

LEARNING ABOUT ELLIPTICAL ORBITS

RECOMMENDED FOR GRADES 9-12

To a rough approximation, everything in the sky seems to move in circles around Earth — the Sun, the Moon, the planets, and even the stars. For thousands of years, astronomers tried to model the motion of the planets using circles or combinations of circles — partly because the circle was such a “perfect” shape. In 1543, Polish astronomer Nicholas Copernicus told us that Earth and the other planets actually orbit the Sun, and that the Moon orbits Earth. But he still described these orbits as circular. Then in 1609, German astronomer Johannes Kepler proved that the actual shape of Mars’ orbit is an *ellipse*. It followed that all of the planets follow elliptical orbits around the Sun, with the Sun at one focus point.

Materials

String, 2 push pins, corrugated cardboard (~8”x11”), pencil, ruler, tape, paper

Directions

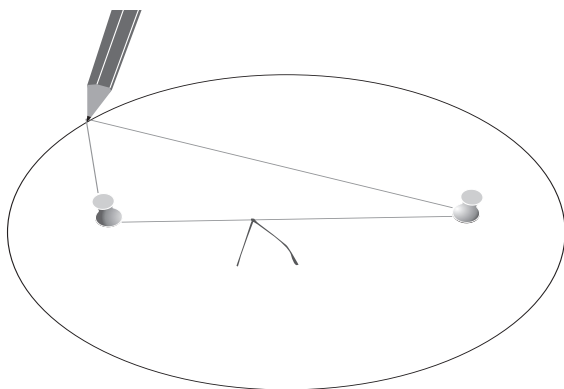
As you do the activity, write your answers for questions A through D on a separate piece of paper that you will turn in to your teacher.

Activity

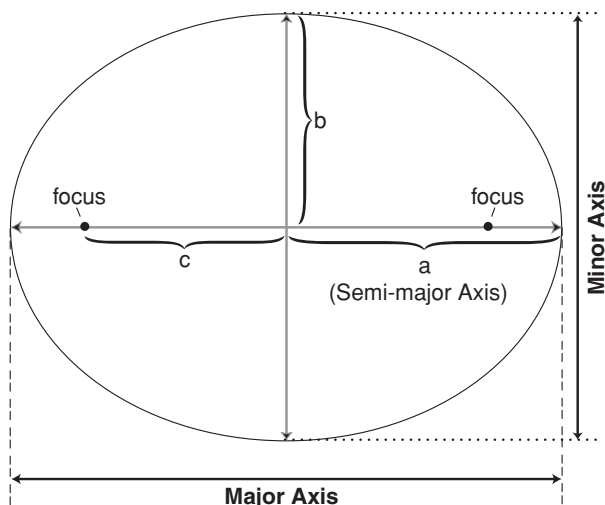
1. Tape one sheet of paper firmly to the cardboard.
2. Tie a piece of string in to a loop (~15-cm in circumference).
3. Push one pushpin into the middle of the surface of the paper.
4. Place the string around the pushpin, place the pencil inside the string and move the pencil around the pin with the string taut at all times (tracing out a circle).

A) Is this a good representation of an orbit of a planet? Why or why not? Record your answers on a separate piece of paper.

5. Place the second pushpin into the surface about 3 cm away from the first pushpin.
6. Place the string around both pushpins, place the pencil inside the string and move the pencil around the pin with the string taut at all times (tracing out an **ellipse**).
7. Move the second pushpin about another 3 cm further away from the first (6 cm total), repeat Step 6



B) Refer to the diagram below and measure the values of a , b , and c for each of the three shapes you have drawn. Record your answers on your paper.



Analysis

Eccentricity, e , indicates how an ellipse deviates from the shape of a circle:

$$e = c/a$$

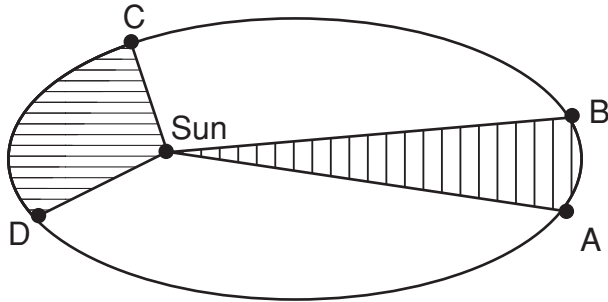
A perfect circle has an eccentricity of zero, while more and more elongated ellipses have higher eccentricities ≤ 1 .

$$[0 \leq e \leq 1]$$

C) Record the values of e for each of your ellipses on your paper.

Extension

Kepler published his first two laws in 1609. The first law states: "The orbit of every planet is an ellipse with the Sun at one of the foci." The second explains why planets move at different speeds at different points in their orbit: "A line joining a planet and the Sun sweeps out equal areas during equal intervals of time." This is easiest to see in a diagram:



The area of A-B-Sun equals the area of C-D-Sun. Therefore, the planet is moving slower going from A to B and faster when going from C to D.

Kepler continued his work on planetary motions, and in 1619 published "Harmony of the Spheres" in which he showed that there is a relationship between a planet's distance from the Sun and the time it takes that planet to go around the Sun (its orbital period). Kepler's third law states "The squares of the orbital periods of planets are directly proportional to the cubes of the semi-major axis of the orbits.":

$$P^2 = a^3$$

(where **P** is orbital period and **a** is the semi-major axis)

This law works very well when we use years as the unit for **P** and astronomical units (AU, the Earth-Sun semi-major axis) as the unit for **a**.

D) The table below shows several objects in the solar system with their eccentricity and semi-major axis listed. Use the semi-major axis values to calculate the orbital period for each object (in years). Record your answers.

Eventually Isaac Newton (1642-1727) refined Kepler's laws so that they could be used for any orbiting objects, like comets and asteroids orbiting the Sun, moons orbiting planets, planets orbiting other stars, two stars orbiting each other — even material orbiting black holes! But it all started with the mathematical talent of Johannes Kepler.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Halley's Comet
Eccentricity	0.20563	0.00677	0.01671	0.09341	0.04839	0.05415	0.04716	0.00858	0.967
Semi-major axis (AU)	0.39	0.72	1.0	1.5	5.2	9.5	19.2	30	17.8

ANSWERS: A: No, planet orbits are elliptical, not circular. B: (These are example measurements [in cm], allow for variation) Circle: $a = 7.5, b = 7.5, c = 0$; Ellipse 1: $a = 5.8, b = 5.6, c = 1.5$; Ellipse 2: $a = 4.4, b = 3.25, c = 2$. C: Circle: $e = 0$; Ellipse 1: $e = 0.26$; Ellipse 2: $e = 0.68$. D: Mercury = 0.24 yr, Venus = 0.61 yr, Earth = 1 yr, Mars = 1.84 yr, Jupiter = 11.86 yr, Saturn = 29.28 yr, Uranus = 84.13 yr, Neptune = 164.3 yr, Comet = 75.1 yr. Note: these values are slightly different than the real-world values due to rounding errors.