _m=0.20 DGP model (blue dotted) have nearly identical Hubble diagrams, but different growth factors as shown in Fig. 1b. The same is true of the SUGRA (green dashed) and _m =0.27 DGP (black double dotted) models.

Growth factor of linear density perturbations for several dark energy and DGP models. Note that the growth factor in the _m=0.27 DGP model is suppressed with respect to that in the LCDM model, which has the same _m.

Dark Energy versus DGP models lensing and CMB Spectra





Lensing convergence power spectra for several dark energy and DGP models: LCDM model is in red solid line; SUGRA model is in green dashed line; _m =0.27 DGP model is in black double dotted line; _m =0.20 DGP model is in blue dotted line;

IMPORTANT POINT: the lensing power spectrum contains information about the growth rate of LSS while the supernova luminosity-distance does not

CMB power spectra for several dark energy and DGP models: LambdaCDM model is in red solid line; SUGRA model is in green dashed line; _m =0.27 DGP model is in black double dotted line; _m=0.20 DGP model is in blue dotted line; We modified the homogeneous evolution in CMBFAST (Zaldarriaga and Seljak 2000) for DGP models

The Cosmological Experiments simulated

- We used 2000 supernovae with maximum redshift 1.7 (and included some of the systematic limits)
- A weak gravitational survey covering 10% of the sky
- 10 weak lensing tomography bins
- CMB experiment: 1 year of data from PLANCK

Implementing the procedure using simulated cosmological observations from near future experiments

- We assume that the true cosmology is that of modified gravity DGP model and generate Supernova magnitudes, Weak Lensing convergence power spectrum, and CMB temperature power spectrum.

- We determine the effective Dark Energy model from the best fit to the supernove data and the CMB power spectrum (we use pairs including the CMB in order to break degeneracies among the model parameters). The pair [Supernova+CMB] probes the expansion history

- This gives a first effective Dark Energy parameter space I: e.g. $\{\Omega_{de}, w_0, w_1\}$

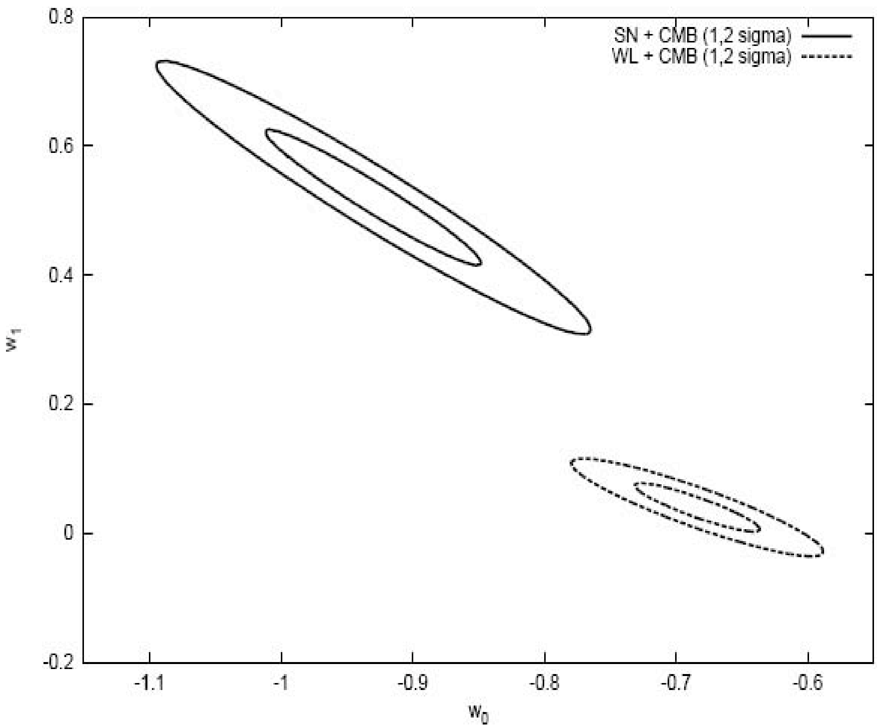
- We determine the effective Dark Energy model from the best fit to the weak lensing convergence spectrum and the CMB power spectrum. The pair [Gravitational Lensing+CMB] probes the growth.

