All HII Regions Great and Small

An overview of C^2-Ray

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Contents

- Photo-ionization Gas Dynamics
- Method (C^2-Ray)
- Some Results for Turbulent HII regions
- Some Results for Reionization

Photo-Ionization Hydrodynamics

- The photo-ionization process is one of the important radiative feedback processes in astrophysics.
- The increase in pressure caused by it can trigger strong dynamic effects: photo-ionization hydrodynamics.
- The challenge in combining hydrodynamics with photo-ionization lies in the difference in time scales between the two process.

Radiative Transfer

- Assuming that:
  - Speed of the ionization front is non-relativistic.
  - Recombinations to the ground level can be incorporated using a modified recombination rate (On The Spot approximation).
- The full radiative transfer equation

$$\frac{\partial I_\nu}{c\partial t} + \nabla \cdot (\Omega I_\nu) = j_\nu - \kappa_\nu I_\nu$$

- Simplifies to

$$\nabla \cdot (\Omega I_\nu) = -\kappa I_\nu$$

Photo-Ionization Chemistry

$$\frac{\partial n(H^+)}{\partial t} = (1 - n(H^+))\Gamma - n(H^+) n_e \alpha(T) + (1 - n(H^+)) n_e C(T)$$

Photo-ionization   Recombinations   Collisional Ionization

$$\Gamma = \frac{1}{4\pi r^2} \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu(r)} \mathrm{d}\nu$$

Photo-Ionization rate

Solution of the radiative transfer equation

$$\tau_\nu(r) = \int_0^r a_\nu n(H^0) \mathrm{d}r$$

Optical depth between source and position *r*

Non-local connection!

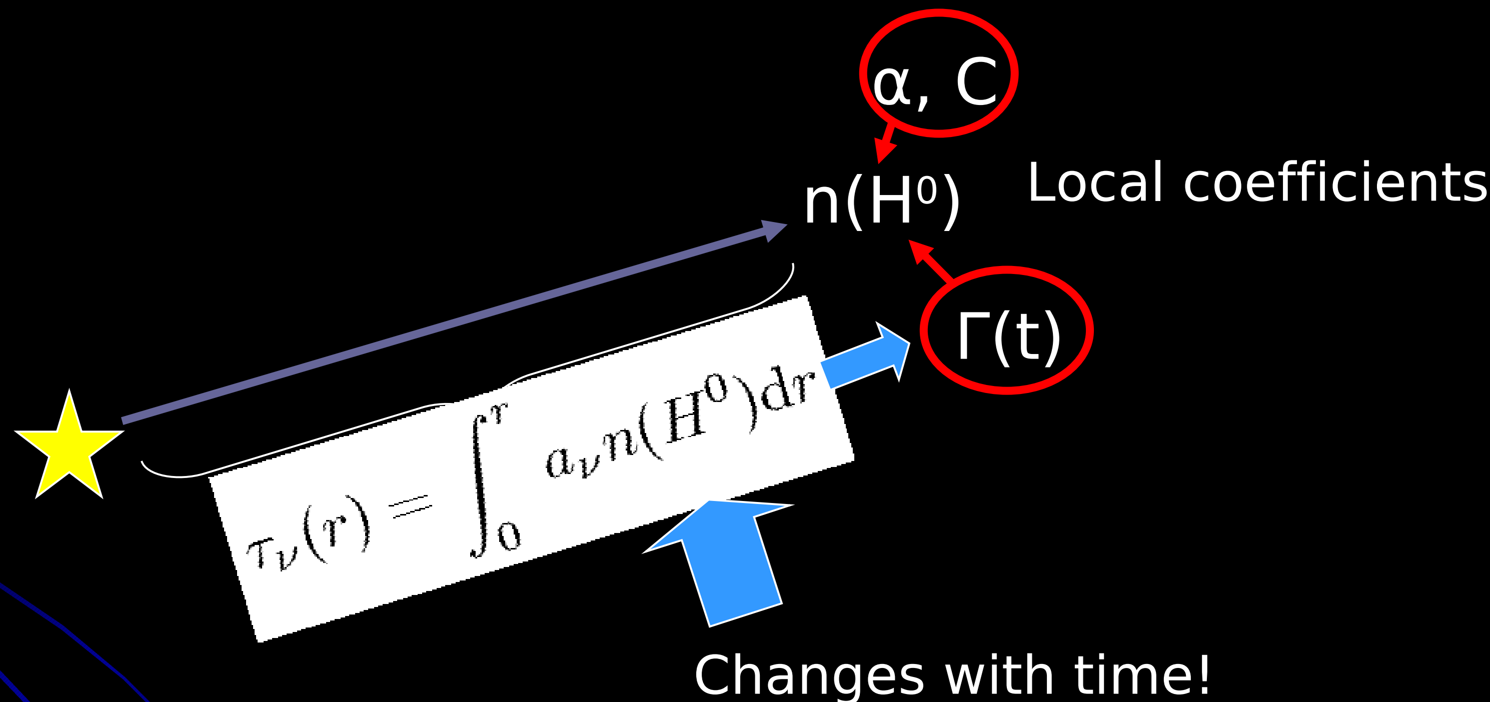

α, C
n(H0)
Local coefficients
Γ(t)
$\tau_\nu(r) = \int_0^r a_\nu n(H^0) \mathrm{d}r$
Changes with time!

Photo-Ionization Heating

$$\mathcal{H} = \frac{n(H^0)}{4\pi r^2} \int_{\nu_0}^{\infty} h(\nu - \nu_0) \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu(r)} \mathrm{d}\nu$$

Energy above the ionization energy

Note: the higher energy photons heat more, and have a lower optical depth :

$$a_\nu = a_0 \left(\frac{\nu_0}{\nu}\right)^3$$

Multi-Scale Problem: Time

- The ionization fraction evolves on a typical time scale **$t_i=1/(x[H^0](\Gamma + C(T)) + x[H^+]n_e\alpha)$**
- In addition Γ depends on non-local effects (τ), so there is also the time scale for **change in τ**, hence in Γ. This is related to speed of the ionization front.

$$\Gamma = \frac{1}{4\pi r^2}\int_{\nu_0}^{\infty}\frac{L_\nu}{h\nu}a_\nu e^{-\tau_\nu(r)}\mathrm{d}\nu$$

- The gas evolves on a typical time scale **$t_{hydro}=\Delta x/(v+v_s)$**
- These time scales are unrelated, and can be orders of magnitude different.
- In a gas dynamic simulation you prefer to use **t_{hydro}**!

