Bayes and the Cosmic Distance Scale

Tom Barnes
McDonald Observatory
The University of Texas at Austin

Collaborators in order of appearance:
Bill Jefferys, Raquel Rodriguez, Jim Berger,
Peter Mueller, Kim Orr, Amy Forestell,
Tom Moffett, & Tom Jefferys

Bayes and the Cosmic Distance Scale

Thomas Bayes (1702 - 1761, England), an astronomer?

"You may remember a few days ago we were speaking of Mr. [Thomas] Simpson's attempt to show the great advantage of taking the mean between several astronomical observations rather than trusting to a single observation carefully made, in order to diminish the errors arising from the imperfection of instrument and the organs of sense."

"Now that the errors arising from the imperfection of the instrument and the organs of sense should be thus reduced to nothing or next to nothing only by multiplying the number of observations seems to me extremely incredible. On the contrary the more observations you make with an imperfect instrument the more it seems to be that the error in your conclusion will be proportional to the imperfection of the instrument made use of ... "

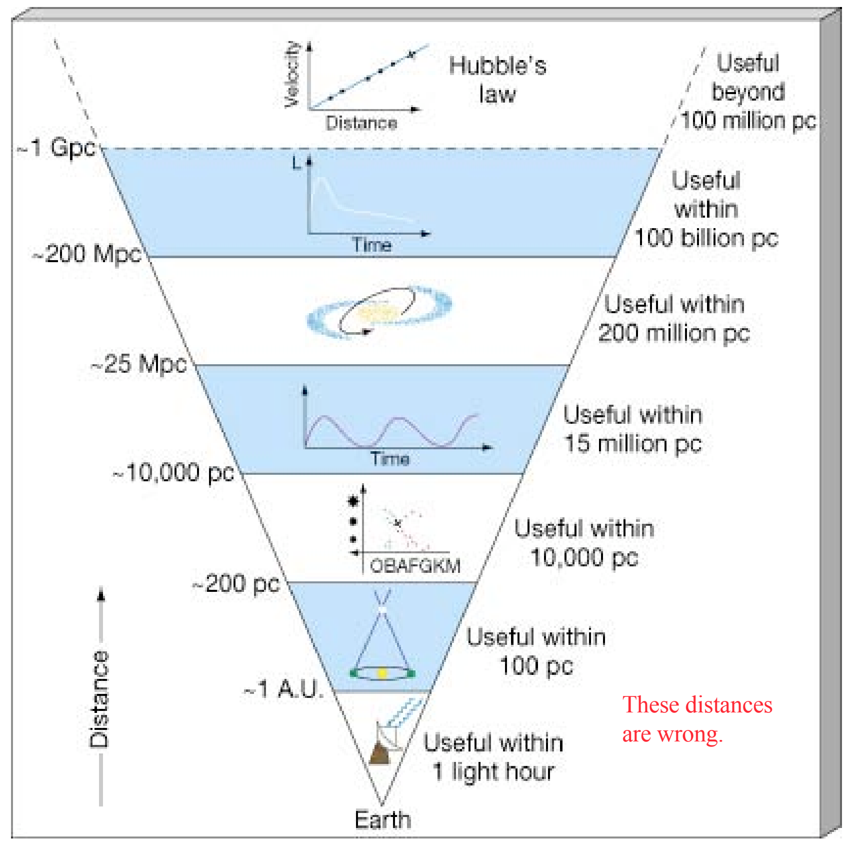
http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Bayes.html

Bayes and the Cosmic Distance Scale

The Cosmic Distance Scale

- Hubble's Law

- Supernovae
- Tully-Fisher relation
- Cepheid variable stars
- Spectroscopic parallax
- (Statistical parallax)
- Trigonometric parallax
  - parsec = parallax of 1 arcsec
  - 1 parsec = 206265 AU
- Radar
  - 1 AU = 1.5 x 10^{11} m



from Chaisson & McMillan (2002)

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The importance of Cepheid variable stars

