

NUCLEAR ASTROPHYSICS - INTRO.

• CONSTITUENTS

• PROTONS

$$S = 1/2$$

(PAULI EXCLUSION PRINCIPLE)

• NEUTRONS

$$S = 1/2$$

FERMIONS

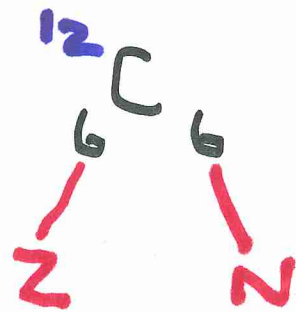
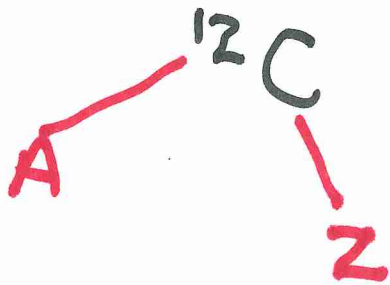
• QUARKS

$$p = uud$$

$$n = udd$$

NUCLEONS

• NOMENCLATURE



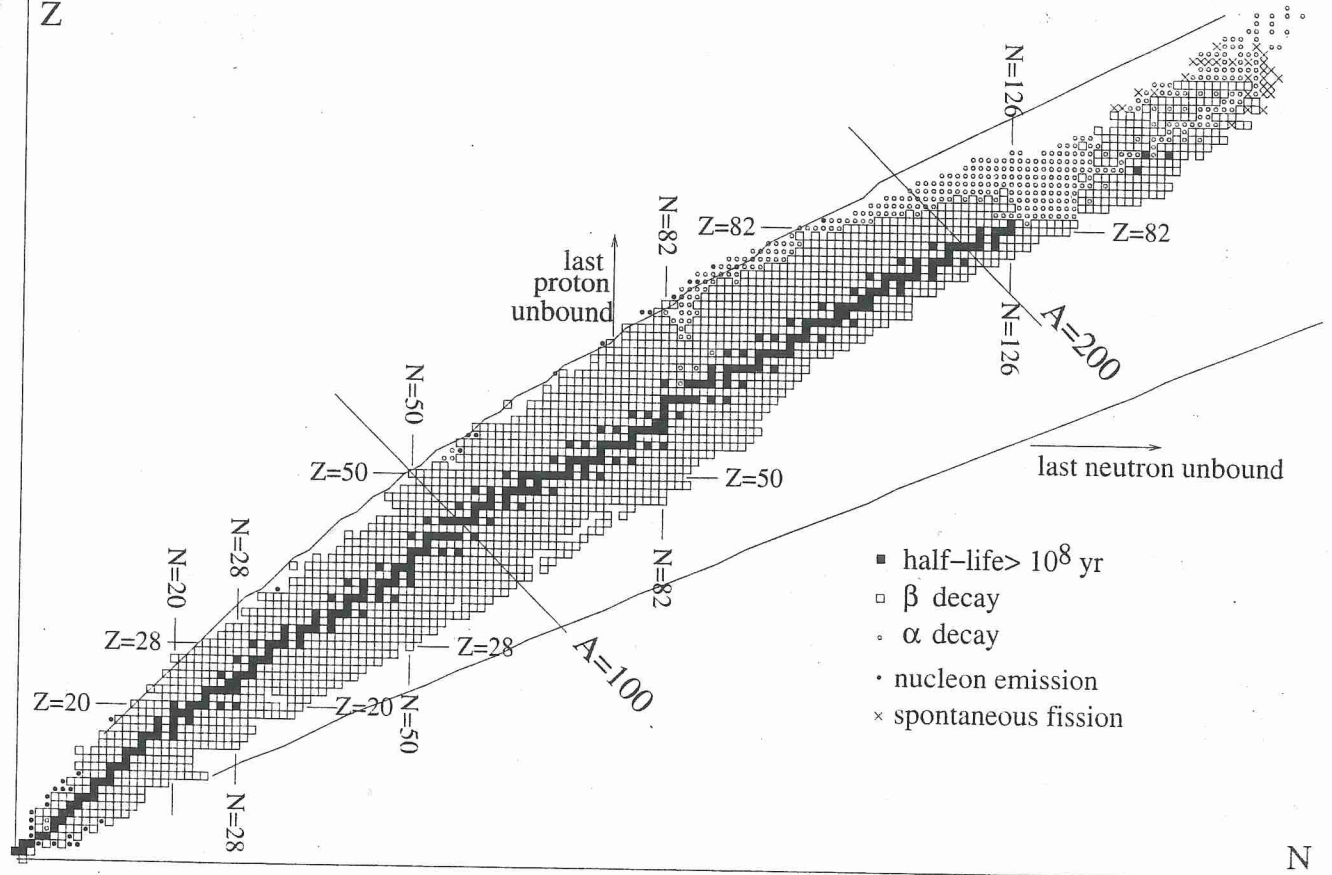
• NUCLIDE

- ISOTOPES

- ISOBARS

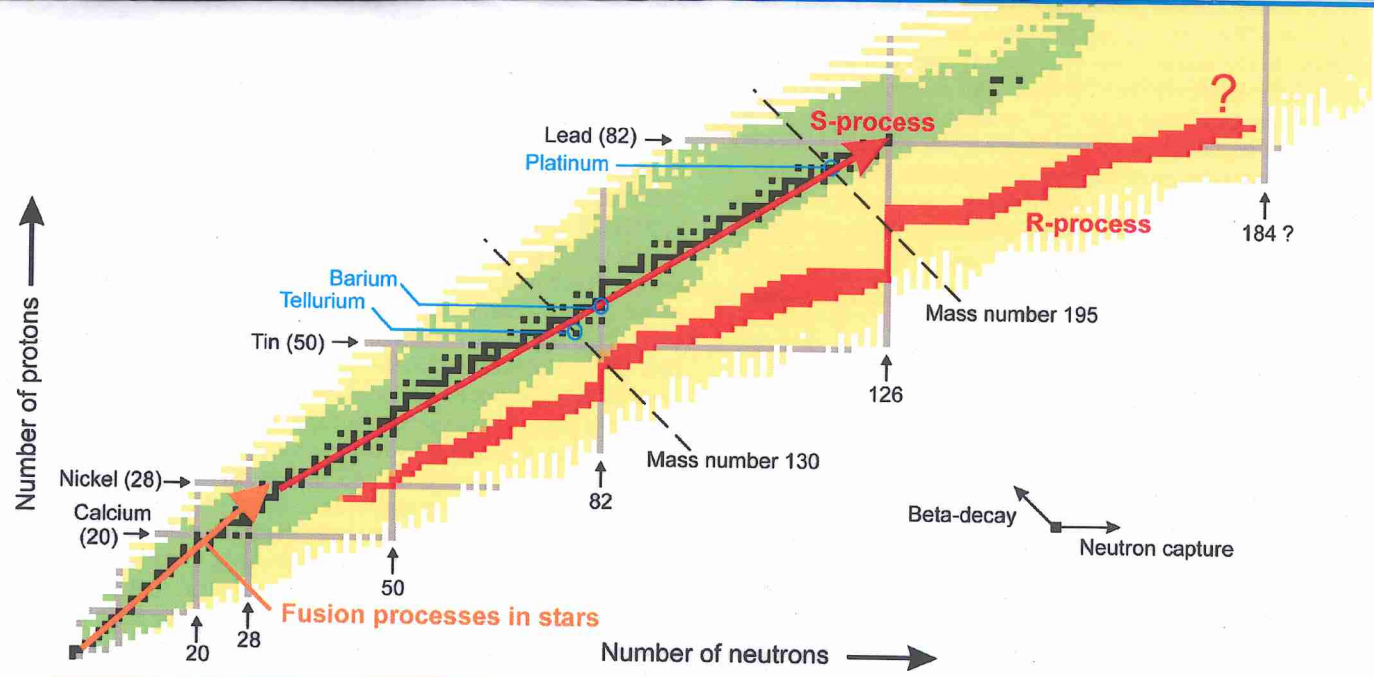
- ISOTONES

Fig. 0.2. The nuclei. The black squares are long-lived nuclei present on Earth. Unbound combinations of (N, Z) lie outside the lines marked "last proton/neutron unbound." Most other nuclei β -decay or α -decay to long-lived nuclei.