Multi-Scale Problem: Length

- If our computational cells are rather large, it may happen that there is considerable optical depth *over* a cell: **$\Delta\tau_\nu = a_\nu n(H^0)\Delta r$**.
- If **$\Delta\tau_\nu \gg 1$** the photo-ionization rate Γ is *not* constant inside the cell.
- Using a value from a particular radius will either under- or overestimate the number of photo-ionizations.

$$\Gamma = \frac{1}{4\pi r^2}\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu(r)} \mathrm{d}\nu$$

Diagnostic: Photon-Conservation

- There is a simple way to check whether you are solving the problem in the correct way: photon-conservation.
- The number of photons sent out by the source within a time step should either be used for ionizations (new, or balancing recombinations), or escape from the domain.
- So by comparing these numbers, the correctness of your solution can be checked.

C^2-Ray

- Conservative, Causal Ray Tracing (GM, Iliev, Alvarez, Shapiro, 2006).
- Uses time- and spatially averaged quantities to allow a photo-ionization calculation at Δt_{hydro} and Δx_{hydro}.
- The radiation transport is done using short characteristics ray tracing.
- With this method we can calculate three-dimensional photo-ionization on regular grids.

- We use a modular approach for developing the code:
  - Can be used as a stand-alone code (R-type fronts, reionization).
  - Can be coupled to hydrodynamics (HII regions).

Photon-Conservation: Space & Time

- Dealing with thick cells: take it to be a volume in (Abel, Norman, Madau 1999).

$$\Gamma = \int_{\nu_0}^{\infty} \frac{L_\nu e^{-\tau_\nu}}{h\nu} \frac{1-e^{-\Delta\tau_\nu}}{(1-x)nV} d\nu$$

- Dealing with long time steps: use the time-averaged optical depth

$$\langle\Delta\tau_\nu\rangle = a_\nu \Delta r \left(1 - x_{\rm eq} + (x_{\rm eq} - x_0)(1 - e^{-\Delta t/t_{\rm i}})\frac{t_{\rm i}}{\Delta t}\right)$$

- Derived from relaxation solution (Schmidt-Voigt & Köppen 1987; Mellema 1993, see Altay et al. 2008 for He version).

$$x(t) = x_{\rm eq} + (x_0 - x_{\rm eq})e^{-\Delta t/t_{\rm i}}$$

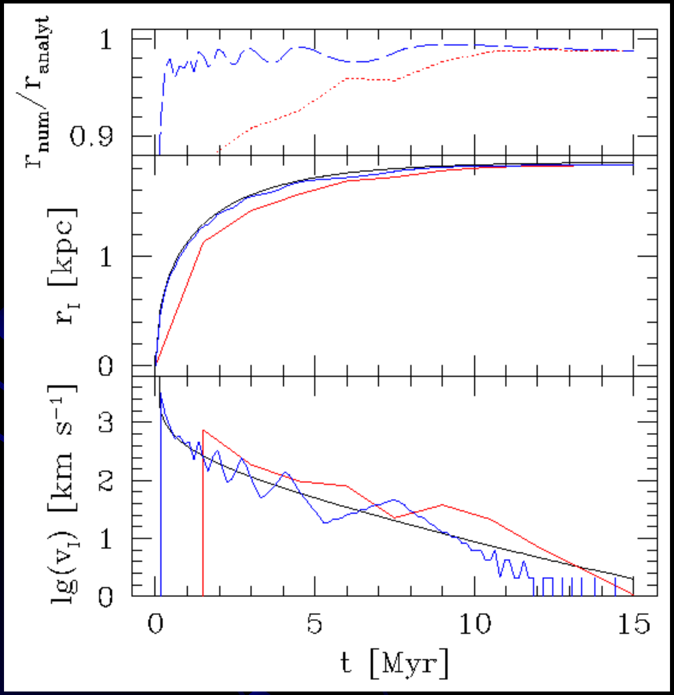
$$t_{\rm i} = 1/(\Gamma + n_{\rm e}\alpha)$$

$$x_{\rm eq} = \frac{\Gamma}{\Gamma + n_{\rm e}\alpha}$$

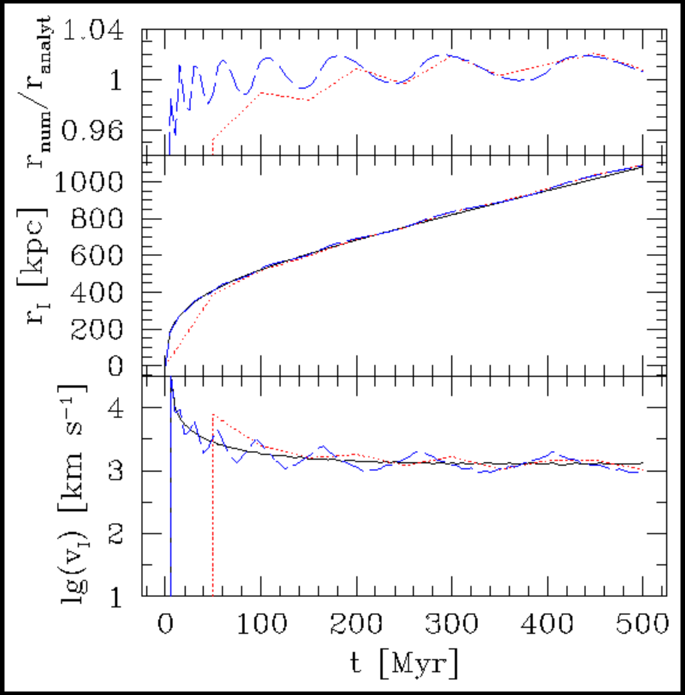
Tests for Ionization

- Ionization fronts in different density environments.
- Worst case: 16 radial cells, 10 or 100 time steps.