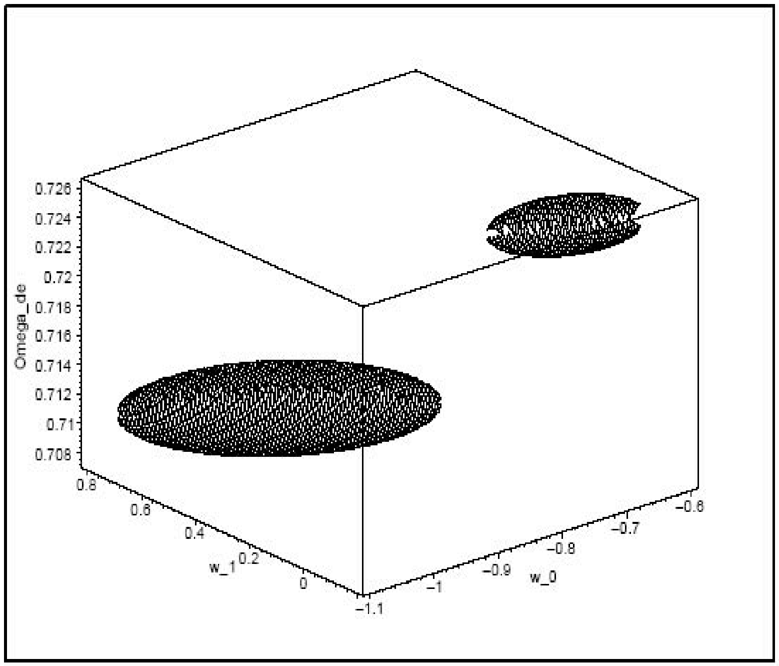
- This gives a second effective Dark Energy parameter space II

- We compare the two effective dark energy parameter spaces to look for inconsistencies within the precision of the combinations of the cosmological probes used

- A significant inconsistency between the dark energy parameter spaces will be a signature of the underlying modified gravity model

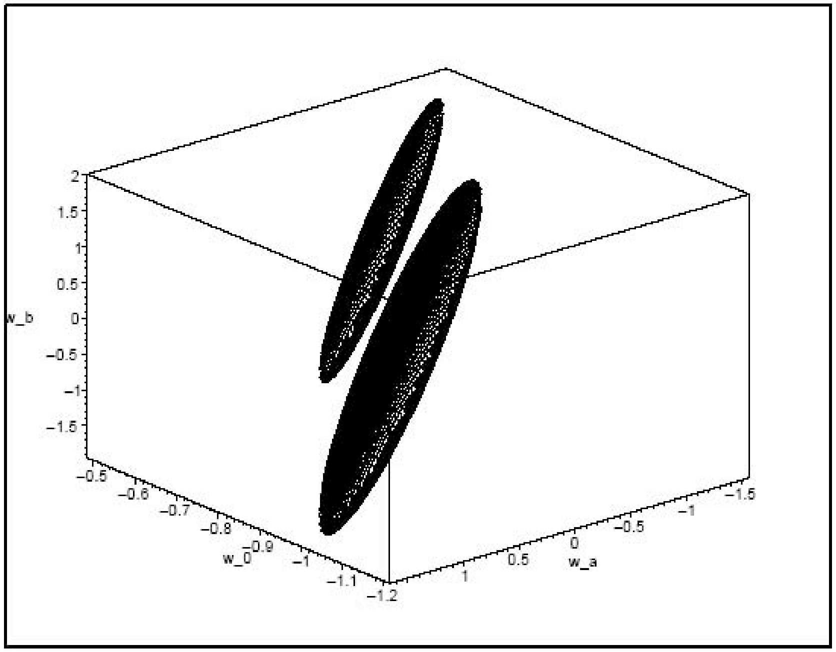
Results: An inconsistency between the two DE parameter spaces. A clear signature of the 5 D modified gravity that we assumed to generate the data





The fact that these dark energy parameter spaces are significantly distinct is an observational signature of the modified gravity model assumed and shows our ability to detect it using future experiments