- Period - luminosity relation

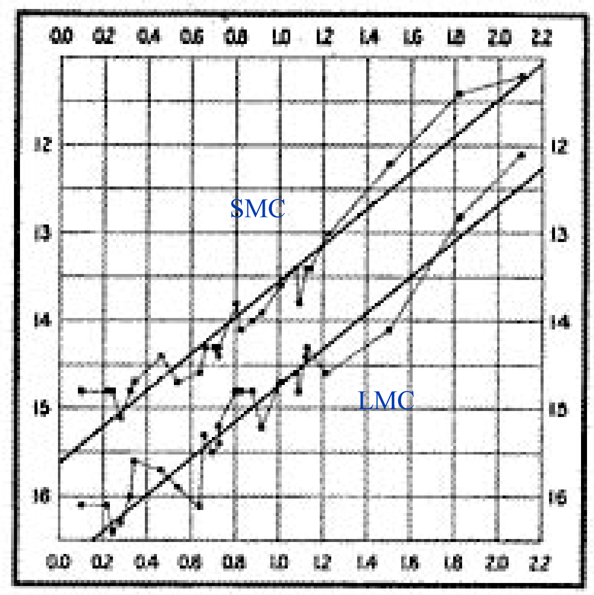
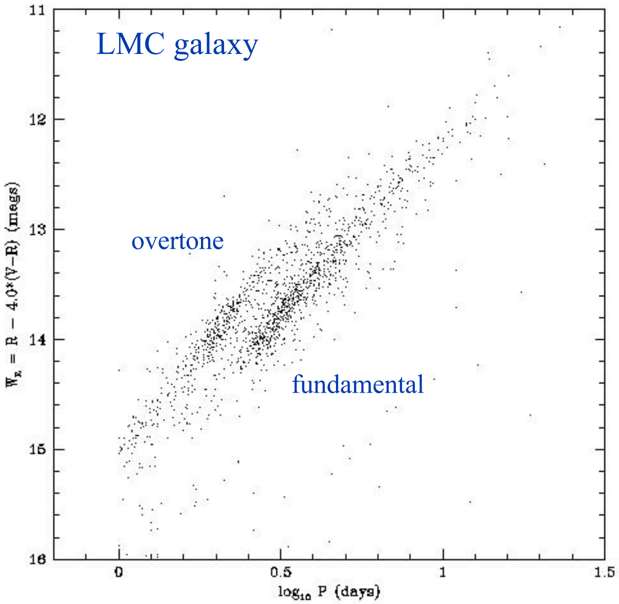


FIG. 2.

Henrietta Leavitt (1912), two galaxies



MACHO (2000), overtone and fundamental

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Distances to Cepheid variable stars

- Convert apparent brightness to luminosity
  - Trigonometric Parallaxes
    - Hubble Space Telescope: Benedict *et al.* (2002)
      - δ Cephei 272 ± 9 pc
      - 10 additional Cepheids being observed by HST
  - Other principal techniques
    - Cluster main-sequence fitting
      Gieren, Fouqué, & Gómez 1997
    - Surface brightness technique
      Barnes & Evans 1976
      Barnes, Jefferys, Berger, Mueller, Orr & Rodriquez 2003

