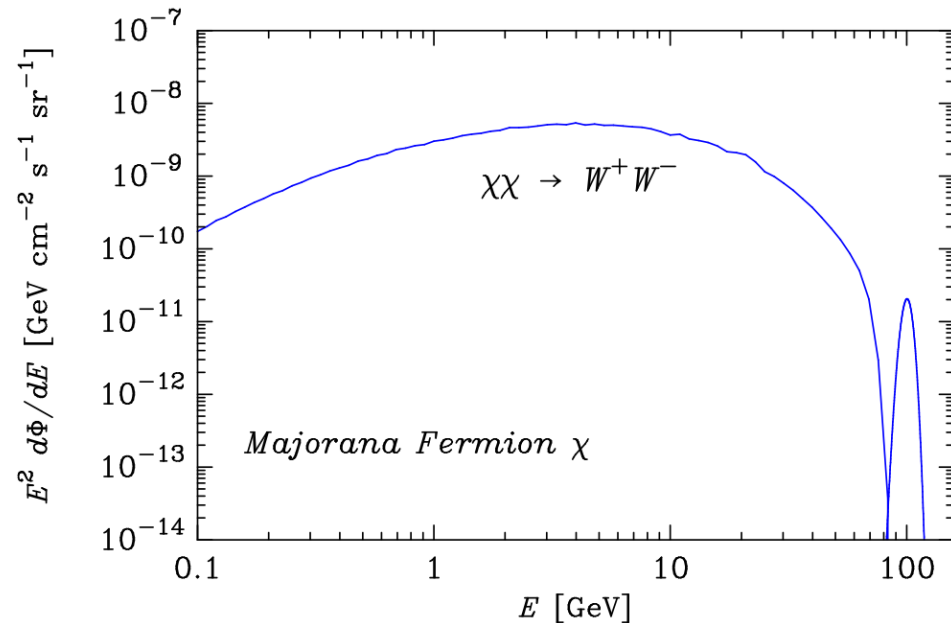
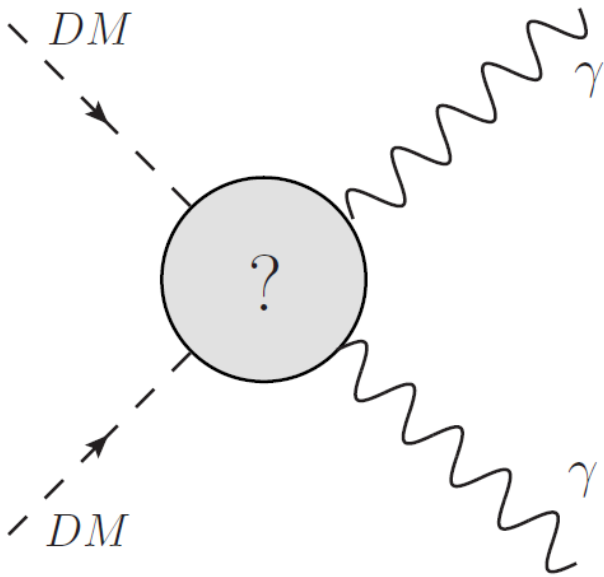


Limits on γ -ray Lines from Unitarity



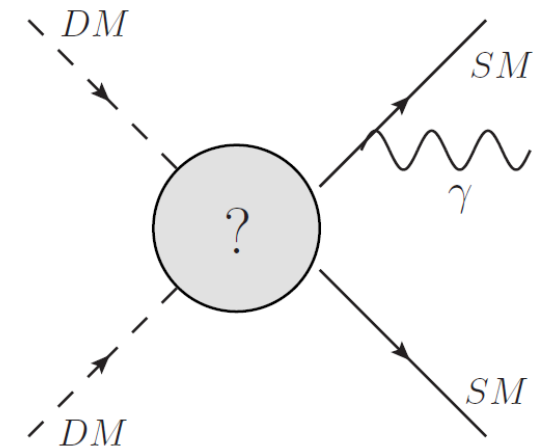
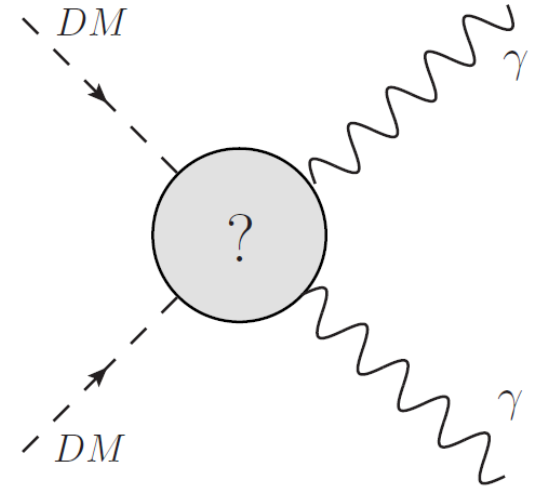
Can Kılıç, UT Austin