SEGRÉ CHART OF THE NUCLIDES

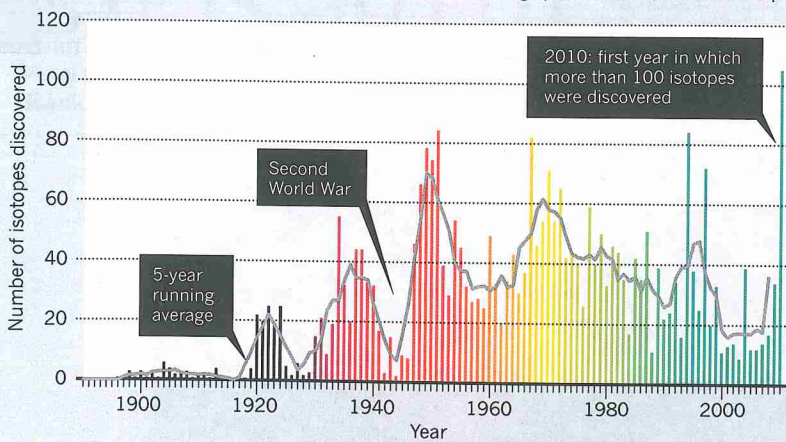
- $N \approx Z$ at low Z to $N \gg Z$ at high Z
- MAGIC NUMBERS for N & Z
- DRIP LINES
- SUPERHEAVY ELEMENTS?
- STEADY EXPLORATION



▲ **Fig. 2:** Typical predicted path [3] of the r-process (in red) on the chart of nuclides. On this chart, the number of neutrons increases column wise to the right, while the number of protons (element number) increases row wise upward. Each box therefore represents a nucleus with a specific number of protons and neutrons. Nuclei within a horizontal row represent the different isotopes of a given chemical element. Marked in grey are the "magic" proton and neutron numbers, for which the respective shells are closed. Shown are nuclei, which are stable or so long lived that they naturally exist (black), unstable nuclei for which the mass is known (green) and all other unstable nuclei that are predicted by nuclear theory to exist (yellow). The diagonal dashed lines mark the mass numbers where the r-process abundance maxima occur (see Fig. 1). Also shown in a very schematic way is the path of the fusion processes in stars (orange arrow) and the s-process (red arrow). For neutron capture N increases by one, while for a beta decay Z is increased by 1 and N is decreased by 1 (see arrows).

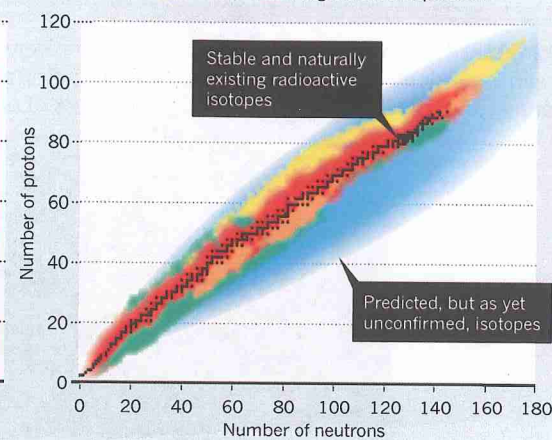
THE NUCLIDE TRAIL

Isotope discovery over the past 100 years (below) has jumped with each introduction of new technology. Some 2,700 radioactive isotopes have been discovered so far (below right), but about 3,000 more are predicted to exist.



Isotope-discovery technique

- Light particle reactions
- Neutron reactions
- Fusion
- Fragmentation/spallation



SOME PROPERTIES OF NUCLIDES

- SIZE
- DENSITY
- MASS
- BINDING ENERGY
- ENERGY LEVELS

RADII

- MANY are ~ spherical with fuzzy edge

e → charge and mass ← n
distributions slightly different

$$V \sim A v_0 \sim \frac{4}{3} \pi r_0^3 A$$

$$\sim \frac{4}{3} \pi R^3$$

$$\rightarrow R = r_0 A^{1/3} \quad \text{with and}$$

$$r_0 \sim 1.2 \text{ fm (charge)}$$

$$[1 \text{ fm} = 10^{-15} \text{ m}]$$

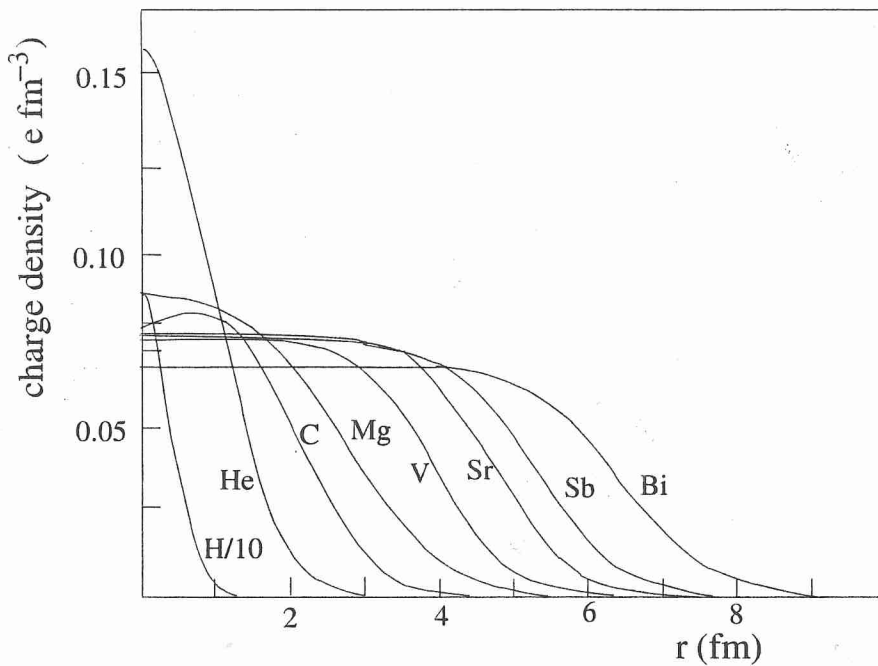
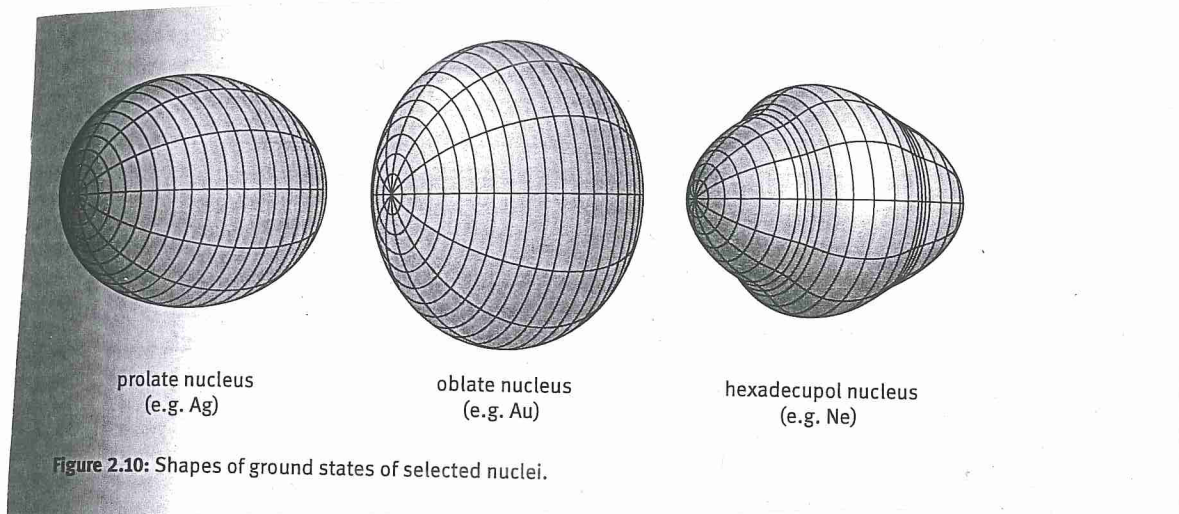


Fig. 1.1. Experimental charge density ($e \text{ fm}^{-3}$) as a function of $r(\text{fm})$ as determined in elastic electron-nucleus scattering [8]. Light nuclei have charge distributions that are peaked at $r = 0$ while heavy nuclei have flat distributions that fall to zero over a distance of $\sim 2 \text{ fm}$.

Table 1.1. Radii of selected nuclei as determined by electron-nucleus scattering [8]. The size of a nucleus is characterized by r_{rms} (1.11) or by the radius R of the uniform sphere that would give the same r_{rms} . For heavy nuclei, the latter is given approximately by (1.9) as indicated in the fourth column. Note the abnormally large radius of ^2H .

nucleus	r_{rms} (fm)	R (fm)	$R/A^{1/3}$ (fm)	nucleus	r_{rms} (fm)	R (fm)	$R/A^{1/3}$ (fm)
^1H	0.77	1.0	1.0	^{16}O	2.64	3.41	1.35
^2H	2.11	2.73	2.16	^{24}Mg	2.98	3.84	1.33
^4He	1.61	2.08	1.31	^{40}Ca	3.52	4.54	1.32
^6Li	2.20	2.8	1.56	^{122}Sb	4.63	5.97	1.20
^7Li	2.20	2.8	1.49	^{181}Ta	5.50	7.10	1.25
^9Be	2.2	2.84	1.37	^{209}Bi	5.52	7.13	1.20
^{12}C	2.37	3.04	1.33				

ALL MEASUREMENTS REFER
TO NUCLIDE'S GROUND STATE?
AND TO STABLE NUCLIDES?



hyperfine structure
isotopic shifts

> wavelength
shifts between
isotopes
↓
isotopic
abundance
ratios
(sometimes?)

MASSES

• UNITS

- atomic mass unit

$$m({}^{12}\text{C})_{\text{ATOM}} = 12 \text{ u}$$

- some nuclide mass measurements

- atomic masses

- nuclear masses

= diff is binding energy of all/some electrons

- see Baudouet et al.
extract for summary
of techniques

- <http://csrwww.in2p3.fr/AMDC/>?