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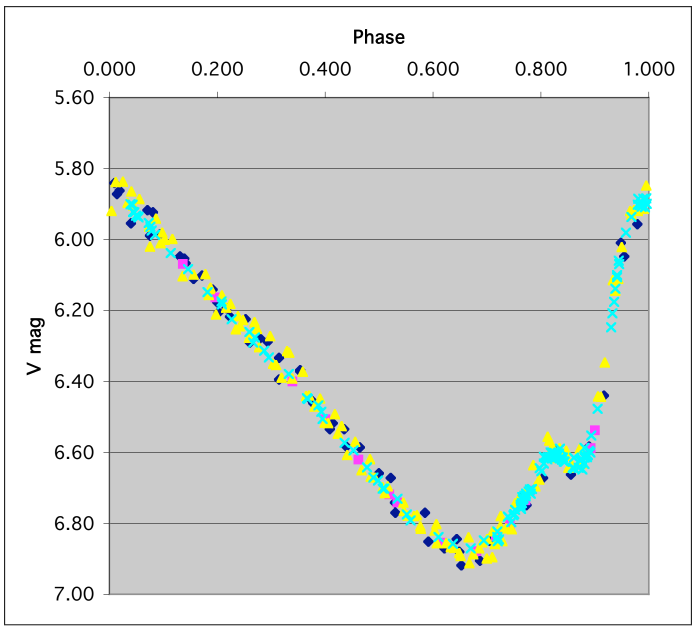
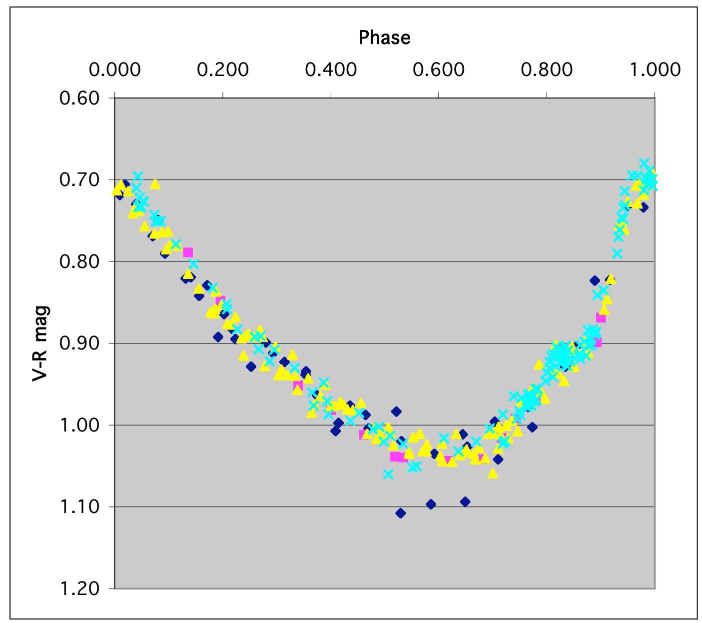
Properties of Cepheid variable stars (X Cyg, P = 16.4 days)

- Observables

Apparent brightness



Color ($\propto$ log(surface brightness))



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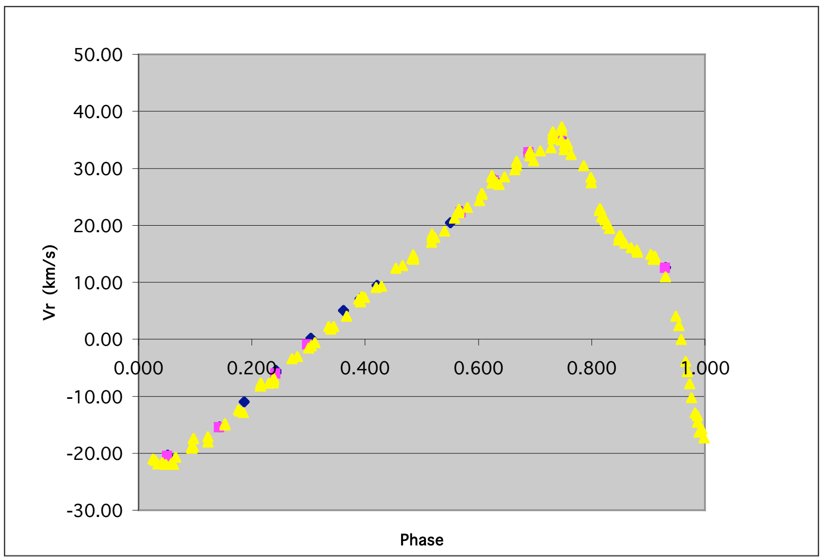
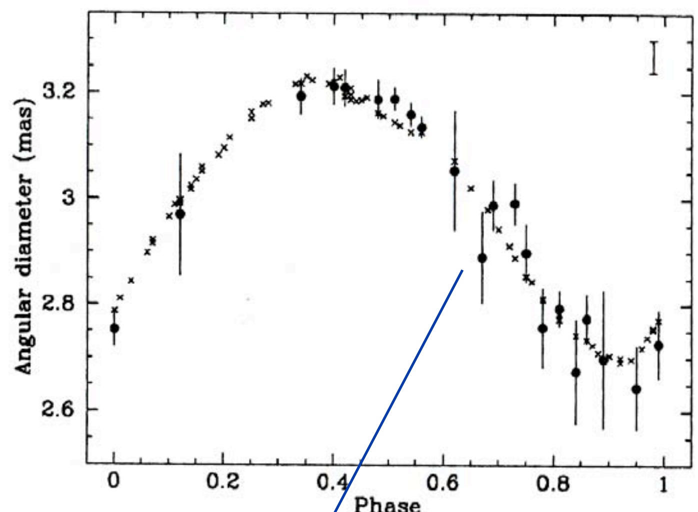
Properties of Cepheid variable stars

- Observables

Radial velocity (X Cyg)



Angular size (l Car)



Observed size

- Very few have measured angular sizes, but ….