1/r density



Expanding Universe



Energy Conservation

- Photon-conservation automatically leads to energy conservation.
- Heating = (photon-energy$_{in}$ – photon-energy$_{out}$) / Δt.
- Large time steps still give correct photo-ionization heating.

$$H = n_{\rm HI} \int_{\nu_{\rm HI}}^{\infty} h(\nu - \nu_{\rm HI}) \frac{L_\nu a_\nu e^{-\tau_\nu}}{h\nu} {\rm d}\nu \,,$$

$$H = \int_{\nu_{\rm HI}}^{\infty} h(\nu - \nu_{\rm HI}) \frac{L_\nu e^{-\langle \tau_\nu \rangle}}{h\nu} \frac{1 - e^{-\langle \Delta\tau_\nu \rangle}}{V_{\rm shell}} {\rm d}\nu \,,$$

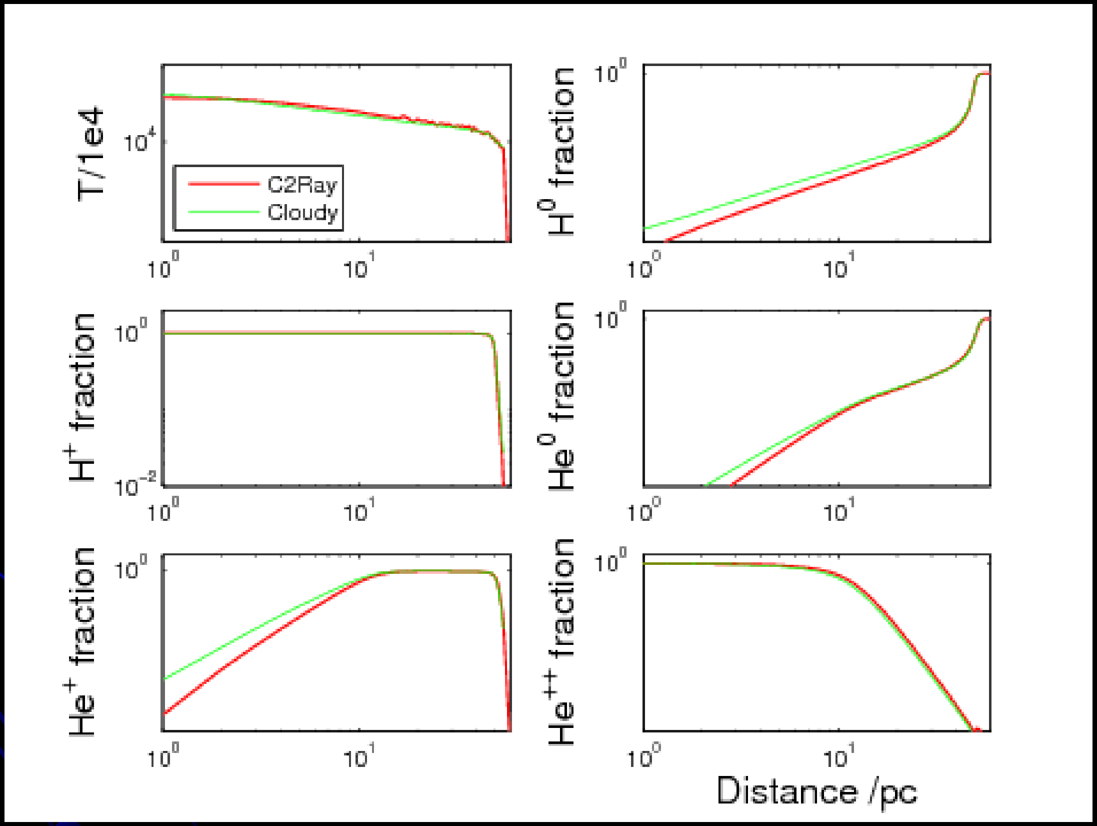
Thermal Update

- Using the photon-conserving heating rate, we update the temperature by applying it together with a cooling rate.
- The cooling rate is determined from a look-up tables. These could be coronal cooling curve, or specified per ion. For the turbulent HII region simulations we used non-equilibrium H^0,H^+ cooling plus approximate metal cooling.
- In order to deal with the stiff thermal equation (and the temperature dependent cooling rates) we subcycle the time step by limiting the change in thermal energy.

In Progress: Adding Helium

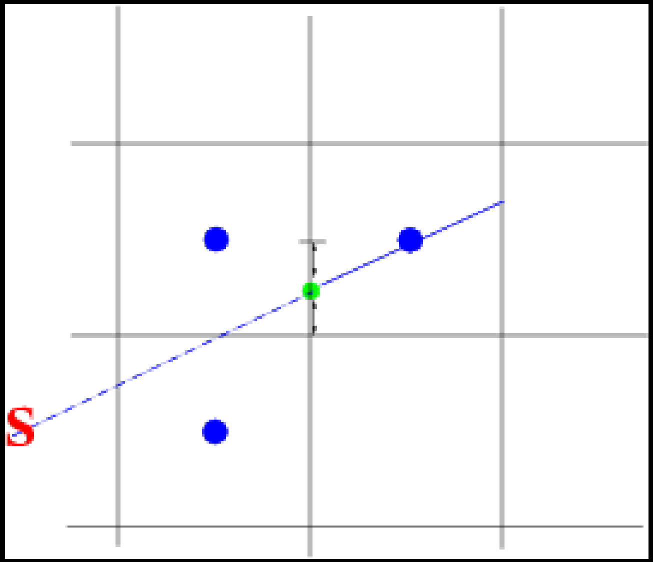
- Martina Friedrich (Stockholm) is working on putting back Helium to C^2-Ray. First 1D test results look promising:

Equilibrium solution



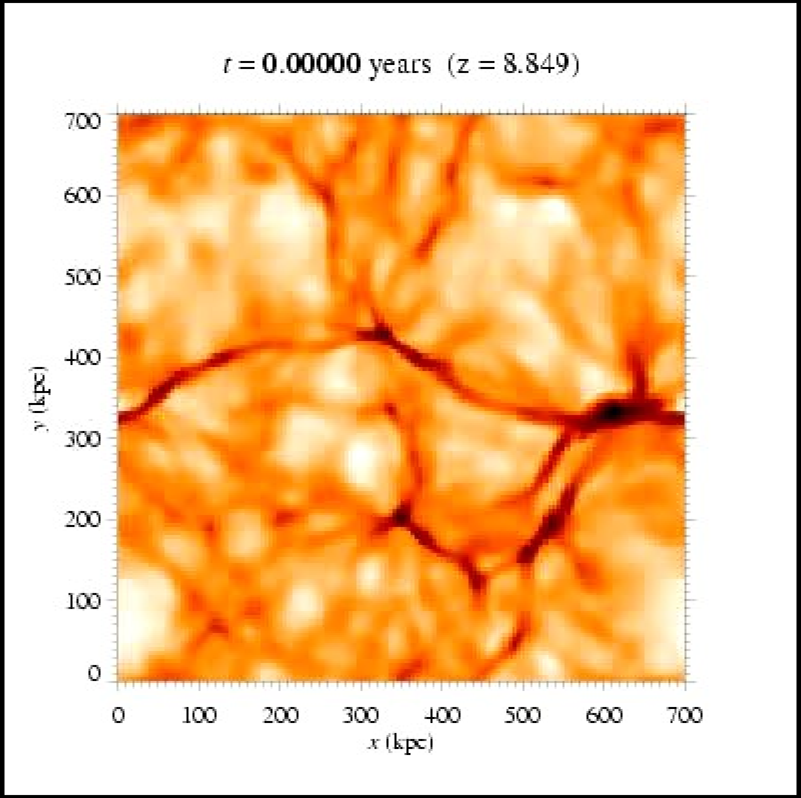
Multi-dimensional Ray-tracing

- To calculate the transfer in 2 or 3 dimensions, one needs to do ray-tracing.
- We developed an efficient 3D ray-tracer for this, based on the technique of short characteristics.
- Optical depth is constructed from the solutions of cells closer to the source ('causal').
- Note: short-characteristics does not guarantee photon-conservation (5% errors).



Multiple Sources

- Handling multiple sources is possible.
- Each source has its own ray-tracing.
- Calculate and add the rates due to each source, but do not apply them.
- Apply the combined rates.
- Iteration needed to make sources ‘feel’ other sources.
- Sources can be processed in parallel (MPI).



Cut through cosmological density field, 16 sources (code comparison project, Test 4).

Photo-Ionization Gas Dynamics

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Continuity Equation

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p$$

Momentum Equation

$$\frac{\partial e}{\partial t} + \nabla \cdot ((e + p)\mathbf{v}) = \mathcal{H} - \mathcal{C}$$

Energy Equation

A multiscale problem, in time and space

Coupling to Hydrodynamics

- Coupling the photo-ionization radiative transfer and chemistry to hydrodynamics involves two points of contact:
  - Advection of the ionized fractions
  - Applying radiative heating and cooling rates to the thermal energy.

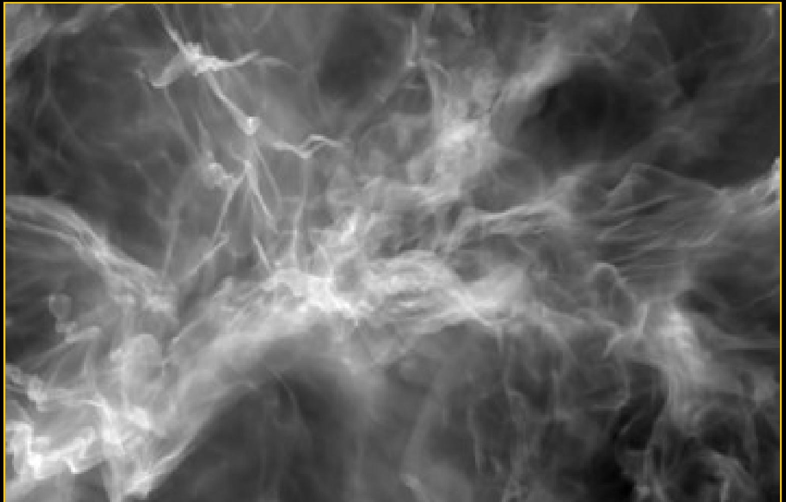
- Currently the 'chemistry' model is still very simplified: only H ionization fractions, H photo-heating, and H + approximate metal ion cooling. Working on: He, molecular cooling.

- C^2-Ray has been coupled to a range of (M)HD codes: Capreole3D, Yguazú-a, Phab, and partly to CubeP^{3}M.

Application 1:
Growing HII Regions in Turbulent Molecular Clouds

Turbulent Molecular Clouds

- Observations show that Molecular Clouds have turbulent structures, with supersonic velocities.
- How this turbulence is maintained is one of the open questions of star formation theory.
- Several groups have been pursuing numerical simulations of driven turbulence (e.g. Padoan, Klessen, Burkert, Vazquez-Semadeni), both with and without magnetic fields.
- These models seem to mimick real MC structures, as well as the Initial Mass Function for stars.
- What does an HII region look like when growing into this? (Mellema et al. 2006)



Padoan et al. (2007)


N90

High Resolution (512^3) Simulations

- Henney, Arthur, Mellema, Vazquez-Semadeni, Iliev (2009)
- Single source
- Coupled with hydrodynamics (Capreole3D).

Initial conditions (Vázquez-Semadeni et al. 2005):
- $\langle n_0 \rangle = 1000\ \mathrm{cm}^{-3}$; $T_0 \sim 5$ K
- $M_{rms} = 10$
- L = 4 pc; $L_J = 4$ pc
- 1 collapsing object

Ionizing source (O7.5, ~30 $M_{\odot}$):
- $T_{eff} = 37000$ K
- $Q_H = 10^{48.5}\ \mathrm{s}^{-1}$

Evolution Movie

Soon at a Planetarium near you!
(if you are in New York...)

Simulating Reionization

- The coming decade should bring us the first observations of the redshifted 21cm signal from reionization
- To better understand the properties of the signal we are studying it with detailed large scale cosmological simulations.
- Three steps:
  - Evolution of IGM density (δ) & (proto-)galaxies from a cosmological simulation.
  - Assign EUV luminosity to (proto-)galaxies.
  - Transfer EUV radiation through the IGM (x_{HI}).
- For large scale simulations galaxy formation is unresolved and baryons and dark matter have the same distribution:
  - Cosmological N-body simulation (for DM).
  - Transfer of EUV radiation can be done in postprocessing mode.