AME 2003 AME 2012

Audi et al. Nucl. Phys A 729 (2003)

<https://www-nds.iaea.org/amdc/>

*
3527 nuclides

Mass units

$$\bullet m(^{12}\text{C atom}) = 12 \text{ u}$$

Table 1.2. Masses and rest energies for some important particles and nuclei explained in the text, mass ratios of charged particles or ions are most accurately determined by using mass spectrometers or Penning trap measurements of cyclotron frequencies. Combinations of ratios of various ions allows one to find the ratio of any mass to that of the ^{12}C atom which is defined as 12 u. Masses can be converted to rest energies accurately by using the theoretically calculable hydrogen atom spectrum. The neutron mass is derived accurately from a determination of the deuteron binding energy.

particle	mass m (u)	mc^2 (MeV)
electron e	$5.48579903(13) \times 10^{-4}$	0.510998902(21)
proton p	1.007276470(12)	938.271998(38)
neutron n	1.008664916(82)	939.56533(4)
deuteron d	2.013553210(80)	1875.612762(75)
^{12}C atom	12 (exact)	$12 \times 931.494013(37)$

$$\Delta m/m \approx 10^{-8}$$

$$\frac{\Delta B}{B} \leq 10^{-6}$$

→ sufficient for astrophysics
sufficient for testing nuclear
models

- mass spectrometers
 - ISOLDE
 - Penning trap
- Bayel plot

NUCLEAR BINDING ENERGY

$$B(Z, N) = [Zm_p + Nm_n - M_{\text{nuc}}(Z, N)]c^2$$

But atomic masses usually measured (nuc + electrons)

$$B_{\text{ab}}(Z, N) = [Z(m_p + m_e) + Nm_n - M_{\text{at}}(Z, N)]c^2 - B_e(Z, N)$$

$$B_e(Z) = 14.4Z^{2.4} - 1.55 \times 10^{-6} Z^{5.4} \quad \text{eV}$$

and 'other neglected'.

mass excess: $\delta = M_{\text{at}} - A$
in μU or keV

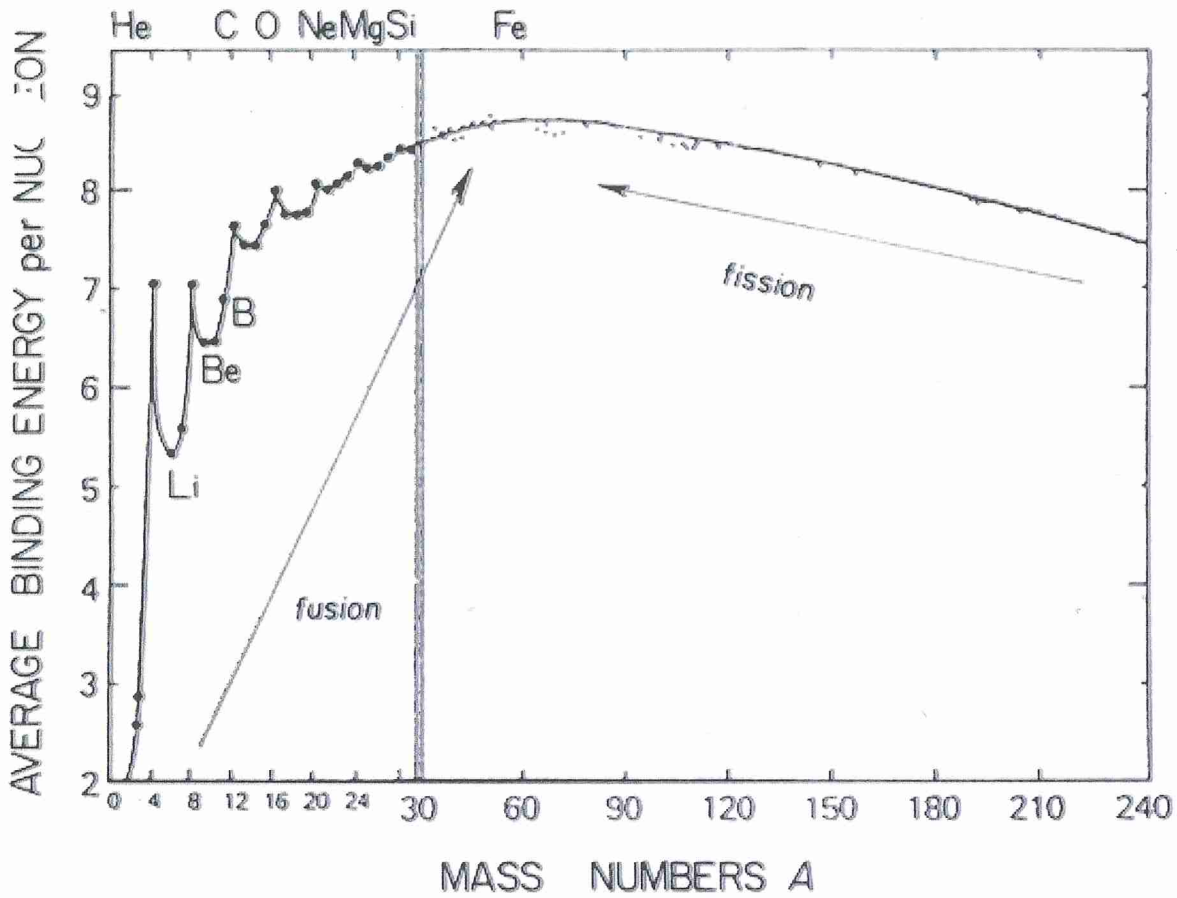
$$\text{mass defect: } \Delta M = M_{\text{at}} - ZM_H - NM_n$$

$$\delta = M_{at} - A \text{ with } A = (N+Z)m_0$$

$$B(z, N) = Z \delta(H) + N \delta(N) - \delta(z, N)$$

with $B_e(z) \cong 0$

12



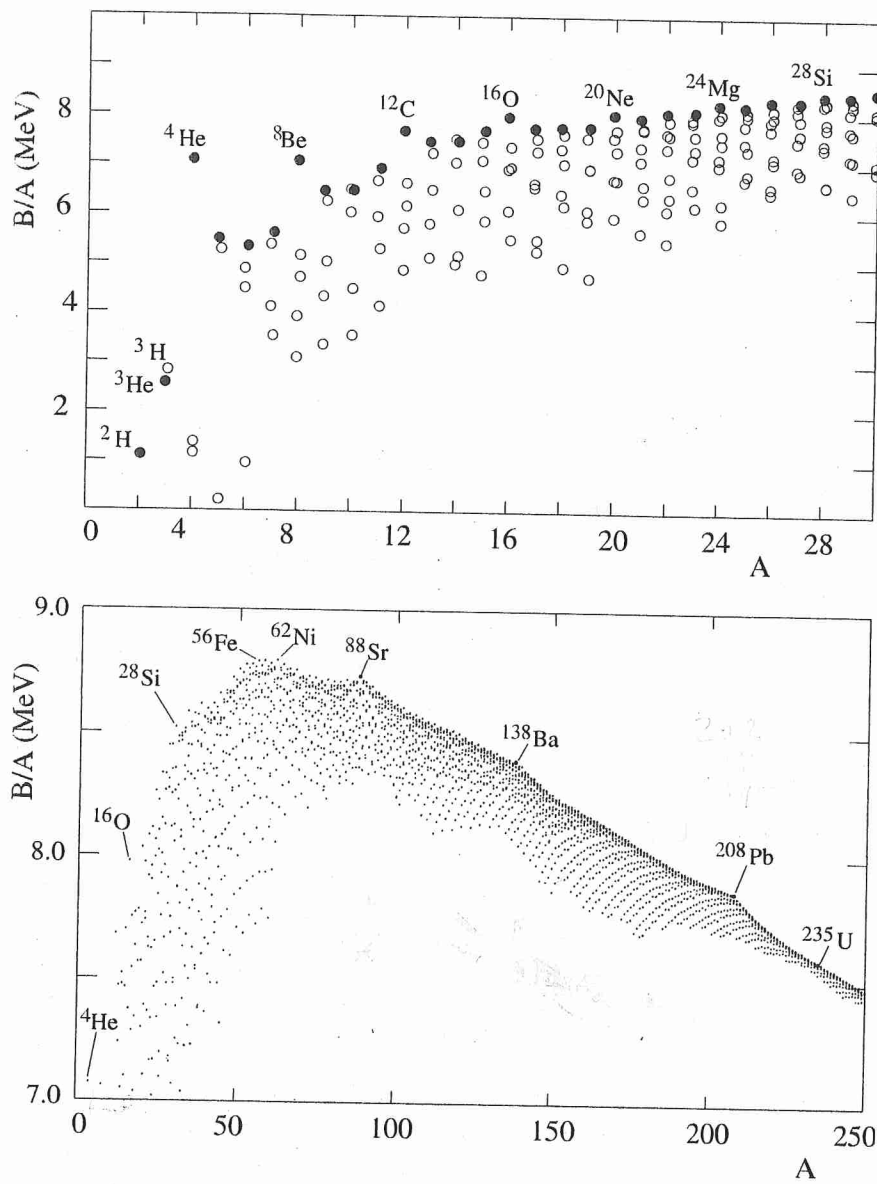


Fig. 1.2. Binding energy per nucleon, $B(A, Z)/A$, as a function of A . The upper panel is a zoom of the low- A region. The filled circles correspond to nuclei that are not β -radioactive (generally the lightest nuclei for a given A). The unfilled circles are unstable (radioactive) nuclei that generally β -decay to the lightest nuclei for a given A .

TABLE 3.1. Atomic masses and binding energies.