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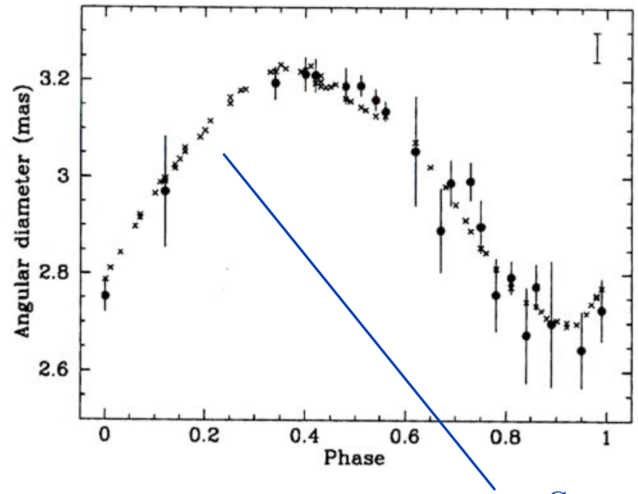
Properties of Cepheid variable stars

- Angular diameter $\phi(\theta)$ from observables V_o , $(V\text{-}R)_o$
  - Definition of $F_v(\theta) = 4.2207 - 0.1V_o(\theta) - 0.5\log\phi(\theta)$ Eq. (1)
  - Empirically , $F_v(\theta) = A + B\,(V\text{-}R)_o(\theta)$ Eq. (2)

    A, B from observed values of $\phi(\theta)$ and $(V\text{-}R)_o(\theta)$
  - Equate (1) & (2): $\log\phi(\theta) = 2\,(4.2207 - 0.1\,V_o(\theta) - A - B\,(V\text{-}R)_o(\theta))$



Computed size

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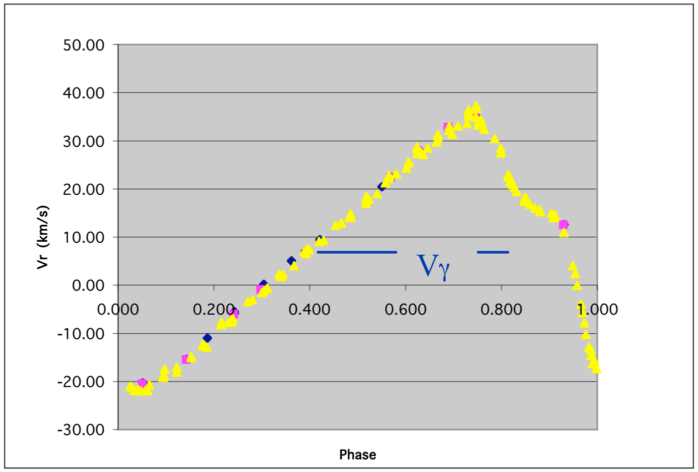
Properties of Cepheid variable stars

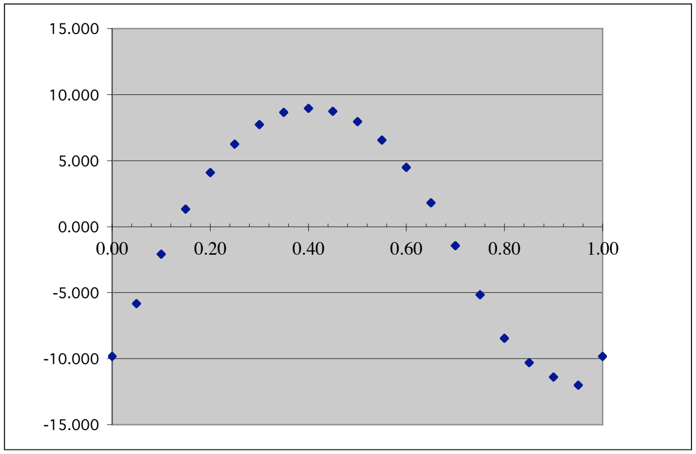
- Linear displacements $\Delta R(\theta)$ from radial velocities
  - $\Delta R(\theta) = - \int p \, (V_r(\theta) - V_\gamma) \, d\theta$

    p converts *radial velocity* to *pulsational velocity*; V_γ is the center of mass velocity