in collaboration with

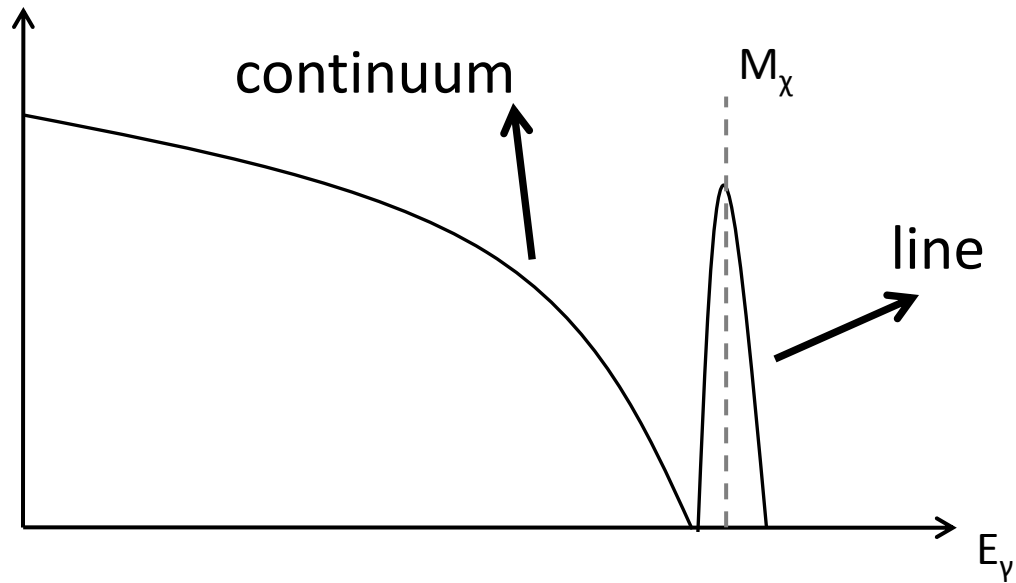
K. Abazajian, P. Agrawal, Z. Chacko

Beacon in the Dark

- Indirect detection at astrophysical distances: gammas are best.
- Direct annihilation gives monoenergetic photons. Rare.
- Bremsstrahlung and hadronic decays give continuum.
- Potential check on anomalies in other indirect detection channels.



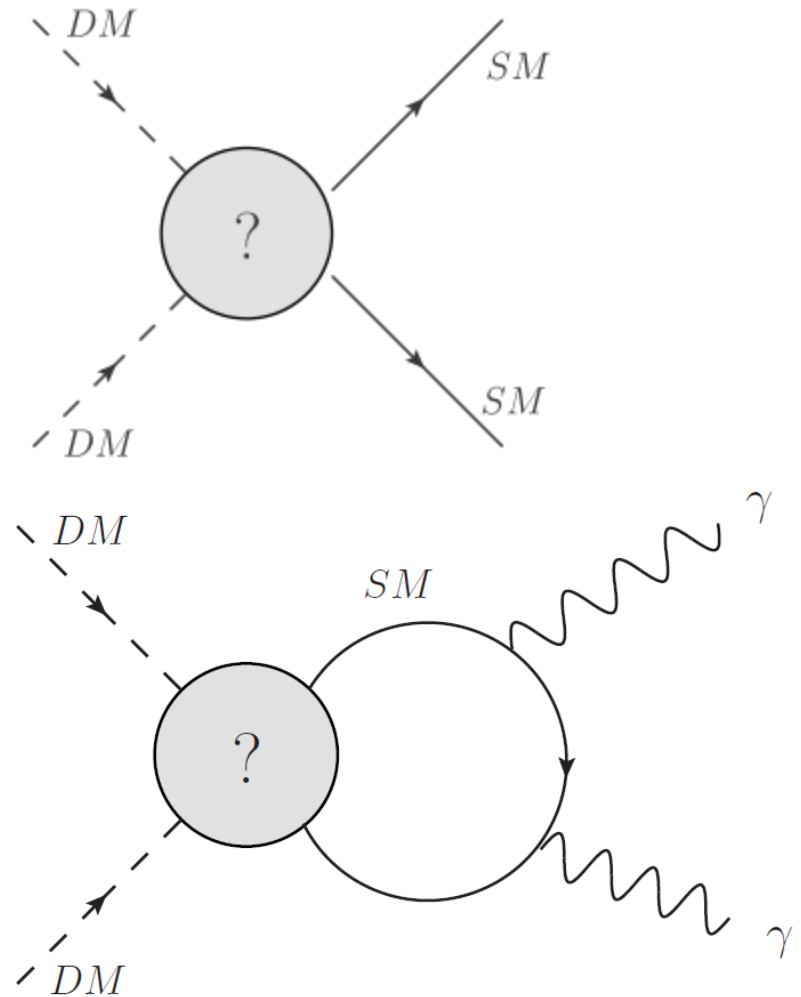
Line and Continuum



Minimum strength for line
with respect to continuum?