Element	Z	N	A	Atomic mass M_A (u)	Mass excess $M_A - A$ (μ u)	Mass defect ΔM_A (μ u)	Binding energy E_B (MeV)	E_B/A (MeV/A)
n-	0	1	1	1.008 665	8 665	0	-	-
H	1	0	1	1.007 825	7 825	0	-	-
D	1	1	2	2.014 102	14 102	-2 388	2.22	1.11
T	1	2	3	3.016 049	16 049	-9 106	8.48	2.83
He	2	1	3	3.016 029	16 029	-8 286	7.72	2.57
He	2	2	4	4.002 603	2 603	-30 377	28.30	7.07
He	2	4	6	6.018 886	18 886	-31 424	29.27	4.88
Li	3	3	6	6.015 121	15 121	-34 348	32.00	5.33
Li	3	4	7	7.016 003	16 003	-42 132	39.25	5.61
Be	4	3	7	7.016 928	16 928	-40 367	37.60	5.37
Be	4	5	9	9.012 182	12 182	-62 442	58.16	6.46
Be	4	6	10	10.013 534	13 534	-69 755	64.98	6.50
B	5	5	10	10.012 937	12 937	-69 513	64.75	6.48
B	5	6	11	11.009 305	9 305	-81 809	76.20	6.93
C	6	6	12	12.000 000	0	-98 940	92.16	7.68
N	7	7	14	14.003 074	3 074	-112 356	104.7	7.48
O	8	8	16	15.994 915	-5 085	-137 005	127.6	7.98
F	9	10	19	18.998 403	-1 597	-158 671	147.8	7.78
Ne	10	10	20	19.992 436	-7 564	-172 464	160.6	8.03
Na	11	12	23	22.989 768	-10 232	-200 287	186.6	8.11
Mg	12	12	24	23.985 042	-14 958	-212 837	198.3	8.26
Al	13	14	27	26.981 539	-18 461	-241 495	225.0	8.33
Si	14	14	28	27.976 927	-23 073	-253 932	236.5	8.45
P	15	16	31	30.973 762	-26 238	-282 252	262.9	8.48
K	19	20	39	38.963 707	-36 293	-358 266	333.7	8.56
Co	27	32	59	58.933 198	-66 802	-555 355	517.3	8.77
Zr	40	54	94	93.906 315	-93 685	-874 591	814.7	8.67
Ce	58	82	140	139.905 433	-94 567	-1 258 941	1 172.7	8.38
Ta	73	108	181	180.947 993	-52 007	-1 559 045	1 452.2	8.02
Hg	80	119	199	198.968 254	-31 746	-1 688 872	1 573.2	7.91
Th	90	142	232	232.038 051	38 051	-1 896 619	1 766.7	7.62
U	92	143	235	235.043 924	43 924	-1 915 060	1 783.9	7.59
U	92	144	236	236.045 563	45 563	-1 922 087	1 790.4	7.59
U	92	146	238	238.050 785	50 785	-1 934 195	1 801.7	7.57
Pu	94	146	240	240.053 808	53 808	-1 946 821	1 813.5	7.56

USE THIS TABLE TO CHECK YOUR
CALCULATIONS OF

- Mass excess
- Mass defect
- Binding energy per nucleon

NUCLEAR ENERGY LEVELS

- QUANTUM STATES
 E , spin J , parity π
 J^π ← TOTAL ANG. MOMENTUM

• EXCITED STATES

GENERALLY decay by γ -emission
 $t_{1/2} = \text{v.v. short}$

\therefore ALL TERRESTRIAL NUCLIDES IN
 GROUND STATE *NOT IN γ S

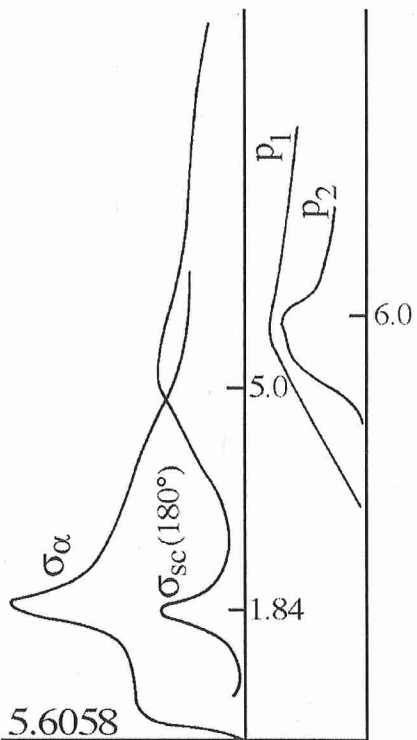
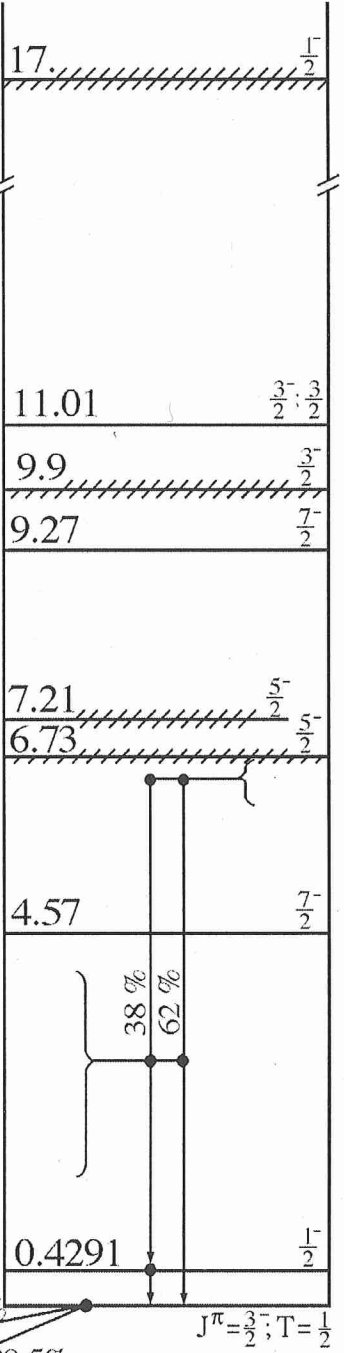
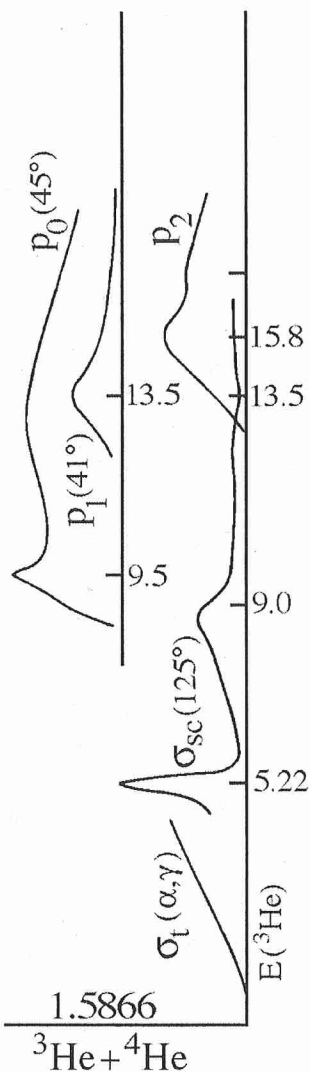
FEW EXCEPTIONS ($t \geq 1^5$)
 = ISOMERS

!