Radial velocity (km/s)

=>



Radial displacement (Ro)

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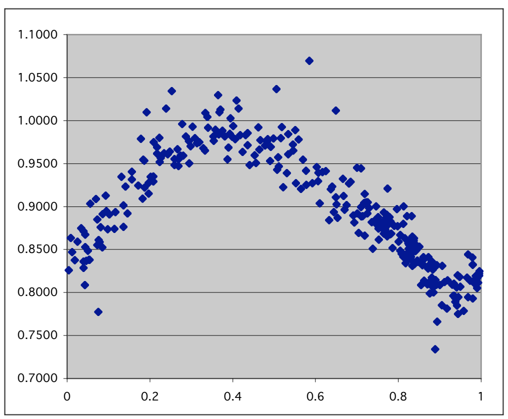
Surface brightness technique for determining distance

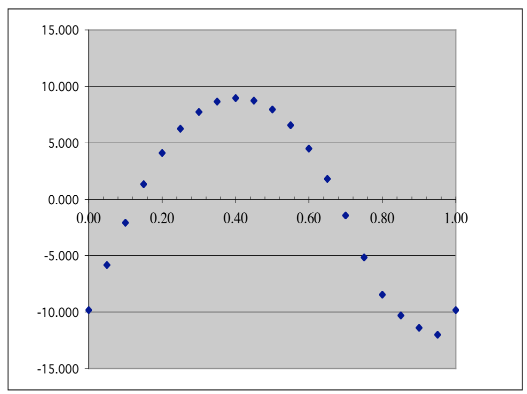
$$\phi(t) = 2\,(\Delta R(\theta + \Delta\theta) + \langle R \rangle\,)\,/\,d$$

- $\phi(\theta)$ and $\Delta R(\theta)$ are computed from observables;
  mean radius $\langle R \rangle$, stellar distance d & $\Delta\theta$ are determined.

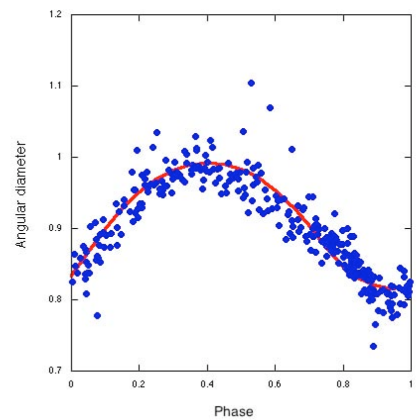
X Cyg



Computed angular size



Computed linear size



Fit at distance

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Surface brightness method

- Limitations in previous surface brightness studies
  - The radial velocity data must be modeled before integration. This creates a model selection problem. Previous attempts were *ad hoc*.

    $\Delta R(\theta) = -\int p\,(V_r(\theta) - V_\gamma)\,d\theta$

  - Both parameters in the fit $\phi(\theta)$ *vs.* $\Delta R(\theta)$ have error in them.
    This is an errors–in–variables problem.

    Previous attempts used LSQ bisector fits.

    $\phi(\theta) = 2\,(\Delta R(\theta + \Delta\theta) + \langle R \rangle)\,/\,d$

  - The uncertainty in $\Delta R(\theta)$ must be properly treated or the uncertainties in d and $\langle R \rangle$ may be underestimated. We'll see that indeed they are.

    $\Delta R(\theta) = -\int p\,(V_r(\theta) - V_\gamma)\,d\theta\,,\quad \phi(\theta) = 2\,(\Delta R(\theta + \Delta\theta) + \langle R \rangle)\,/\,d$

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Bayesian analysis

- Advantages over previous attempts
  - model selection on $V_r(\theta)$ and $V_o(\theta)$
    - M, N - lengths of Fourier series
    - Objectively chosen, Fourier coefficients determined.
  - errors–in–variables, $\phi(\theta)$ *and* $\Delta R(\theta)$
    - Properly handled
  - uncertainty in $\Delta R(\theta)$
    - Properly handled

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Bayesian statistical analysis

Bayesian mantra:

- Posterior probability distribution $\propto$ likelihood x prior
  - pdf(d,<R>,$\Delta\theta$, a, b ... I data) = posterior probability distribution

    a and *b* are M- and N- dimensional vectors of Fourier coefficients for $V_r(\theta)$ and $V_o(\theta)$ of unknown length
  - p(data I model) = likelihood function
  - p(model) = prior knowledge of the models