Line Bound From Unitarity

- Strength of line is related to the primary annihilation mode.
- No model independent bound for the full amplitude.
- Imaginary part of loop is much more robust.
- Ratio to continuum also model-independent.



Unitarity

- S matrix is unitary

$$S^\dagger S = 1$$

- $S=1+iT$

$$-i(T - T^\dagger) = T^\dagger T$$

- put in intermediate states

$$-i\langle f|(T - T^\dagger)|i\rangle = \sum_m \langle f|T^\dagger|m\rangle \langle m|T|i\rangle$$

Y DM SM

- CP

$$-2i\text{Im}\langle f|T|i\rangle = \sum_m \langle f|T^\dagger|m\rangle \langle m|T|i\rangle$$

- single channel

$$4|\text{Im}\langle f|T|i\rangle|^2 = |\langle f|T^\dagger|m\rangle|^2 |\langle m|T|i\rangle|^2$$

Methods

- Use $|J,M;L,S\rangle$ basis.
- Map annihilation into decay process.
- Calculate imaginary part of loop amplitude.
- Bound is

$$\frac{\sigma_{IM} \left(\begin{array}{c} \chi \\ \chi \end{array} \begin{array}{c} \bar{X} \\ X \end{array} \begin{array}{c} \gamma \\ \gamma \end{array} \right)}{\sigma \left(\begin{array}{c} \chi \\ \chi \end{array} \begin{array}{c} \bar{X} \\ X \end{array} \right)} = \frac{\Gamma_{Im} \left(\begin{array}{c} \Phi \\ \Phi \end{array} \begin{array}{c} \bar{X} \\ X \end{array} \begin{array}{c} \gamma \\ \gamma \end{array} \right)}{\Gamma \left(\begin{array}{c} \Phi \\ \Phi \end{array} \begin{array}{c} \bar{X} \\ X \end{array} \right)}$$

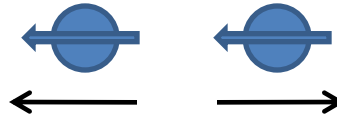
- Can also translate to line / continuum.

Case of Spin-0 Dark Matter



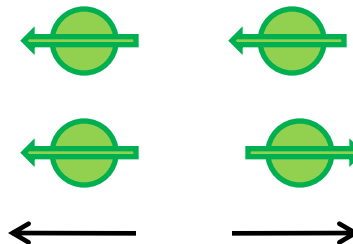
$J=0$, CP even

spin 1/2



chirally suppressed,
heavy preferred
CP forces $S=1$, $L=1$

spin 1



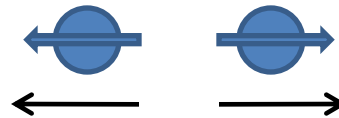
CP allows $S=2$, $L=2$
as well as $S=0$, $L=0$
latter preferred in
non-relativistic limit

Case of Spin-1/2 (Majorana) DM



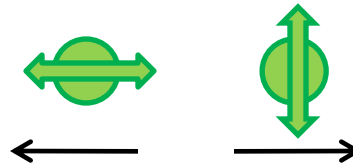
antisymmetry forces
 $S=0, J=0, CP$ odd

spin 1/2



heavy preferred
CP forces $S=0, L=0$

spin 1



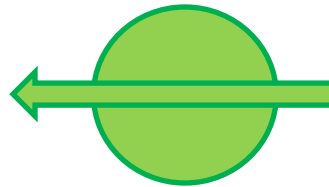
CP allows $S=1, L=1$ only

Case of Spin-1/2 (Dirac) DM

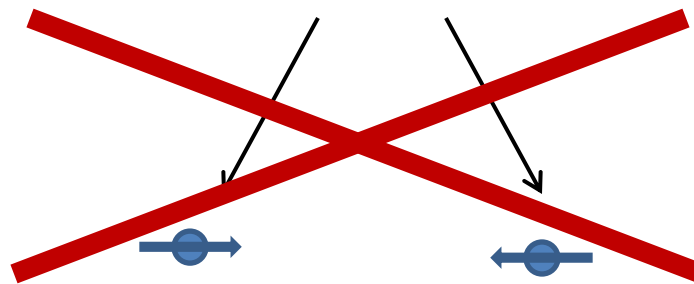


J can be 0,1

take conservative case?