Application 2:
Large Scale Simulations of Reionization

Notation

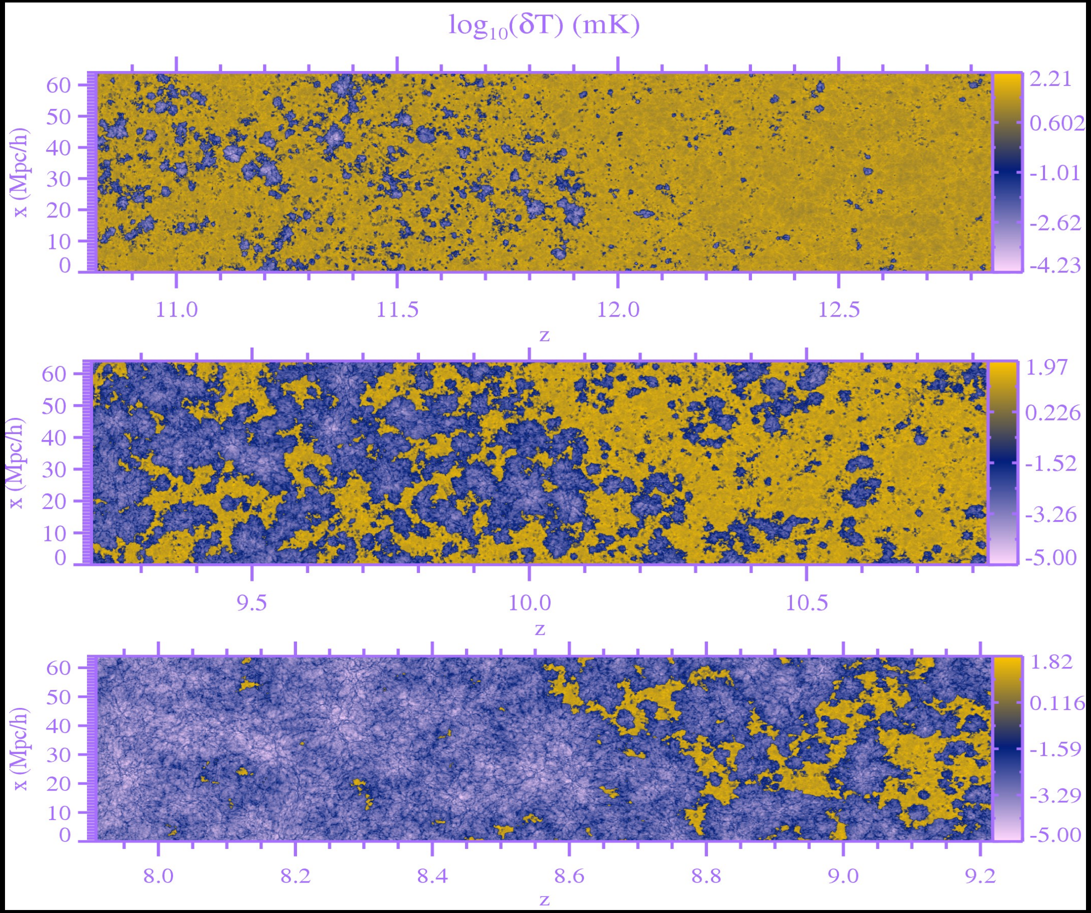
- Our simulations are characterized by

64Mpc_f100C_f250S_432

- 64Mpc: Boxsize = 64/h Mpc
- f100: High mass sources efficiency
- C: Clumping
- f250: Low mass sources efficiency
- S: Low mass sources suppression
- 432: RT grid = 432^3

21cm LOS Reionization History

- From the calculated evolution of the ionized densities we can construct reionization histories along the line of sight.
- Frequency/redshift direction contains evolutionary, geometrical and velocity information.
- Simulation shown: 64Mpc_f100_f250S_432 (64/h Mpc, $M_{halo} > 10^8 M_{\odot}$), with feedback on low mass sources.



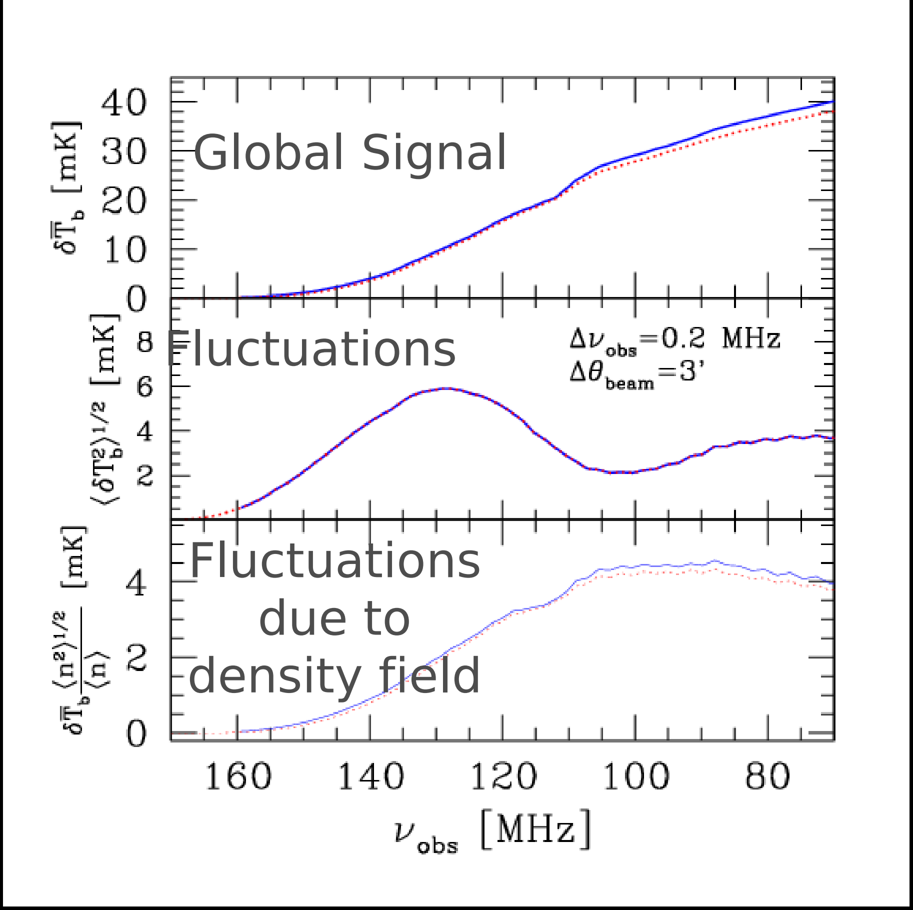
redshift

Statistical Measurements

- The sensitivity of the upcoming EoR experiments will be too low to image 21cm from reionization pixel by pixel: Statistical measurements needed.
  - **First goal**: to reliably detect signatures from reionization (and separate them from foreground and instrumental effects).
  - **Second goal**: to interpret them in terms of astrophysics (source population and properties).
- Luckily, the 21cm line signal is rich in properties:
  - Global signals: mean signal, fluctuations.
  - Angular properties: power spectra
  - Frequency properties: Kaiser effect, correlation length.
  - Non-Gaussianity: skewness