- Priors
  - Conventional priors on precision parameters $\propto 1 / \tau$
  - Flat prior on $\Delta\theta$, <R>

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- Bayesian statistical analysis
  - More on priors
    - Prior on distance *d*

$$p(d) \propto d^2 \exp\left[-\frac{|d \sin\beta|}{z_0}\right]$$

Eliminates for Lutz-Kelker bias

    - Prior on Fourier coefficients *a, b*

$$p(a) \propto \exp\left[-\frac{\tau_a}{2} a'X'Xa\right]$$

with τ_a a hyperparameter and X the design matrix of sines and cosines.
The values τ_a will tell us if the scatter matches the claimed errors.

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- Priors
  - Prior on $\tau_a \sim 1 / \tau_a^{3/2}$

- Sampling scheme
  - Markov Chain Monte Carlo, reversible-jump Gibbs and Metropolis–Hastings sampling of the full posterior probability distribution
  - 10,000 samples per star

- Data sample: 13 Galactic Cepheids
  - Data typical to that shown for X Cyg

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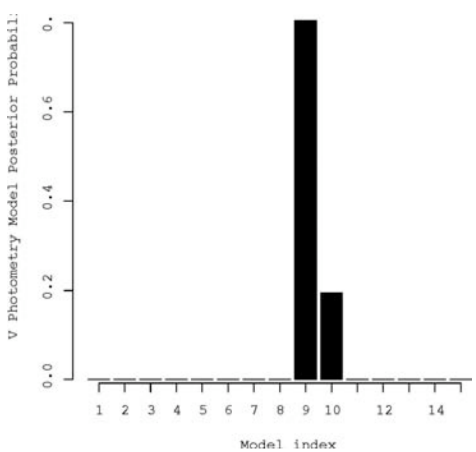
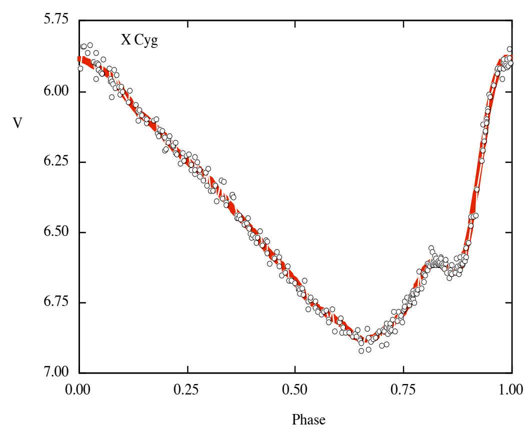
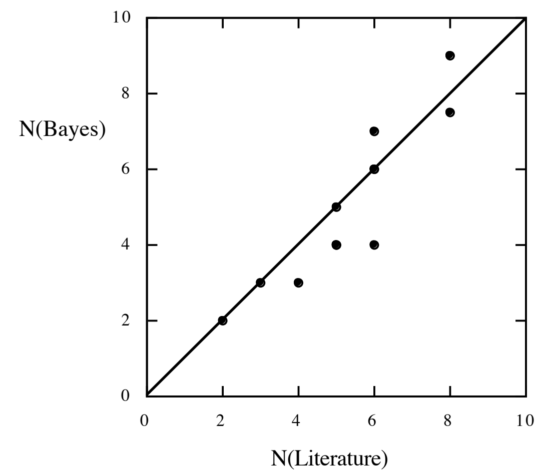
- Results on M, N [lengths of FS on $V_r(\theta)$, $V_o(\theta)$]



N for X Cyg



Fit for N=9



Bayesian N vs. previous N

- Results are marginalized over all values of M,N
  - Max. likelihood picks a single value of M, N for the solution.