→ ^{180m}Ta with $\tau \sim 10^{15}$ yr
 ^{180}Ta ($E=0$) $\tau \sim 8$ hr

$$\frac{10.6758}{^6\text{Be}+n}$$

$$\frac{8.77}{^5\text{Li}+d}$$



$$\frac{5.6058}{^6\text{Li}+p}$$

$$\frac{3.3812}{^6\text{Li}+d-n}$$

$$\frac{1.1454}{^{10}\text{B}+p-\alpha}$$

$$\frac{0.1123}{^6\text{Li}+^3\text{He}-d}$$

$$\frac{-0.8804}{^7\text{Li}+^3\text{He}-t}$$

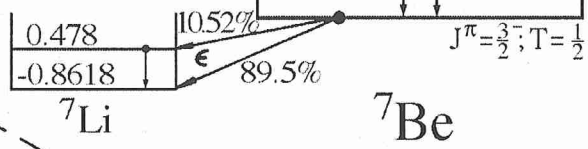
$$\frac{-1.6442}{^7\text{Li}+p-n}$$

$$\frac{-4.3696}{^7\text{Li}+^6\text{Li}-^6\text{He}}$$

$$\frac{-12.0827}{^9\text{Be}+p-t}$$

$$\frac{-14.2080}{^6\text{Li}+\alpha-t}$$

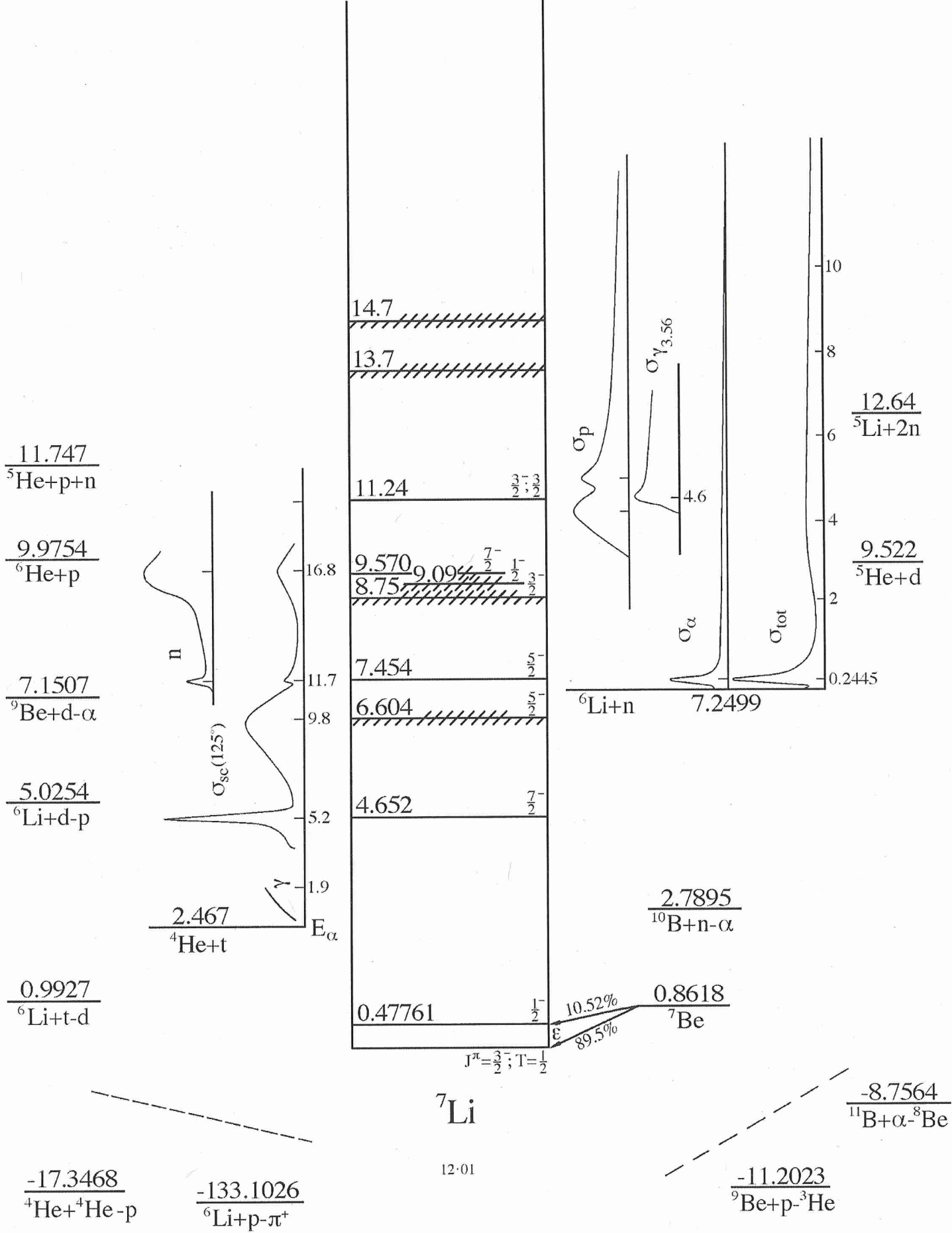
$$\frac{-18.991}{^4\text{He}+\alpha-n}$$



^7Be

12-01

T = isospin



${}^7\text{Li}$

12-01

NUCLEAR ENERGY LEVELS

RADIATIVE DECAYS

SELECTION RULES as in ATOMIC SP.

DECAY PROB. \sim ATOMIC SP
"BEYOND THE SCOPE OF THIS BOOK"

Table 4.1. Selection rules for radiative transitions

type	symbol	angular momentum change $ \Delta J \leq$	parity change
electric dipole	E1	1	yes
magnetic dipole	M1	1	no
electric quadrupole	E2	2	no
magnetic quadrupole	M2	2	yes
electric octopole	E3	3	yes
magnetic octopole	M3	3	no
electric 16-pole	E4	4	no
magnetic 16-pole	M4	4	yes

INTERNAL CONVERSION

- γ -ENERGY \rightarrow K, L, M electron and electron ejected

INTERNAL PAIR CONVERSION

- STABILIZATION OF ^{12}C

Fig 4.2

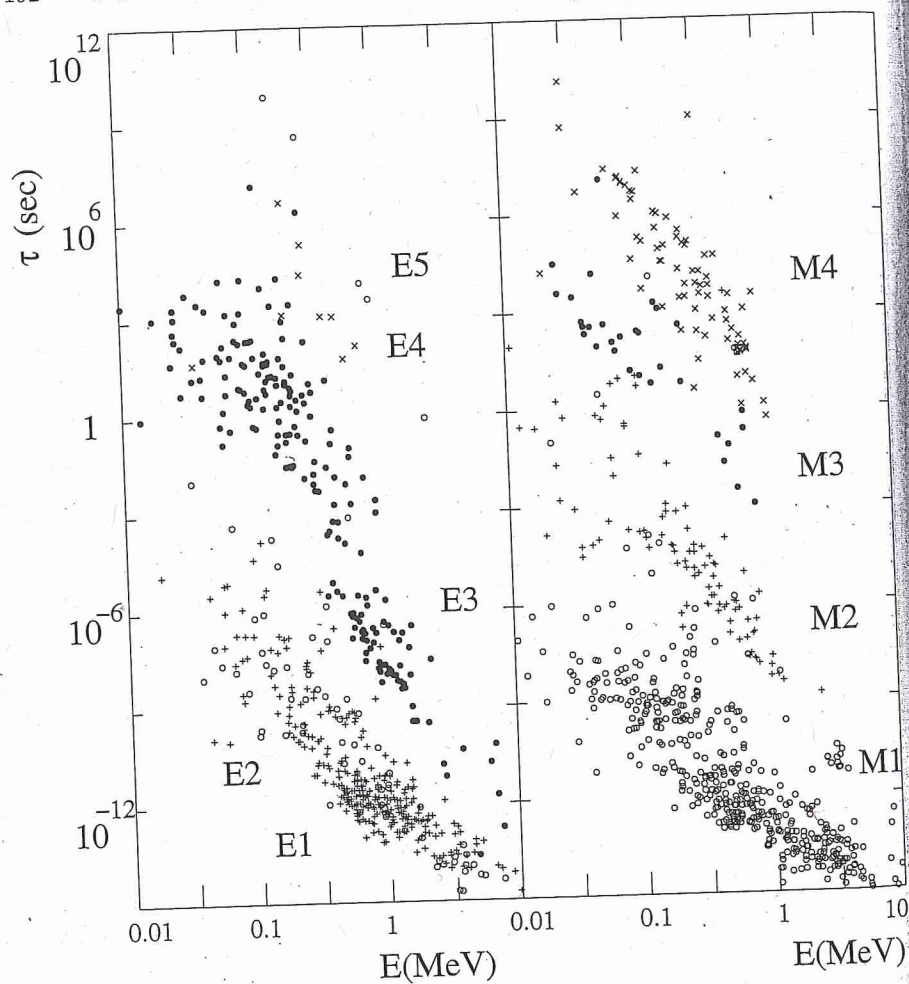


Fig. 4.5. Lifetimes of excited nuclear states as a function of E_γ for various electric and magnetic multipoles. The various multipoles separate relatively well except for the E1 (open circles) and E2 (crosses) transitions that have similar lifetimes. (For clarity, only 10% of the available E1 and E2 transitions appear in the plot.) The surprising strength of the E2 transitions is because they are generally due to collective quadrupole motions of several nucleons, whereas E1 transitions can often be viewed as single nucleon transitions.

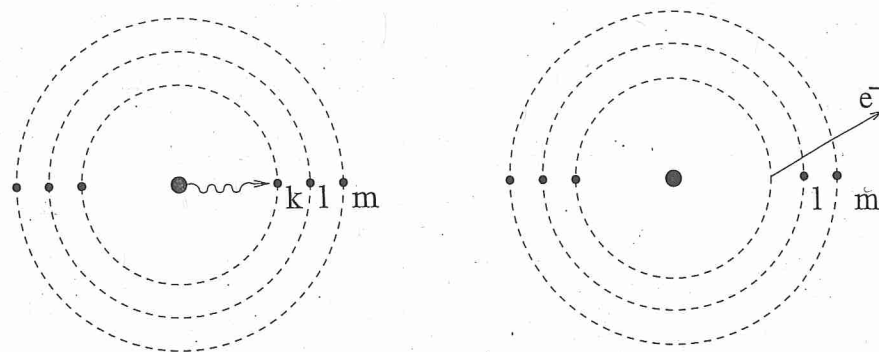


Fig. 4.6. An excited nucleus can transfer its energy to an atomic electron which is subsequently ejected from the atom. The process is called "internal conversion." The ejected electron can come from any of the atomic orbitals. In the figure, an electron from the deepest orbital is ejected, so-called K-conversion. Ejection of electrons in higher orbitals (L-, M- ... conversion) are generally less probable.

