Limits on $\gamma$-ray Lines from Unitarity

Can Kılıç, UT Austin

in collaboration with

K. Abazajian, P. Agrawal, Z. Chacko
Beacon in the Dark

- Indirect detection at astrophysical distances: gammas are best.
- Direct annihilation gives monoenergetic photons. Rare.
- Bremsstrahlung and hadronic decays give continuum.
- Potential check on anomalies in other indirect detection channels.

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Minimum strength for line with respect to continuum?
Line Bound From Unitarity

- Strength of line is related to the primary annihilation mode.
- No model independent bound for the full amplitude.
- Imaginary part of loop is much more robust.
- Ratio to continuum also model-independent.
Unitarity

- S matrix is unitary
  \[ S^\dagger S = 1 \]
- S=1+iT
- put in intermediate states
  \[ -i\langle f|(T-T^\dagger)|i\rangle = \sum_m \langle f|T^\dagger|m\rangle\langle m|T|i\rangle \]
- CP
  \[ -2i\text{Im}\langle f|T|i\rangle = \sum_m \langle f|T^\dagger|m\rangle\langle m|T|i\rangle \]
- single channel
  \[ 4|\text{Im}\langle f|T|i\rangle|^2 = |\langle f|T^\dagger|m\rangle|^2 |\langle m|T|i\rangle|^2 \]
Methods

• Use $|J,M;L,S>$ basis.
• Map annihilation into decay process.
• Calculate imaginary part of loop amplitude.
• Bound is

$$\frac{\sigma_{IM}}{\sigma} = \frac{\Gamma_{Im}}{\Gamma}$$

• Can also translate to line / continuum.
Case of Spin-0 Dark Matter

J=0, CP even

chirally suppressed, heavy preferred
CP forces S=1, L=1

spin 1/2

CP allows S=2, L=2
as well as S=0, L=0
latter preferred in non-relativistic limit
Case of Spin-1/2 (Majorana) DM

Spin 1/2

antisymmetry forces S=0, J=0, CP odd

Heavy preferred CP forces S=0, L=0

Spin 1

CP allows S=1, L=1 only

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Case of Spin-1/2 (Dirac) DM

J can be 0, 1

take conservative case?

Landau-Yang theorem

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Case of Spin-1 (real) DM

- Spin 1/2
  - Symmetry forces $J=0, 2$
  - $J=0$ already covered
  - Light is now OK.
  - CP forces $S=1$
  - $L$ can be \{1, 2, 3\}
- Spin 1
  - $S=0, L=2$ or
  - $S=2, L=\{0, 1, 2, 3, 4\}$

Bound only in kinematic limits

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<table>
<thead>
<tr>
<th>Dark Matter</th>
<th>Initial spin</th>
<th>Annihilation Channel</th>
<th>Mode</th>
<th>Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar</td>
<td>$J = 0$</td>
<td>$WW$</td>
<td>$L = 0, S = 0$</td>
<td>In NR / UR limits.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L = 2, S = 2$</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>$f \bar{f}$</td>
<td>$L = 1, S = 1$</td>
<td>✓</td>
</tr>
<tr>
<td>Majorana Fermion</td>
<td>$J = 0$</td>
<td>$WW$</td>
<td>$L = 1, S = 1$</td>
<td>✓</td>
</tr>
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<td></td>
<td></td>
<td>$f \bar{f}$</td>
<td>$L = 0, S = 0$</td>
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</tr>
<tr>
<td>Dirac Fermion</td>
<td>$J = 0$</td>
<td>$WW$</td>
<td>$L = 1, S = 1$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>$f \bar{f}$</td>
<td>$L = 0, S = 0$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>$J = 1$</td>
<td></td>
<td></td>
<td>Forbidden</td>
</tr>
<tr>
<td>Real Vector Boson</td>
<td>$J = 0$</td>
<td>$WW$</td>
<td>$L = 0, S = 0$</td>
<td>In NR / UR limits.</td>
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<td>$L = 0, S = 0$</td>
<td>✓</td>
</tr>
<tr>
<td></td>
<td>$J = 2$</td>
<td>$WW$</td>
<td>$L = 2, S = 0$</td>
<td>In NR limit.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L = {0, 1, 2, 3, 4}, S = 2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$f \bar{f}$</td>
<td>$L = {1, 2, 3}, S = 1$</td>
<td>In NR / UR limits.</td>
</tr>
</tbody>
</table>
Results – Scalar DM

Can be represented as decay of heavy scalar.

To fermions:

$$\mathcal{L}_{int} = \lambda \bar{f} f \phi$$

$$\frac{\Gamma_{\text{Im}}(\phi \to \gamma \gamma)}{\Gamma(\phi \to f \bar{f})} = \frac{N_c Q^4 e^4 m_f^2}{32\pi^2 m_\chi^2} \beta \left[ \tanh^{-1} \beta \right]^2$$

To W’s:

$$\mathcal{L}_{int} = \frac{1}{\Lambda} \phi \text{Tr} [F_{\mu\nu} F^{\mu\nu}]$$

$$\frac{\Gamma_{\text{Im}}(\phi \to \gamma \gamma)}{\Gamma(\phi \to WW)} = \frac{3 e^4}{64\pi^2 \beta} \quad \text{(NR)}$$

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Results – Scalar DM

To W’s, ultra-relativistic regime

Ultra-relativistic: Use equivalence theorem to separate transverse and longitudinal modes.