Landau-Yang theorem

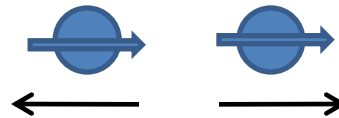


Case of Spin-1 (real) DM



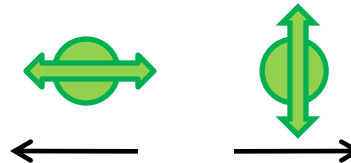
symmetry forces $J=0,2$
 $J=0$ already covered

spin 1/2



light is now OK.
CP forces $S=1$
 L can be $\{1,2,3\}$

spin 1



$S=0, L=2$ or
 $S=2, L=\{0,1,2,3,4\}$

bound only in kinematic limits

Summary of Results

Dark Matter	Initial spin	Annihilation		Bound
		Channel	Mode	
Scalar	$J = 0$	WW	$L = 0, S = 0$ $L = 2, S = 2$	In NR / UR limits.
		$f\bar{f}$	$L = 1, S = 1$	✓
Majorana Fermion	$J = 0$	WW	$L = 1, S = 1$	✓
		$f\bar{f}$	$L = 0, S = 0$	✓
Dirac Fermion	$J = 0$	WW	$L = 1, S = 1$	✓
		$f\bar{f}$	$L = 0, S = 0$	✓
	$J = 1$	Forbidden		
Real Vector Boson	$J = 0$	WW	$L = 0, S = 0$ $L = 2, S = 2$	In NR / UR limits.
		$f\bar{f}$	$L = 0, S = 0$	✓
	$J = 2$	WW	$L = 2, S = 0$ $L = \{0, 1, 2, 3, 4\}, S = 2$	In NR limit.
		$f\bar{f}$	$L = \{1, 2, 3\}, S = 1$	In NR / UR limits.

Results – Scalar DM

Can be represented as decay of heavy scalar.

To fermions : $\mathcal{L}_{int} = \lambda \bar{f} f \phi$

$$\frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma\gamma)}{\Gamma(\phi \rightarrow f\bar{f})} = \frac{N_c Q^4 e^4 m_f^2}{32\pi^2 m_\chi^2} \beta [\tanh^{-1} \beta]^2$$

To W's : $\mathcal{L}_{int} = \frac{1}{\Lambda} \phi \text{Tr} [F_{\mu\nu} F^{\mu\nu}]$

$$\frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma\gamma)}{\Gamma(\phi \rightarrow WW)} = \frac{3e^4}{64\pi^2} \beta \quad (\text{NR})$$

Results – Scalar DM

To W 's, ultra-relativistic regime

Ultra-relativistic: Use equivalence theorem to separate transverse and longitudinal modes.

- Longitudinal state is unique.

$$\mathcal{L}_{int} = \alpha \phi H^\dagger H \qquad \frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma\gamma)}{\Gamma(\phi \rightarrow WW)} \sim \frac{e^4}{16\pi^2} \frac{m_W^4}{m_\chi^4} \left[\log \left(\frac{4m_\chi^2}{m_W^2} \right) \right]^2$$

- Transverse state as well, once CP is taken into account.

$$\mathcal{L}_{int} = \frac{1}{\Lambda} \phi \text{Tr} [F_{\mu\nu} F^{\mu\nu}] \qquad \frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma\gamma)}{\Gamma(\phi \rightarrow WW)} = \frac{e^4}{32\pi^2} \left[\log \left(\frac{4m_\chi^2}{m_W^2} \right) \right]^2$$

- Combine:
$$\frac{\Gamma_{\text{Im}}(\phi \rightarrow \gamma\gamma)}{\Gamma(\phi \rightarrow WW)} = F_{\text{T}} \frac{e^4}{32\pi^2} \left[\log \left(\frac{4m_\chi^2}{m_W^2} \right) \right]^2$$

Results – Majorana Fermion DM

Can be represented as decay of heavy pseudoscalar.

To fermions : $\mathcal{L}_{int} = i\lambda \bar{f} \gamma^5 f \varphi$

$$\frac{\Gamma_{\text{Im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow f\bar{f})} = \frac{N_c Q^4 e^4 m_f^2}{32\pi^2 m_\chi^2} \frac{1}{\beta} [\tanh^{-1} \beta]^2$$

To W's : $\mathcal{L}_{int} = \frac{1}{\Lambda} \varphi \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})$

$$\frac{\Gamma_{\text{Im}}(\varphi \rightarrow \gamma\gamma)}{\Gamma(\varphi \rightarrow WW)} = \frac{e^4}{8\pi^2} \beta [\tanh^{-1} \beta]^2$$

both cases consistent with known SUSY results.

Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)

Can be represented as decay of heavy spin-2 particle

To fermions : Non-relativistic limit. Single species assumed.

$$\mathcal{L}_{int} = -\frac{\kappa}{2} h^{\mu\nu} \bar{f} i\gamma_\mu \partial_\nu f$$

$$\frac{\Gamma_{\text{Im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow f\bar{f})} \Big|_{J=2} = \frac{N_c Q^4 e^4 \beta^3}{120\pi^2} \quad \text{p-wave, weak limit}$$

Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)

Can be represented as decay of heavy spin-2 particle

To fermions : Ultra-relativistic limit.

If there are multiple final states and no phases,
then bound still applies.

$$\mathcal{L}_{int} = -\frac{\kappa}{2} h^{\mu\nu} \bar{f} i \bar{\sigma}_\mu \partial_\nu f$$

$$\frac{\Gamma_{\text{Im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow f\bar{f})} \Big|_{J=2} = \frac{N_f N_c Q^4 e^4}{144\pi^2} \quad \text{J=0 suppressed. bound applies.}$$

Results – Real Vector DM

J=0 case already covered, consider J=2
(more conservative bound applies)

Can be represented as decay of heavy spin-2 particle

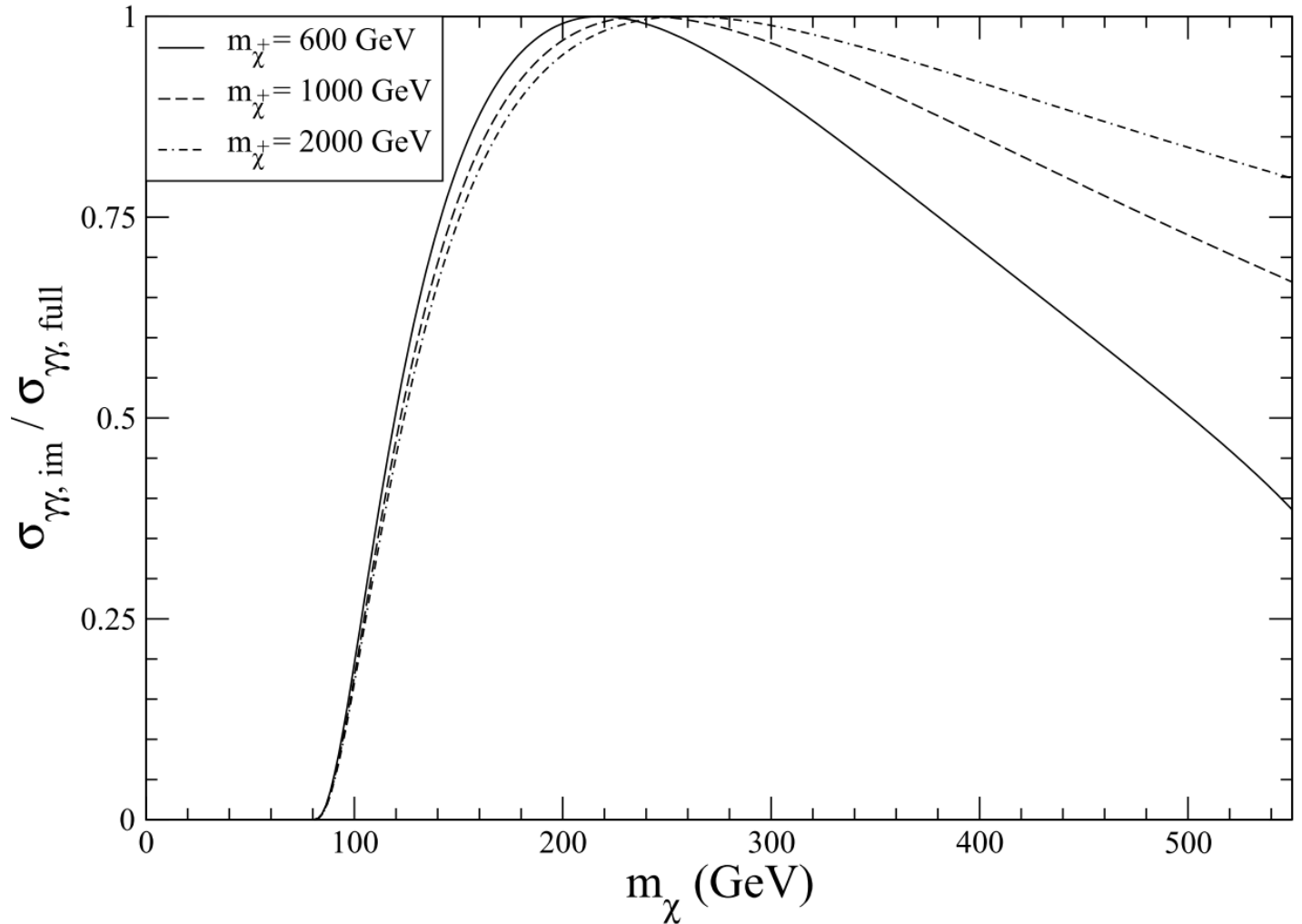
To W's : Non-relativistic limit.