Equations of state with 3 terms in the Taylor expansion



The inconsistency between the two equations of state persists even when we consider a third term in the Taylor expansion of the equation of state

Discussion, conclusion, and future work

- The observed inconsistency in the figure is a consequence of our hypothesis that the true cosmological model is a modified gravity DGP model
- Thus, the inconsistency constitute an observational detection of the assumed underlying modified gravity model
- Finding two significantly different equations of state implies that these are not true EOSs but “forced” ones
- Last figure shows that the inconsistency between the two equations of state persists even when we consider a third term in the Taylor expansion of the equation of state. Robust to the functional form of the EOS
- Future work is needed in order to make these tests more generic and reliable: To consider other dark energy models (with couplings, unusual sound speeds), other modified gravity models, comparison with systematic effects of the of the probes.
- The procedure is based on the comparison of measurements of the expansion history and measurements of the growth rate of large scale structure and shows that we can go beyond the equation of state analysis. Weak gravitational lensing is very promising for that (LSST, JDEM/SNAP)
- The procedure allows one to distinguish between some dark energy models and modified gravity models. Being able to distinguish between the two possibilities is and important step in the quest to understand cosmic acceleration

DETF final report: section III 2a) on page 7.

III. Goals and Methodology for Studying Dark Energy

1. The goal is to determine the very nature of the dark energy that causes the Universe to accelerate and seems to comprise most of the mass-energy of the Universe.

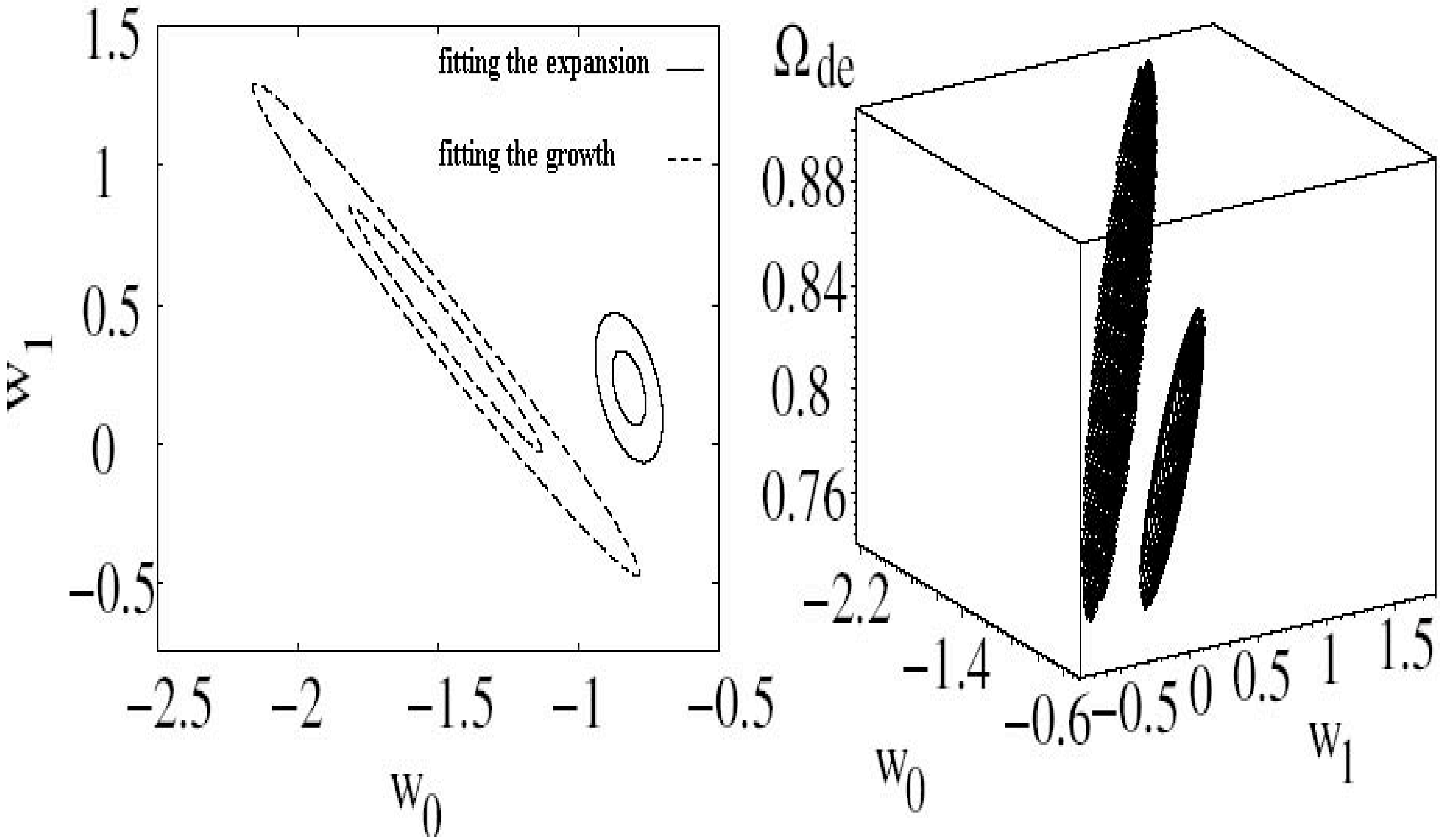
2. Toward this goal, our observational program must
   a. Determine as well as possible whether the accelerating expansion is consistent with being due to a cosmological constant.
   b. If the acceleration is not due to a cosmological constant, probe the underlying dynamics by measuring as well as possible the time evolution of the dark energy by determining the function $w(a)$.
   c. Search for a possible failure of general relativity through comparison of the effect of dark energy on cosmic expansion with the effect of dark energy on the growth of cosmological structures like galaxies or galaxy clusters.