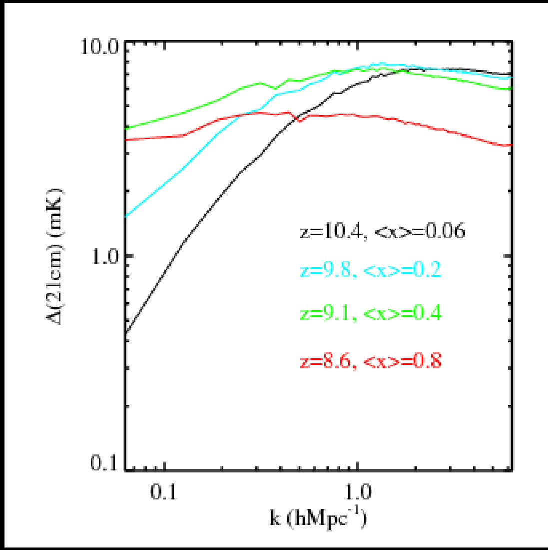
Global Signals

- A single dish telescope could measure the change of the global signal with frequency: simulations do not show a sharp transition.
- The corresponding measurement by an interferometer would be the change of the 21cm (rms) fluctuations. Peak of these falls around 60-70% ionized.
- Simulations shown: 64Mpc_f100_f250S_432 and 64Mpc_f100_f250S_216

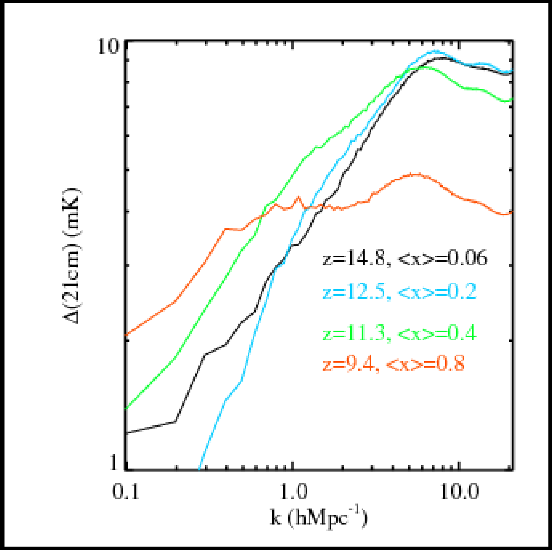


Power Spectra

- Information about the length scales can be obtained from the power spectra.
- Simulations show clear trends of shifting power to larger scales as reionization progresses, and a flattening of the power spectra.
- Note that the angular power spectrum is measured directly by an interferometer, the multipole l is equivalent to $\sqrt{(u^2+v^2)}$ in a visibility map.