$$\mathcal{L}_{int} = \frac{\kappa}{2} h^{\mu\nu} \left([(\partial_\mu W^{+\rho} - \partial^\rho W_\mu^+)(\partial_\nu W_\rho^- - \partial_\rho W_\nu^-) - m_W^2 W_\mu^+ W_\nu^-] + \mu \leftrightarrow \nu \right)$$

$$\frac{\Gamma_{\text{Im}}(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow WW)} \Big|_{J=2} = \frac{e^4}{20\pi^2} \beta \quad \begin{array}{l} J=0 \text{ bound applies} \\ \text{(More conservative)} \end{array}$$

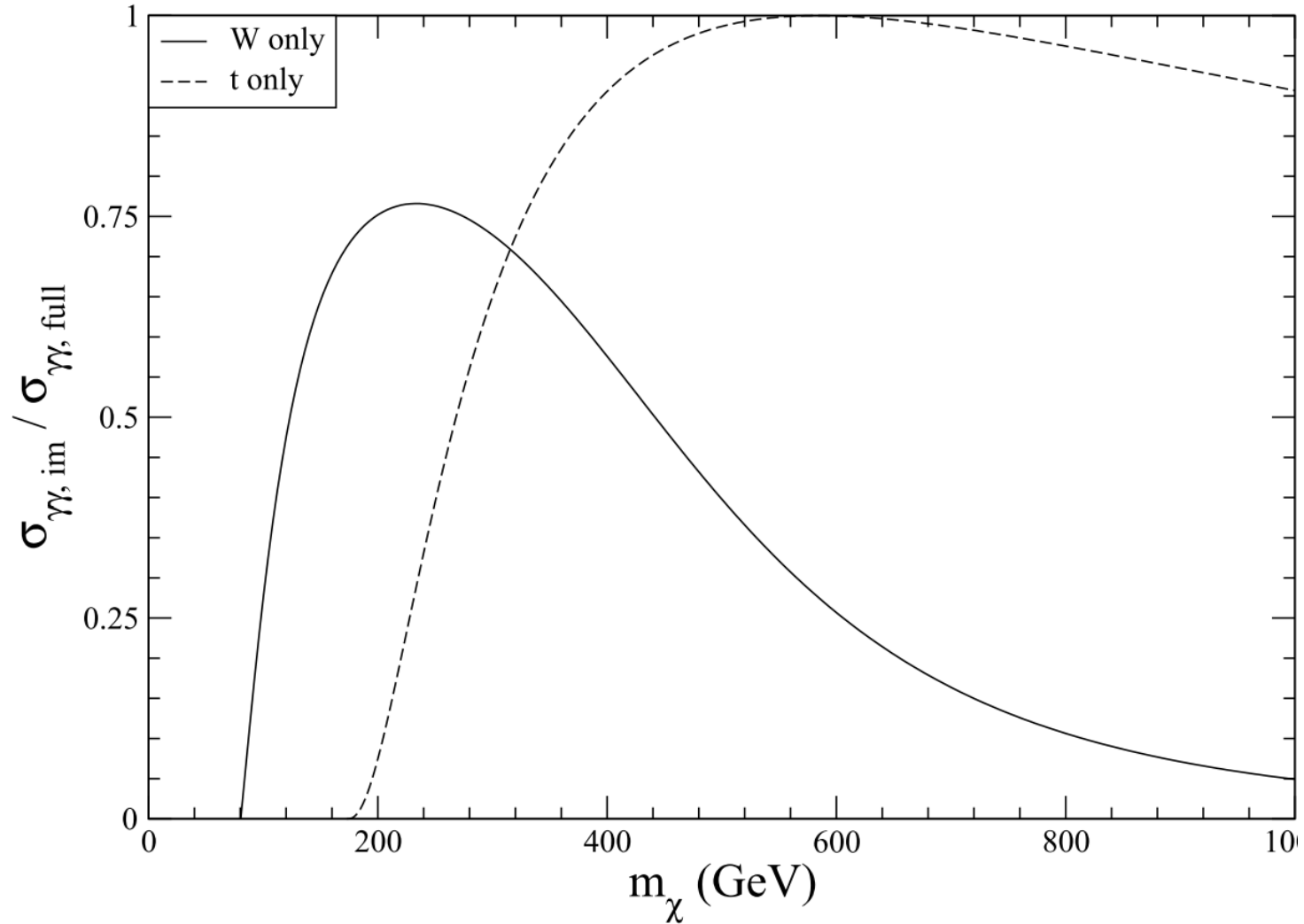
Comparison With Known Cases

SUSY
 $\chi\chi \rightarrow WW$



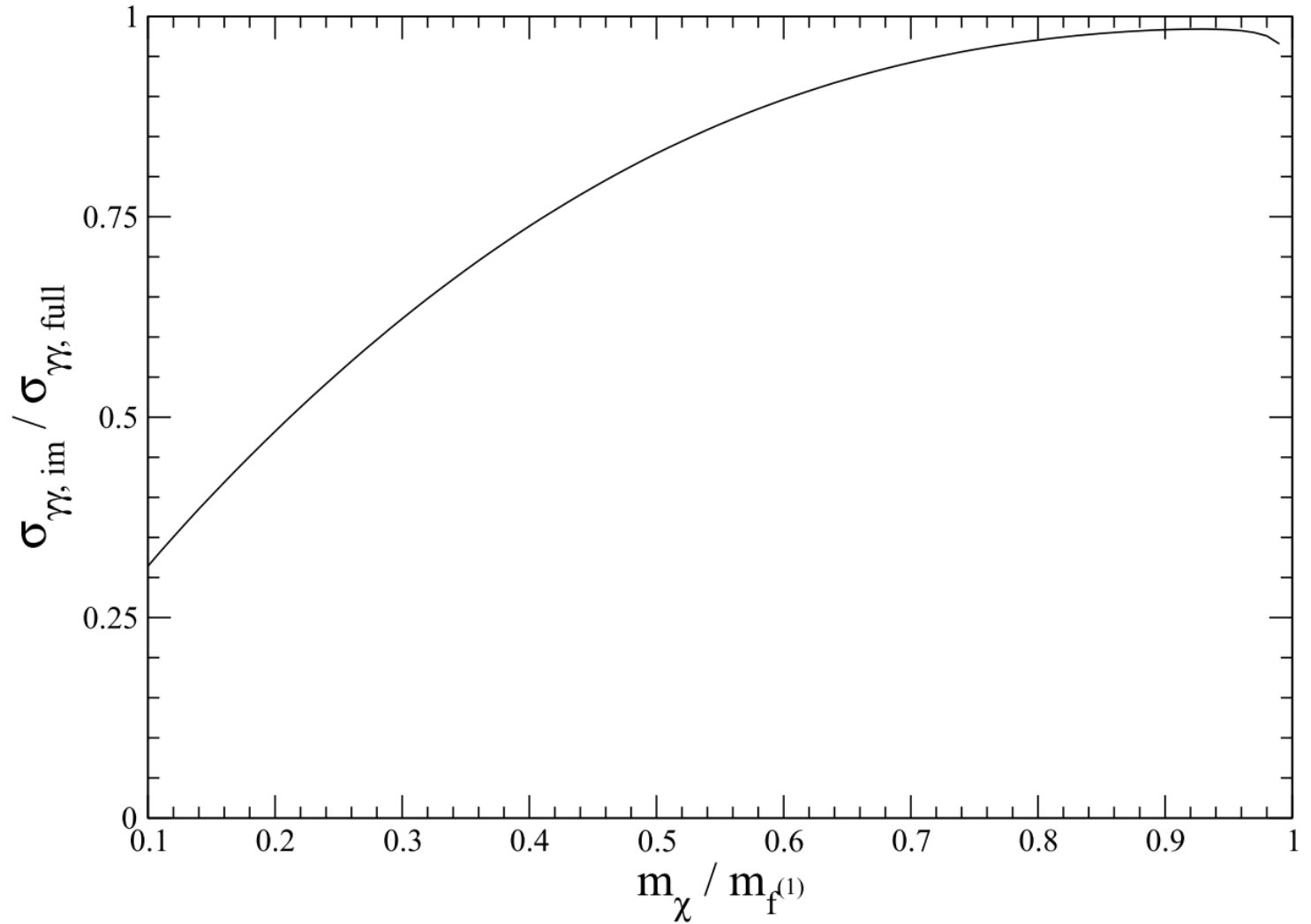
Comparison With Known Cases

Little Higgs
 $\chi\chi \rightarrow (h)$
 $\rightarrow tt, WW$



Comparison With Known Cases

UED
 $\chi\chi \rightarrow f\bar{f}$



Comparison with Continuum Bound

For Lines: $\frac{d\Phi}{dE} = \frac{\langle \sigma_A v \rangle}{8\pi m_\chi^2} \frac{\mathcal{J}}{J_0} \frac{dN}{dE}$ where $\frac{dN}{dE} = 2\delta(E_\gamma - m_\chi)$

Search region includes caps

$$|b| > 10^\circ$$

and Galactic center

$$|b| < 10^\circ \quad |\ell| < 10^\circ$$

Choose Einasto DM profile with parameters to minimize signal

specifically $\rho_{\text{Einasto}}(r) = \rho_s \exp \left[-\frac{2}{\alpha} \left(\left[\frac{r}{r_s} \right]^\alpha - 1 \right) \right]$

$$\begin{aligned} \alpha &= 0.22 \\ r_s &= 21 \text{ kpc} \\ r_\odot &= 8.28 \text{ kpc} \\ \rho_\odot &= 0.385 \text{ GeV cm}^{-3} \end{aligned}$$