3. Since $w(a)$ is a continuous function with an infinite number of values at infinitesimally separated points, $w(a)$ must be modeled using just a few parameters whose values are determined by fitting to observations. No single

Procedure proposed to find an inconsistency

- Assume that the true universe is described by a modified gravity model (for example, we used the DGP model).
- So in this case, the true expansion history and growth rate functions are those of a DGP model.
- However, assume that the cosmological data are analyzed using dark energy models instead
- Determine the effective Dark Energy model from the best fit to measurements of the expansion history
- This gives a first effective Dark Energy parameter space I: $\{\Omega_{de}, w_0, w_1\}$
- Determine the effective Dark Energy model from the best fit to measurements of the growth rate function
- This gives a second effective Dark Energy parameter space II
- Compare the two effective parameter spaces to look for inconsistencies
- A significant inconsistency between the dark energy parameter spaces will be a signature of the underlying modified gravity model

A first pedagogical example, and it works! A clear inconsistency



The distortion matrix

Light rays traveling to us from background galaxies get deflected by mass fluctuations in large scale structures. This results in distortions of the sizes and shapes of these galaxies that can be described by the transformation matrix

$$A_{ij} \equiv \frac{\partial \theta_s^i}{\partial \theta^j} = \begin{pmatrix} 1-\kappa-\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \tag{6}$$

where θ_s is the angular position in the source plane; θ is the angular position in the image plane; κ is the convergence and describes the magnification of the size of the image; γ_1 and γ_2 are the components of the complex shear and describe the distortion of the shape of the image. In the weak gravitational lensing limit, $|\kappa|, |\gamma| \ll 1$.

The convergence power spectrum

The convergence is given by a weighted projection of the matter energy density fluctuations $\delta \equiv \delta\rho/\rho$ along the line of sight between the source and the observer,

$$\kappa(\hat{\theta}) = \int_0^{x_H} W(\chi)\delta(\chi, \chi\hat{\theta})d\chi \qquad (7)$$

The convergence scalar field can be decomposed into multipole moments of the spherical harmonics as

$$\kappa(\hat{\theta}) = \sum_{lm} \kappa_{lm} Y_l^m(\hat{\theta}),$$

where

$$\kappa_{lm} = \int d\hat{\theta}\kappa(\hat{\theta}, \chi)Y_l^{m*}(\hat{\theta}).$$

The convergence power spectrum P_l^κ is then defined by

$$\langle \kappa_{lm}\kappa_{l'm'} \rangle = \delta_{ll'}\delta_{mm'}P_l^\kappa$$

and we will use it as our weak lensing statistic.