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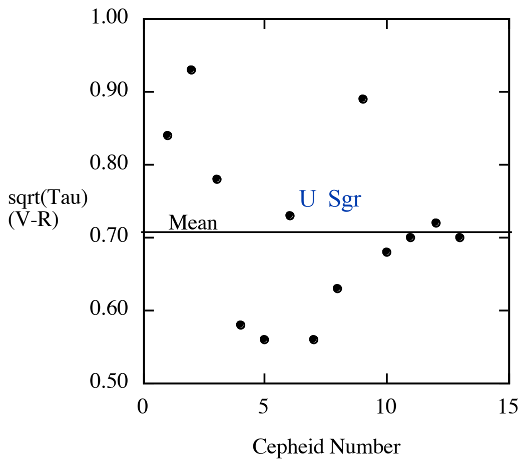
Diagnostic Results on hyperparameters τ_a

- **Hyperparameters** - checking the claimed uncertainties



U Sgr example:

$\sigma_V = \pm 0.016 \rightarrow \pm 0.030$ mag.

$\sigma_{V\text{-}R} = \pm 0.018 \rightarrow \pm 0.024$ mag.

$\sigma_{Vel} = \pm 1.56 \rightarrow \pm 2.08$ km/s.

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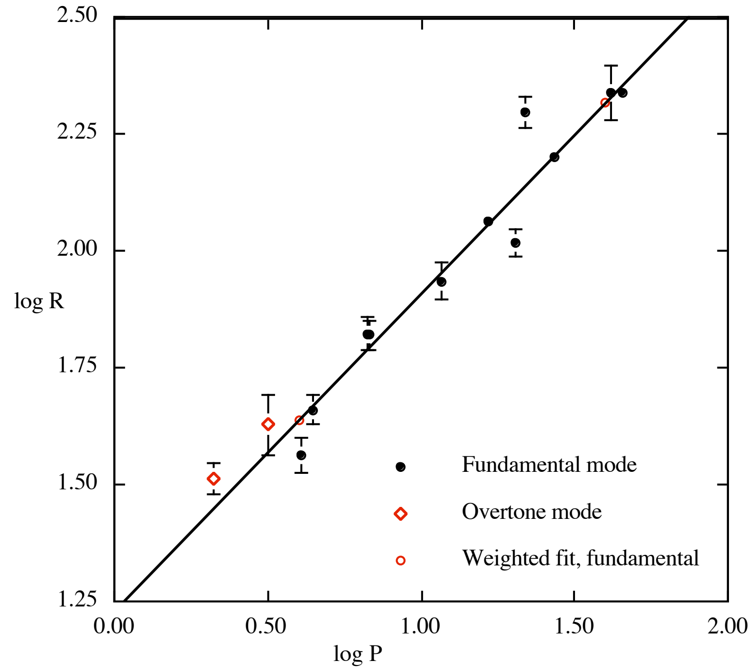
Radius results



- log <R> = 0.693 (±0.037)(log P -1.2) +2.014 (±0.047)
  - compare with log <R> = 0.690 (log P-1.2) +1.979

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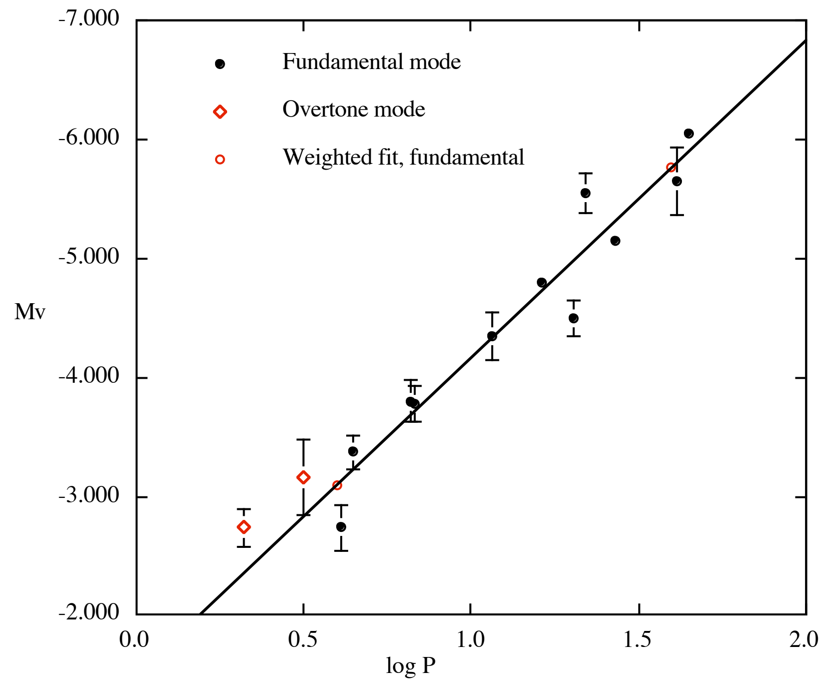
Distance results