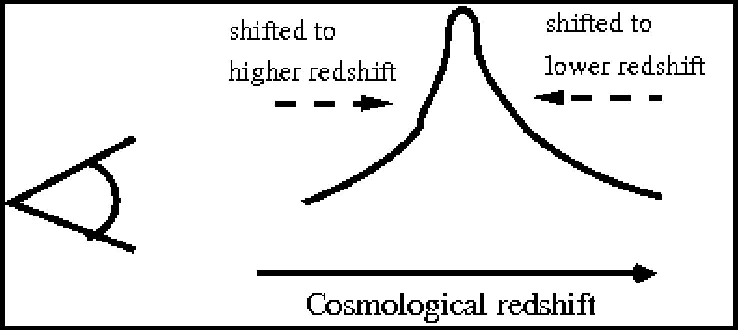
100Mpc_f250C_f0_203



64Mpc_f100_f250S_432

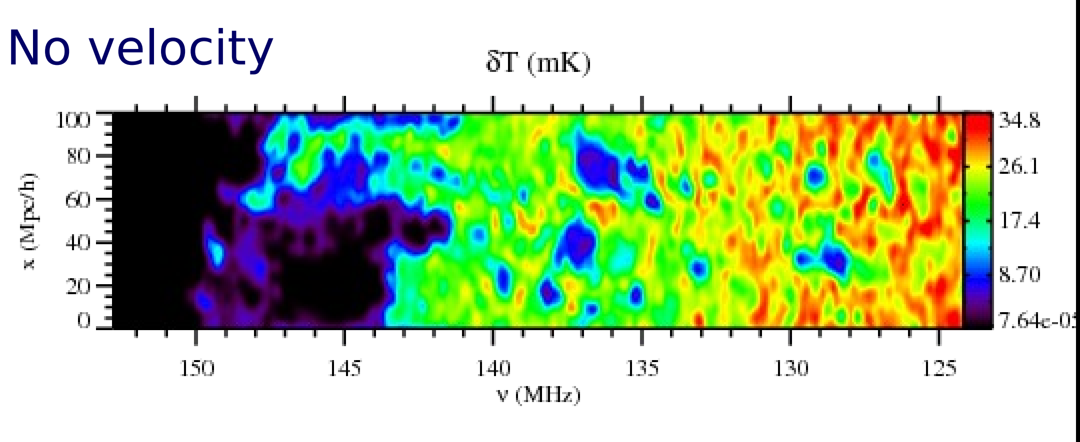
Velocity Distortions

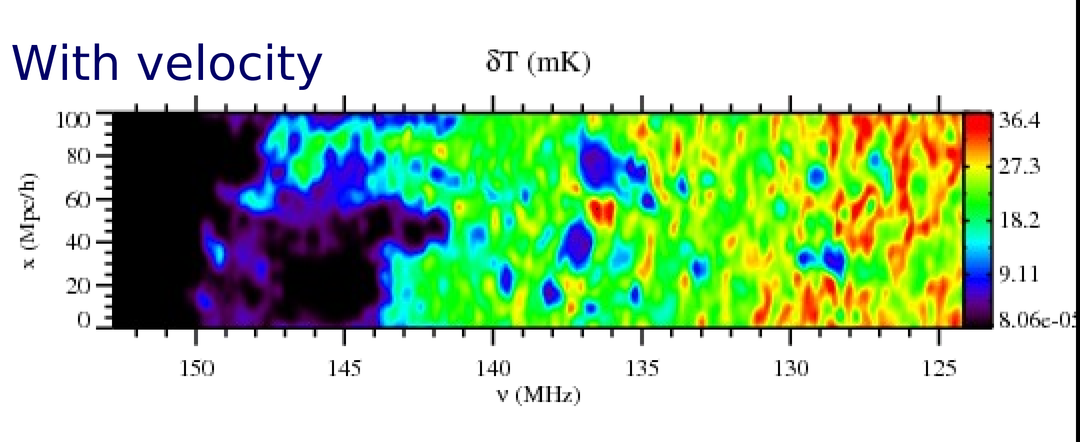
- Due to the peculiar velocity field, the signal can be displaced from its cosmological redshift.
- \`Kaiser effect´ or \`velocity compression´: due to infall, signal concentrates at the high density peaks.
- This is clearly seen in the simulations and gives ~30% increase in fluctuations (and up to a factor of 2).
- This shows that fluctuations calculated from density and ionization fractions alone miss some power → better to use image cubes.



Effect of the Velocity Field

- Adding the velocity distortions visibly increases the fluctuations in the neutral medium.
- Maximum value also larger.
- The effect remains noticeable even at LOFAR-like resolution (3’, 200 kHz).
- Simulation shown: 100Mpc_f250C_f0_203.



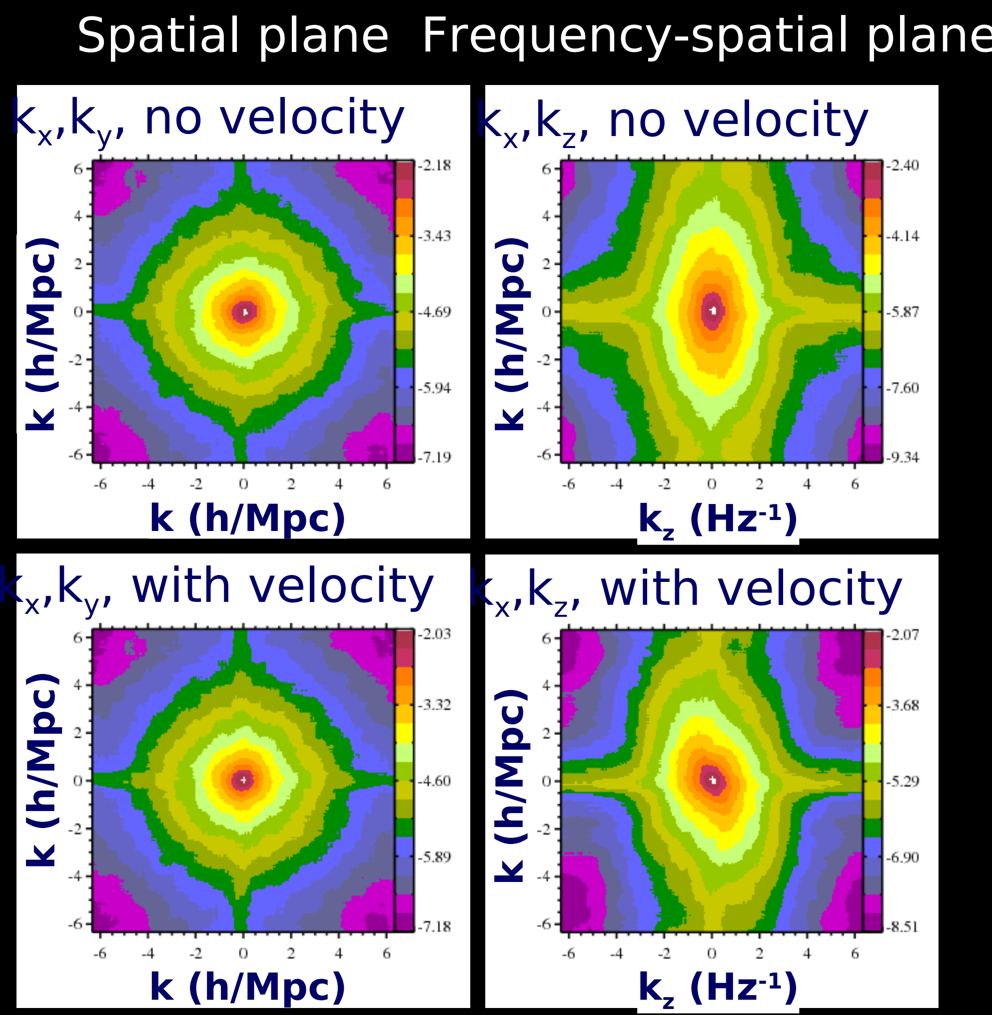


And in Fourier Space...

- Since the velocity gradients responsible for the distortions are related to the density field, one can write the total (3D) power P(**k**) in terms of a polynomial in $\mu=\cos(\theta_k)$, the angle between the LOS and the **k** vectors (see Barkana & Loeb 2005).
- $P(\mathbf{k}) = (P_{\delta\delta} - 2P_{x\delta} + P_{xx}) + 2(P_{\delta\delta} - P_{x\delta})\mu^2 + P_{\delta\delta}\mu^4$
- Since $P_{\delta\delta}$ goes with μ^4, it should be possible to separate it from the dependence on the ionized fraction x, and thus to directly find the density power spectrum.