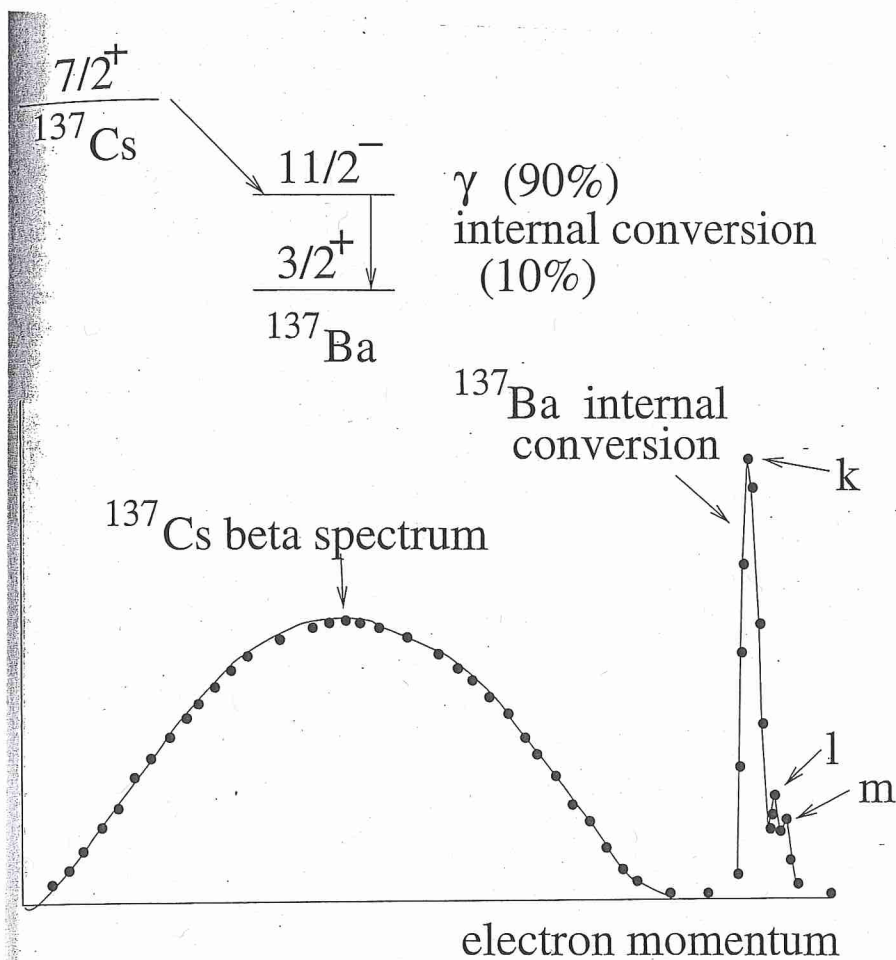


Fig. 4.7. The β -spectrum of ^{137}Cs and the internal conversion lines from the decay of the first excited state of ^{137}Ba [40]. Captures from the K, L and M orbitals are seen.

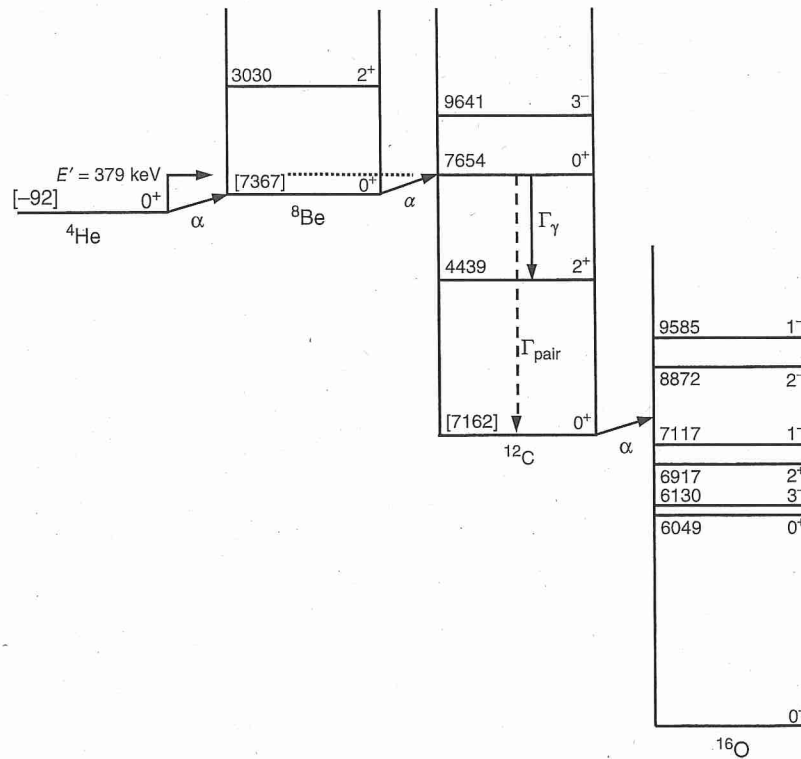


Figure 5.21 Energy level diagrams for the most important nuclides participating in helium burning. The numbers in square brackets represent the energy of the ground state of the system $^A_Z X_N + ^4_2\text{He}_2$ with respect to the ground state of the nucleus $^{A+4}_{Z+2} Y_{N+2}$

(i.e., the Q-value of the (α, γ) reaction on $^A_Z X_N$ or the α -particle separation energy of $^{A+4}_{Z+2} Y_{N+2}$). All information is adopted from Ajzenberg-Selove (1990) and Wang *et al.* (2012).

EXAMPLE of internal pair formation

$$\frac{\Gamma_{\text{pair}}}{\Gamma_{\gamma}} = \text{small}$$

can be released by the “fusion” of light nuclei into heavier ones, or by the “fission” of heavy nuclei into lighter ones.

As for nuclear volumes, it is observed that for stable nuclei which are not too small, say for $A > 12$, the binding energy B is in first approximation *additive*, i.e. proportional to the number of nucleons :

$$B(A, Z) \simeq A \times 8 \text{ MeV} ,$$

or more precisely

$$7.7 \text{ MeV} < B(A, Z)/A < 8.8 \text{ MeV} \quad 12 < A < 225 .$$

The numerical value of ~ 8 MeV per nucleon is worth remembering!

The additivity of binding energies is quite different from what happens in atomic physics where the binding energy of an atom with Z electrons increases as $Z^{7/3}$, i.e. $Z^{4/3}$ per electron. The nuclear additivity is again a manifestation of the saturation of nuclear forces mentioned above. It is surprising from the quantum mechanical point of view. In fact, since the binding energy arises from the pairwise nucleon–nucleon interactions, one might expect that $B(A, Z)/A$ should increase with the number of nucleon pairs $A(A-1)/2$.¹ The additivity confirms that nucleons only interact strongly with their nearest neighbors.

The additivity of binding energies and of volumes are related via the uncertainty principle. If we place A nucleons in a sphere of radius R , we can say that each nucleon occupies a volume $4\pi R^3 A/3$, i.e. it is confined to a linear dimension of order $\Delta x \sim A^{-1/3} R$. The uncertainty principle² then implies an uncertainty $\Delta p_i \sim \hbar A^{1/3}/R$ for each momentum component. For a bound nucleon, the expectation value of p_i must vanish, $\langle p_i \rangle = 0$, implying a relation between the momentum squared and the momentum uncertainty

$$(\Delta p_i)^2 = \langle p_i^2 \rangle - \langle p_i \rangle^2 = \langle p_i^2 \rangle . \quad (1.13)$$

Apart from numerical factors, the uncertainty principle then relates the mean nucleon kinetic energy with its position uncertainty

$$\left\langle \frac{p^2}{2m} \right\rangle \sim \frac{\hbar^2}{2m} \frac{A^{2/3}}{R^2} . \quad (1.14)$$

Since $R \simeq r_0 A^{1/3}$ this implies that the average kinetic energy per nucleon should be approximately the same for all nuclei. It is then not surprising that the same is true for the binding energy per nucleon.³ We will see in Chap. 2 how this comes about.

¹ In the case of atoms with Z electrons, it increases as $Z^{4/3}$. In the case of pairwise harmonic interactions between A fermions, the energy per particle varies as $A^{5/6}$.

² See for instance J.-L. Basdevant and J. Dalibard, *Quantum mechanics*, chapter 16, Springer-Verlag, 2002.

³ The virial theorem only guarantees that for power-law potentials these two energies are of the same order. Since the nuclear potential is not a power law, exceptions occur. For example, many nuclei decay by dissociation, e.g. ${}^8\text{Be} \rightarrow {}^4\text{He} {}^4\text{He}$. Considered as a “bound” state of two ${}^4\text{He}$ nuclei, the binding energy is, in fact,

As we can see from Fig. 1.2, some nuclei are exceptionally strongly bound compared to nuclei of similar A . This is the case for ${}^4\text{He}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$. As we shall see, this comes from a filled shell phenomenon, similar to the case of noble gases in atomic physics.

1.2.3 Mass units and measurements

The binding energies of the previous section were defined (1.12) in terms of nuclear and nucleon masses. Most masses are now measured with a precision of $\sim 10^{-8}$ so binding energies can be determined with a precision of $\sim 10^{-6}$. This is sufficiently precise to test the most sophisticated nuclear models that can predict binding energies at the level of 10^{-4} at best.

Three units are commonly used to describe nuclear masses: the atomic mass unit (u), the kilogram (kg), and the electron-volt (eV) for rest energies, mc^2 . In this book we generally use the energy unit eV since energy is a more general concept than mass and is hence more practical in calculations involving nuclear reactions.