- $M_v = -2.690(\pm 0.169)(\log P - 1.2) - 4.699(\pm 0.216)$
  - compare with $M_v = -2.851 (\log P - 1.2) - 4.812$
  - H_o distance scale $M_v = -2.760 (\log P - 1.2) - 4.770$

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- Results on uncertainties in d, <R>
  - Comparison to recent maximum likelihood analysis

    (Storm et al. 2004, A&A, 415, 531) - data courtesy Jesper Storm (Potsdam Univ.)
    - Everything the same [data , reddening, *p*, F_V - (V-K),…]
    - Tests on η Aql (P~7 days) and T Mon (P~27 days)
      - Distances

η Aql	T Mon	Method
250 ± 5 pc	1455 ± 37 pc	Max. likelihood
259 ± 16	1466 ± 73	Bayesian MCMC

      - Radii

48.4 ± 1.0 R_o	149.5 ± 3.8 R_o	Max. likelihood
50.6 ± 3.1	153.1 ± 7.6	Bayesian MCMC

    - Conclusions:
      - Within the uncertainties, the same distances and radii
      - Maximum likelihood uncertainties are underestimated by 2-3 times!
      - Only two stars; full sample of 32 in progress

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- Bayesian analysis provides wealth of information
  - Bayesian advantages vs. maximum likelihood
    - Eliminates Lutz-Kelker bias in distances
    - Objective selection of the model (N, M)
    - Marginalizes over all the possibilities
    - Finds incorrectly assigned observational uncertainties
  - Radius results are similar to previous
    - A suggestion of larger radii; but 1 σ result
    - Max. likelihood errors underestimated; but only two stars
  - Distance results are similar to previous
    - Max. likelihood errors underestimated; but only two stars

Bayes and the Cosmic Distance Scale

Statistical parallax problem

- Maximum likelihood approach
  Hawley, Jefferys, Barnes & Wan 1986 on RR Lyrae variables
  Wilson, Barnes, Hawley & Jefferys 1991 on Cepheid variables
  - First rigorous, non-Bayesian solution to statistical parallax problem.
  - First use of simplex optimization in astronomy.

- Bayesian approach
  T. R. Jefferys, W. H. Jefferys, & T. G. Barnes 2004
  - Hierarchical Bayes model
  - Metropolis within-Gibbs sampler, MCMC
  - Observed proper motions, radial velocities, apparent luminosities and metallicities
  - Determines luminosities, distances, motions

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Statistical parallax problem

- Test of hierarchical Bayesian *vs.* maximum likelihood on RR Lyr stars (apparent magnitudes corrected per Barnes & Hawley 1986)
  - M_V (141 fundamental mode pulsators)
    - This study 0.71 ± 0.11
    - Hawley et al. 0.68 ± 0.14
  - M_V (17 overtone pulsators)
    - This study 0.67 ± 0.27
    - Hawley et al. 1.01 ± 0.38
  - Recovers previous result when sample is large
  - Reveals physically more likely result for small sample
  - Suggests overestimation of max. likelihood errors
  - Capable of estimating metallicity dependence

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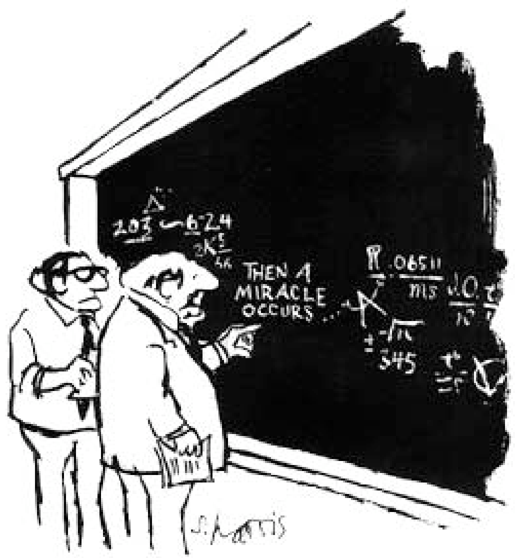
- Conclusions



"I think you should be more explicit here in step two."

(copyright Sidney Harris)

Bill, thanks for all the math miracles!