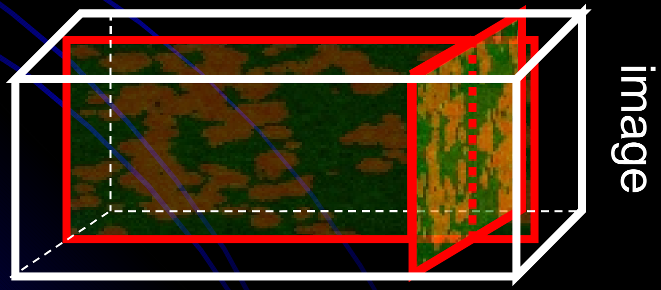
And in Fourier Space...

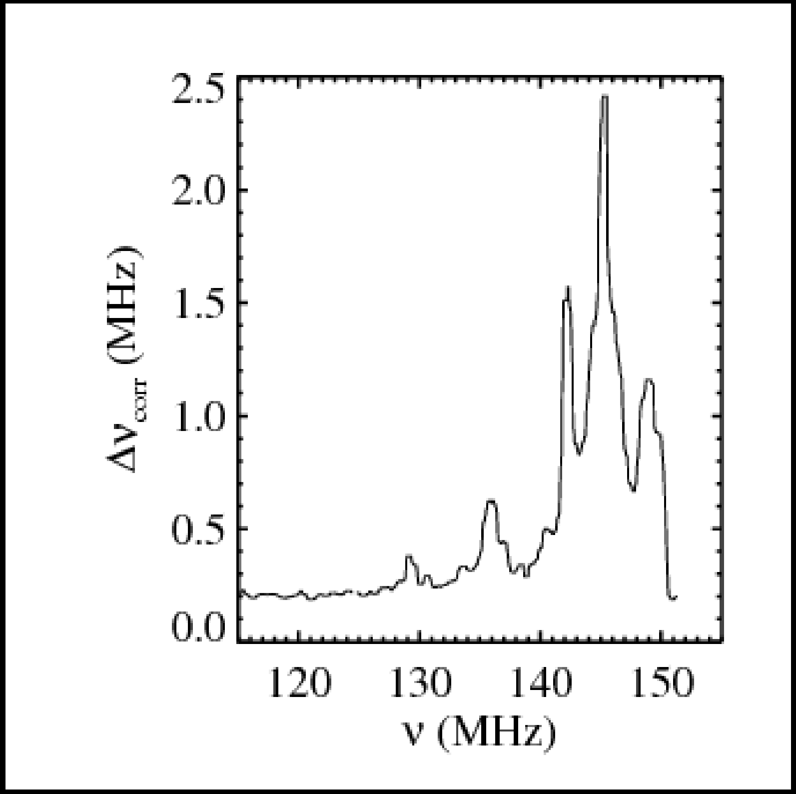
- 3D Fourier transform of (part) of the image cube shows the effect of velocities.
- Without velocity distortions contours are spherical in the spatial plane and (here) elliptical in the frequency-spatial plane (due to transformation from spatial coordinate to frequency).
- With velocity distortions the 1st is still spherical, but the 2nd shows tilted contours.



Correlation Length

- Reionization changes the correlation length along the frequency axis in a characteristic way: formation of large HII regions increases correlation length from ~200 kHz to ~1 MHz.
- Still substantially shorter than for the continuum foregrounds, so this can be used as a test for proper foreground subtraction.

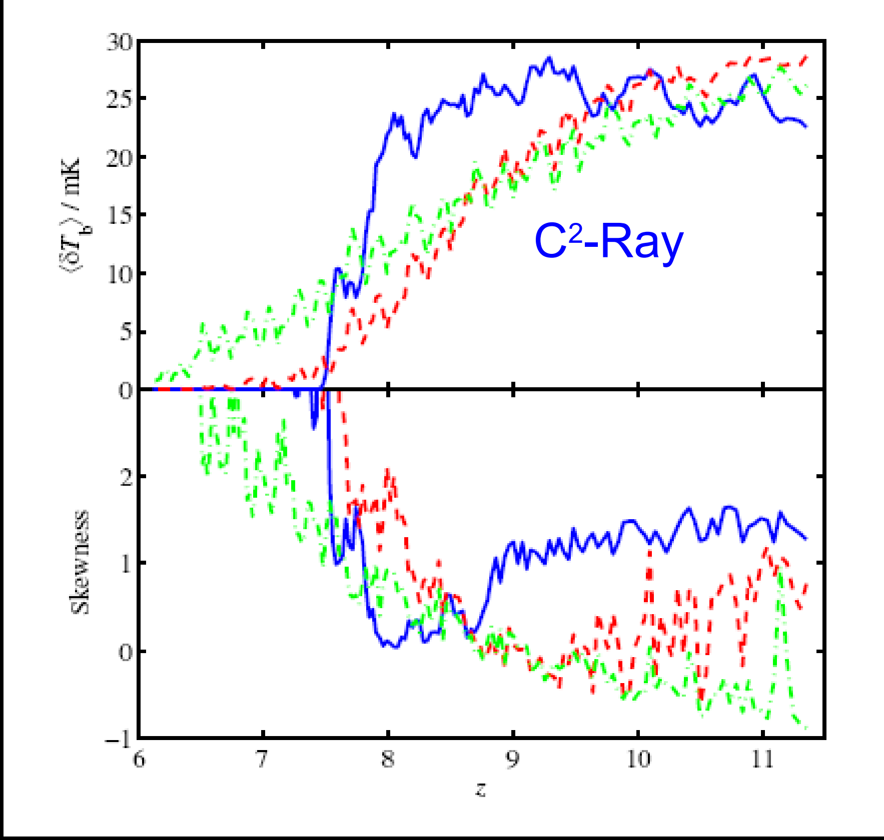




Simulation
100Mpc_f250C_f0_203

Skewness

- The 21cm signal is strongly non-Gaussian.
- This suggests that measuring the skewness could be an interesting diagnostic.
- The simulations show a characteristic evolution of skewness with increasing ionization: constant → decreasing → increasing.
- Skewness may offer an good way to detect/confirm the signal, if remnants of foreground subtractions and other effects are dominantly Gaussian (cf. Harker et al. 2008).



Conclusions

- The C^2-Ray methodology (finite volume/time-averaged optical depth) allows for efficient calculation of photo-ionization inside or outside a hydrodynamic calculation
- HII regions growing in the density field of a turbulent molecular cloud resemble observed HII regions.
- The 21cm signal from reionization is characterized by a range of statistical properties which should help in (confirming) its detection.