It is worth taking some time to explain clearly the differences between the three systems. The atomic mass unit is a purely microscopic unit in that the mass of a ${}^{12}\text{C}$ atom is defined to be 12 u:

$$m({}^{12}\text{C atom}) \equiv 12 \text{ u} . \quad (1.15)$$

The masses of other atoms, nuclei or particles are found by measuring ratios of masses. On the other hand, the kilogram is a macroscopic unit, being defined as the mass of a certain platinum-iridium bar housed in Sèvres, a suburb of Paris. Atomic masses on the kilogram scale can be found by assembling a known (macroscopic) number of atoms and comparing the mass of the assembly with that of the bar. Finally, the eV is a hybrid microscopic-macroscopic unit, being defined as the kinetic energy of an electron after being accelerated from rest through a potential difference of 1 V.

Some important and very accurately known masses are listed in Table 1.2.

Mass spectrometers and ion traps. Because of its purely microscopic character, it is not surprising that masses of atoms, nuclei and particles are most accurately determined on the atomic mass scale. Traditionally, this has been done with mass spectrometers where ions are accelerated by an electrostatic potential difference and then deviated in a magnetic field. As illustrated in Fig. 1.3, mass spectrometers also provide the data used to determine the isotopic abundances that are discussed in Chap. 8.

The radius of curvature R of the trajectory of an ion in a magnetic field B after having been accelerated from rest through a potential difference V is

negative and ${}^8\text{Be}$ exists for a short time ($\sim 10^{-16}\text{s}$) only because there is an energy barrier through which the ${}^4\text{He}$ must tunnel.

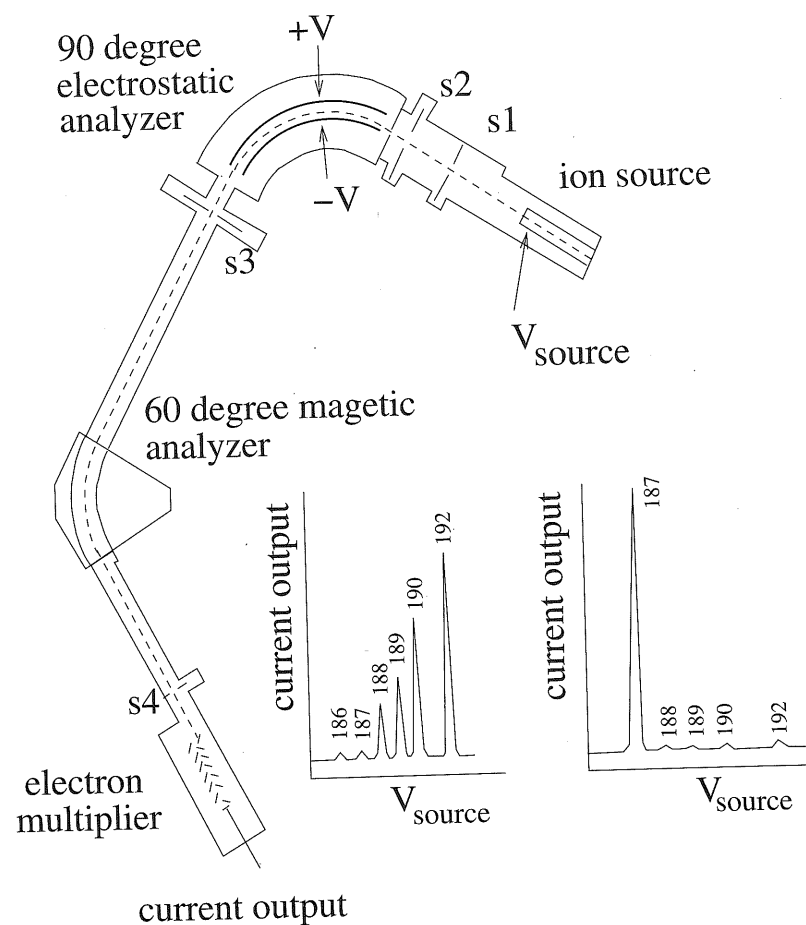


Fig. 1.3. A schematic of a “double-focusing” mass spectrometer [9]. Ions are accelerated from the source at potential V_{source} through the beam defining slit S_2 at ground potential. The ions are then electrostatically deviated through 90 deg and then magnetically deviated through 60 deg before impinging on the detector at slit S_4 . This combination of fields is “double focusing” in the sense that ions of a given mass are focused at S_4 independent of their energy and direction at the ion source. Mass ratios of two ions are equal to the voltage ratios leading to the same trajectories. The inset shows two mass spectra [10] obtained with sources of OsO_2 with the spectrometer adjusted to focus singly ionized molecules OsO_2^+ . The spectra show the output current as a function of accelerating potential and show peaks corresponding to the masses of the long-lived osmium isotopes, ^{186}Os – ^{192}Os . The spectrum on the left is for a sample of terrestrial osmium and the heights of the peaks correspond to the natural abundances listed in Appendix G. The spectrum on the right is for a sample of osmium extracted from a mineral containing rhenium but little natural osmium. In this case the spectrum is dominated by ^{187}Os from the β -decay $^{187}\text{Re} \rightarrow ^{187}\text{Os} + \bar{\nu}_e$ with $t_{1/2} = 4.15 \times 10^{10}$ yr (see Exercise 1.15).

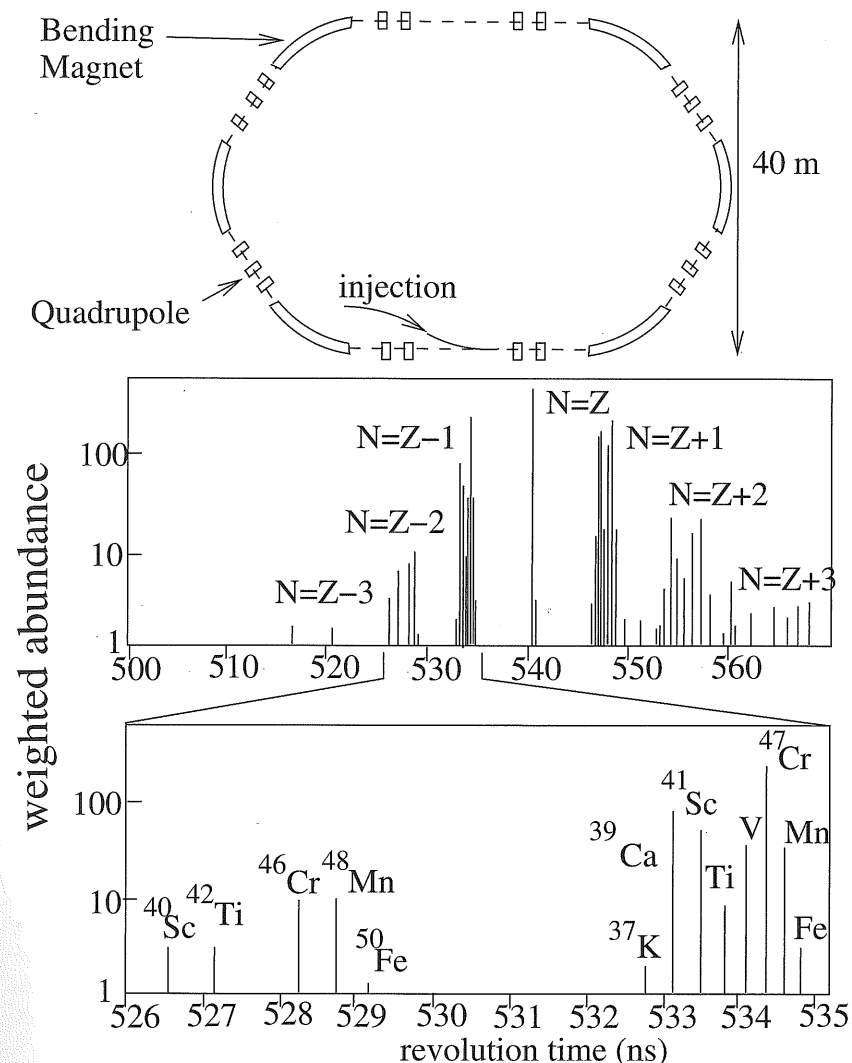


Fig. 1.4. Measurement of nuclear masses with isochronous mass spectroscopy [11]. Nuclei produced by fragmentation of 460 MeV/u ^{84}Kr on a beryllium target at GSI laboratory are momentum selected [12] and then injected into a storage ring [13]. About 10 fully ionized ions are injected into the ring where they are stored for several hundred revolutions before they are ejected and a new group of ions injected. A thin carbon foil ($17 \mu\text{g cm}^{-2}$) placed in the ring emits electrons each time it is traversed by an ion. The detection of these electrons measures the ion’s time of passage with a precision of ~ 100 ps. The periodicity of the signals determines the revolution period for each ion. The figure shows the spectrum of periods for many injections. The storage ring is run in a mode such that the non-relativistic relation for the period, $T \propto q/m$ is respected in spite of the fact that the ions are relativistic. The positions of the peaks for different q/m determine nuclide masses with a precision of ~ 200 keV (Exercise 1.16).