Comparison with Continuum Bound

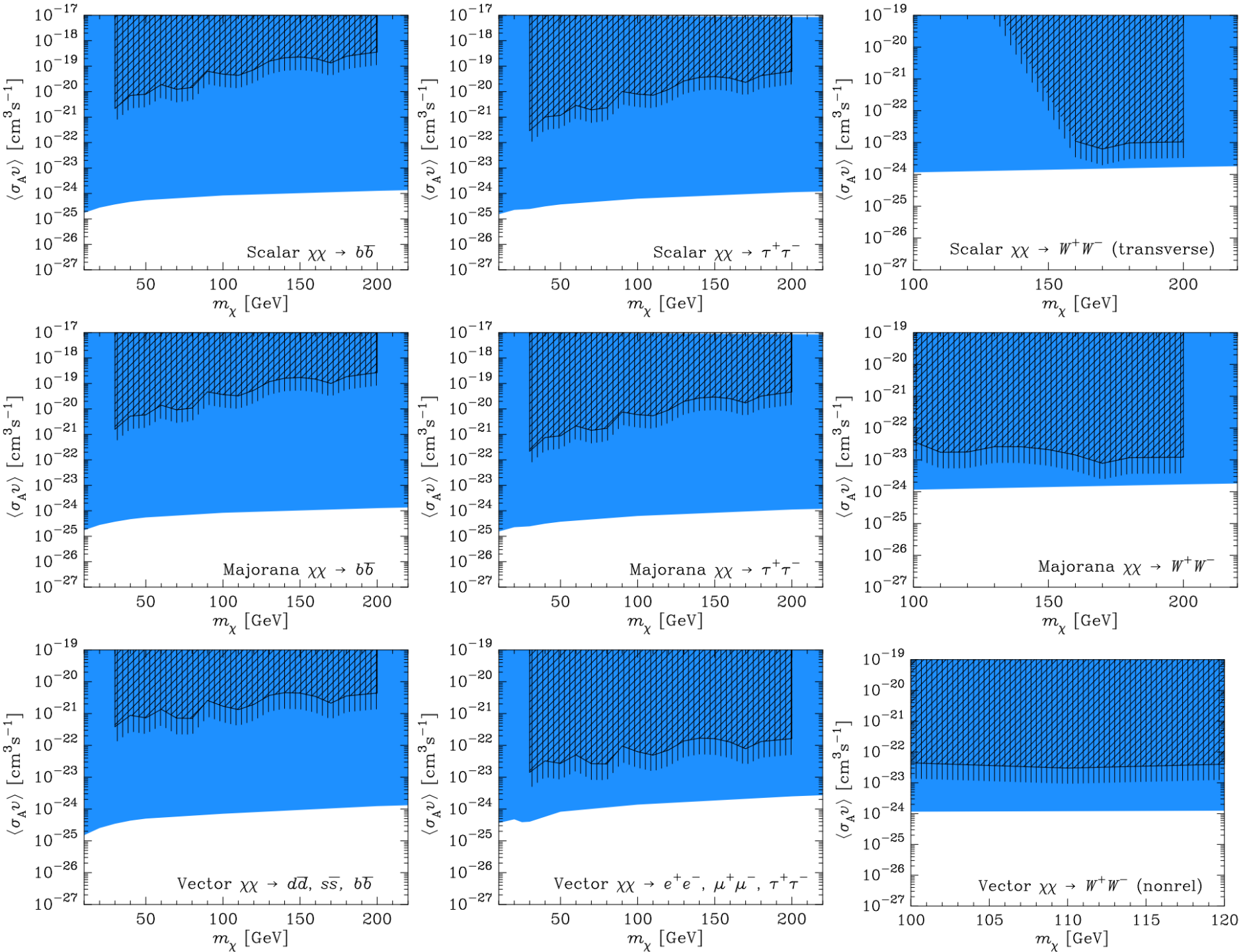
For Continuum:

Isotropic Diffuse Gamma Rays

(Galactic + Extragalactic)

Conservative, dwarf galaxy limits could be an order of magnitude stronger.

Conservative boost factor (2.3)



Conclusions

- Robust bounds obtained for gamma ray lines from DM annihilation through unitarity considerations.
- Minimal line bounds apply to some cases without restriction and to most other cases in certain kinematic regimes.
- In selected cases, the minimal bounds are found to be an $O(1)$ fraction of the full cross section.
- Experimental limits are less stringent than continuum limits, but useful to identify when full calculation is important.

Backup Slides