• Longitudinal state is unique.

\[ \mathcal{L}_{\text{int}} = \alpha \phi \, H^\dagger H \]

\[ \frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma \gamma)}{\Gamma(\phi \rightarrow WW)} \sim \frac{e^4}{16\pi^2} \frac{m_W^4}{m_\chi^4} \left[ \log \left( \frac{4m_\chi^2}{m_W^2} \right) \right]^2 \]

• Transverse state as well, once CP is taken into account.

\[ \mathcal{L}_{\text{int}} = \frac{1}{\Lambda} \phi \, \text{Tr} \left[ F_{\mu\nu} F^{\mu\nu} \right] \]

\[ \frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma \gamma)}{\Gamma(\phi \rightarrow WW)} = \frac{e^4}{32\pi^2} \left[ \log \left( \frac{4m_\chi^2}{m_W^2} \right) \right]^2 \]

• Combine:

\[ \frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma \gamma)}{\Gamma(\phi \rightarrow WW)} = F_T \frac{e^4}{32\pi^2} \left[ \log \left( \frac{4m_\chi^2}{m_W^2} \right) \right]^2 \]

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Results – Majorana Fermion DM

Can be represented as decay of heavy pseudoscalar.

To fermions:

\[
L_{\text{int}} = i \lambda \bar{f} \gamma^5 f \varphi
\]

\[
\frac{\Gamma_{\text{Im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow f\bar{f})} = \frac{N_c Q^4 e^4 m_f^2}{32\pi^2 m_\chi^2} \frac{1}{\beta} [\tanh^{-1} \beta]^2
\]

To W’s:

\[
L_{\text{int}} = \frac{1}{\Lambda} \varphi \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})
\]

\[
\frac{\Gamma_{\text{Im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow WW)} = \frac{e^4}{8\pi^2 \beta} [\tanh^{-1} \beta]^2
\]

both cases consistent with known SUSY results.
Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)
Can be represented as decay of heavy spin-2 particle

To fermions: Non-relativistic limit. Single species assumed.

\[ \mathcal{L}_{int} = -\frac{\kappa}{2} h^{\mu\nu} \bar{f} i\gamma_\mu \partial_\nu f \]

\[ \frac{\Gamma_{\text{Im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow f\bar{f})} \bigg|_{J=2} = \frac{N_c Q^4 e^4 \beta^3}{120\pi^2} \]

p-wave, weak limit
Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)
Can be represented as decay of heavy spin-2 particle

To fermions: Ultra-relativistic limit.
If there are multiple final states and no phases, then bound still applies.

\[ \mathcal{L}_{\text{int}} = -\frac{\kappa}{2} h^{\mu\nu} \bar{f} i\sigma_\mu \partial_\nu f \]

\[ \frac{\Gamma_{\text{Im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow f\bar{f})} \bigg|_{J=2} = \frac{N_f N_c Q^4 e^4}{144\pi^2} \]

J=0 suppressed. Bound applies.
Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)
Can be represented as decay of heavy spin-2 particle

To W’s : Non-relativistic limit.

\[ \mathcal{L}_{\text{int}} = \frac{\kappa}{2} h^{\mu\nu} \left( \left[ (\partial_\mu W^{\mu+\rho} - \partial^\rho W^{+}_\mu)(\partial_\nu W^{-}_\rho - \partial_\rho W^{-}_\nu) \right. \right. \\
\left. \left. - m_W^2 W^{+}_\mu W^{-}_\nu \right] + \mu \leftrightarrow \nu \right) \]

\[ \frac{\Gamma_{1\text{m}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow WW)} \bigg|_{J=2} = \frac{e^4}{20\pi^2} \beta \]

J=0 bound applies
(More conservative)
Comparison With Known Cases

SUSY

\[ \chi \chi \rightarrow WW \]
Comparison With Known Cases

Little Higgs
\( \chi \chi \rightarrow (h) \rightarrow tt,WW \)
Comparison With Known Cases

UED

\( \chi \chi \rightarrow ff \)
Comparison with Continuum Bound

For Lines: \[
\frac{d\Phi}{dE} = \frac{\langle \sigma_A v \rangle}{8\pi m^2_c} \frac{J}{J_0} \frac{dN}{dE}
\]
where \[
\frac{dN}{dE} = 2\delta(E_\gamma - m_\chi)
\]

Search region includes caps \[|b| > 10^\circ\]

and Galactic center \[|b| < 10^\circ \quad |\ell| < 10^\circ\]

Choose Einasto DM profile with parameters to minimize signal

specifically \[
\rho_{\text{Einasto}}(r) = \rho_s \exp\left[-\frac{2}{\alpha} \left(\left[\frac{r}{r_s}\right]^\alpha - 1\right)\right]
\]
\[
\alpha = 0.22 \quad r_s = 21 \text{ kpc} \quad \rho_\odot = 0.385 \text{ GeV cm}^{-3}
\]
\[
r_\odot = 8.28 \text{ kpc}
\]

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Comparison with Continuum Bound

For Continuum:
Isotropic Diffuse Gamma Rays
(Galactic + Extragalactic)
Conservative, dwarf galaxy limits could be an order of magnitude stronger.
Conservative boost factor (2.3)

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Conclusions

• Robust bounds obtained for gamma ray lines from DM annihilation through unitarity considerations.
• Minimal line bounds apply to some cases without restriction and to most other cases in certain kinematic regimes.
• In selected cases, the minimal bounds are found to be an O(1) fraction of the full cross section.
• Experimental limits are less stringent than continuum limits, but useful to identify when full calculation is important.
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