Table 1.2. Masses and rest energies for some important particles and nuclei. As explained in the text, mass ratios of charged particles or ions are most accurately determined by using mass spectrometers or Penning trap measurements of cyclotron frequencies. Combinations of ratios of various ions allows one to find the ratio of any mass to that of the ^{12}C atom which is defined as 12 u. Masses can be converted to rest energies accurately by using the theoretically calculable hydrogen atomic spectrum. The neutron mass is derived accurately from a determination of the deuteron binding energy.

particle	mass m (u)	mc^2 (MeV)
electron e	$5.485\,799\,03\,(13) \times 10^{-4}$	0.510 998 902 (21)
proton p	1.007 276 470 (12)	938.271 998 (38)
neutron n	1.008 664 916 (82)	939.565 33 (4)
deuteron d	2.013 553 210 (80)	1875.612 762 (75)
^{12}C atom	12 (exact)	$12 \times 931.494\,013\,(37)$

$$R = \frac{\sqrt{2Em}}{qB} = \frac{\sqrt{2V}}{B} \sqrt{\frac{m}{q}}, \quad (1.16)$$

where $E = qV$ is the ion's kinetic energy and q and m are its charge and mass. To measure the mass ratio between two ions, one measures the potential difference needed for each ion that yields the same trajectory in the magnetic field, i.e. the same R . The ratio of the values of q/m of the two ions is the ratio of the two potential differences. Knowledge of the charge state of each ion then yields the mass ratio.

Precisions of order 10^{-8} can be obtained with double-focusing mass spectrometers if one takes pairs of ions with similar charge-to-mass ratios. In this case, the trajectories of the two ions are nearly the same in an electromagnetic field so there is only a small difference in the potentials yielding the same trajectory. For example, we can express the ratio of the deuteron and proton masses as

$$\begin{aligned} \frac{m_d}{m_p} &= 2 \frac{m_d}{2m_p + m_e - m_e} \\ &= 2 \left[\frac{m_d}{2m_p + m_e} \right] \left[1 - \frac{m_e/m_p}{2(1 + m_e/m_p)} \right]^{-1}. \end{aligned} \quad (1.17)$$

The first factor in brackets, $m_d/(2m_p + m_e)$, is the mass ratio between a deuterium ion and singly ionized hydrogen molecule.⁴ The charge-to-mass ratio of these two objects is nearly the same and can therefore be very accurately measured with a mass spectrometer. The second bracketed term contains a

⁴ We ignore the small (\sim eV) electron binding energy.

small correction depending on the ratio of the electron and proton masses. As explained below, this ratio can be accurately measured by comparing the electron and proton cyclotron frequencies. Equation (1.17) then yields m_d/m_p .

Similarly, the ratio between m_d and the mass of the ^{12}C atom ($= 12$ u) can be accurately determined by comparing the mass of the doubly ionized carbon atom with that of the singly ionized $^2\text{H}_3$ molecule (a molecule containing 3 deuterons). These two objects have, again, similar values of q/m so their mass ratio can be determined accurately with a mass spectrometer. The details of this comparison are the subject of Exercise 1.7. The comparison gives the mass of the deuteron in atomic-mass units since, by definition, this is the deuteron- ^{12}C atom ratio. Once m_d is known, m_p is then determined by (1.17).

Armed with m_e , m_p , m_d and $m(^{12}\text{C atom}) \equiv 12$ u it is simple to find the masses of other atoms and molecules by considering other pairs of ions and measuring their mass ratios in a mass spectrometer.

The traditional mass-spectrometer techniques for measuring mass ratios are difficult to apply to very short-lived nuclides produced at accelerators. While the radius of curvature in a magnetic field of ions can be measured, the relation (1.16) cannot be applied unless the kinetic energy is known. For non-relativistic ions orbiting in a magnetic field, this problem can be avoided by measuring the orbital period $T = m/qB$. Ratios of orbital periods for different ions then yield ratios of charge-to-mass ratios. An example of this technique applied to short-lived nuclides is illustrated in Fig. 1.4.

The most precise mass measurements for both stable and unstable species are now made through the measurement of ionic cyclotron frequencies,

$$\omega_c = \frac{qB}{m}. \quad (1.18)$$

For the proton, this turns out to be $9.578 \times 10^7 \text{ rad s}^{-1} \text{ T}^{-1}$. It is possible to measure ω_c of individual particles bound in a Penning trap. The basic configuration of such a trap is shown in Fig. 1.5. The electrodes and the external magnetic field of a Penning trap are such that a charged particle oscillates about the trap center. The eigenfrequencies correspond to oscillations in the z direction, cyclotron-like motion in the plane perpendicular to the z direction, and a slower radial oscillation. It turns out that the cyclotron frequency is sum of the two latter frequencies.

The eigenfrequencies can be determined by driving the corresponding motions with oscillating dipole fields and then detecting the change in motional amplitudes with external pickup devices or by releasing the ions and measuring their velocities. The frequencies yielding the greatest energy absorptions are the eigenfrequencies.

If two species of ions are placed in the trap, the system will exhibit the eigenfrequencies of the two ions and the two cyclotron frequencies determined.

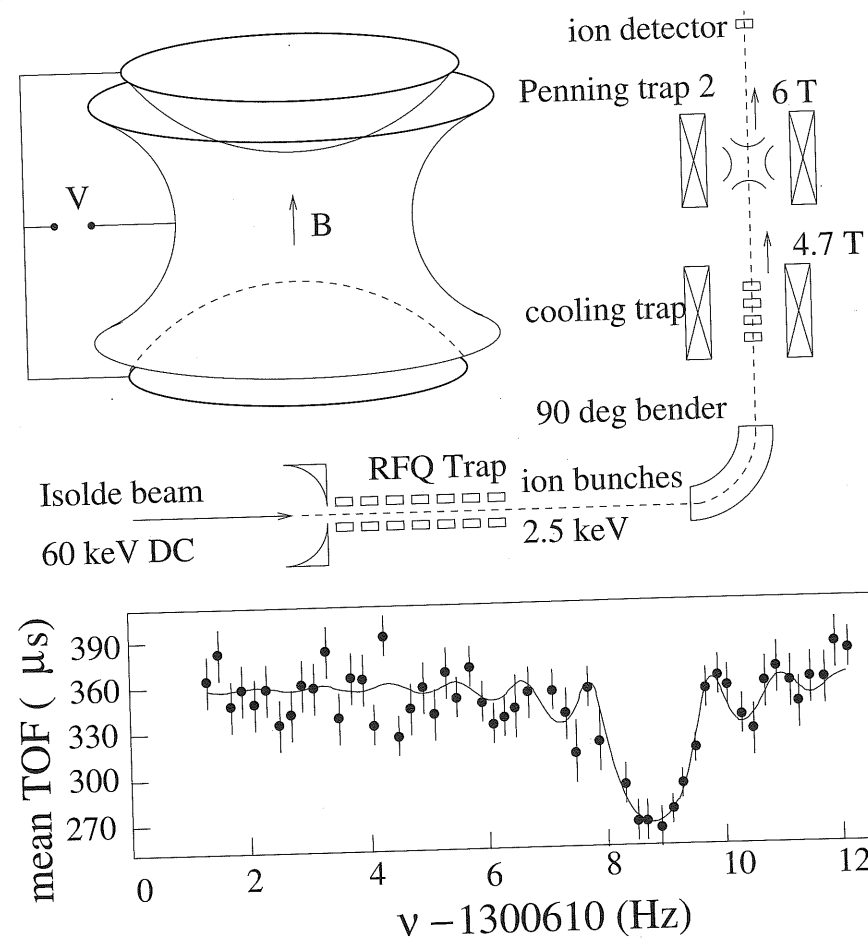


Fig. 1.5. The Isoltrap facility at CERN for the measurement of ion masses. The basic configuration of a Penning trap is shown in the upper left. It consists of two end-cap electrodes and one ring electrode at a potential difference. The whole trap is immersed in an external magnetic field. A charged particle oscillates about the center of the trap. The cyclotron frequency, qB/m can be derived from the eigenfrequencies of this oscillation and knowledge of the magnetic field allows one to derive the charge-to-mass ratio. In Isoltrap, the 60 keV beam of radioactive ions is decelerated to 3 keV and then cooled and isotope selected (e.g. by selective ionization by laser spectroscopy) in a first trap. The selected ions are then released into the second trap where they are subjected to an RF field. After a time of order 1 s, the ions are released and detected. If the field is tuned to one of the eigenfrequencies, the ions gain energy in the trap and the flight time from trap to detector is reduced. The scan in frequency on the bottom panel, for singly-ionized ^{70}Cu , $t_{1/2} = 95.5$ s [15], demonstrates that frequency precisions of order 10^{-8} can be obtained.

The ratio of the frequencies gives the ratio of the masses. Precisions in mass ratios of 10^{-9} have been obtained [14].

The neutron mass. The one essential mass that cannot be determined with these techniques is that of the neutron. Its mass can be most simply derived from the proton and deuteron masses and the deuteron binding energy, $B(2,1)$

$$m_d = m_p + m_n - B(2,1)/c^2. \quad (1.19)$$

The deuteron binding energy can be deduced from the energy of the photon emitted in the capture of neutrons by protons,



For slow (thermal) neutrons captured by stationary protons, the initial kinetic energies are negligible (compared to the nucleon rest energies) so to very good approximation, the ${}^2\text{H}$ binding energy is just the energy of the final state photon (Exercise 1.8):

$$B(2,1) = E_\gamma \left(1 + \frac{E_\gamma}{2m_d c^2} \right). \quad (1.21)$$

The correction in parenthesis comes from the fact that the ${}^2\text{H}$ recoils from the photon and therefore carries some energy. Neglecting this correction, we have

$$m_n = m_d - m_p + E_\gamma/c^2. \quad (1.22)$$

Thus, to measure the neutron mass we need the energy of the photon emitted in neutron capture by protons.

The photon energy can be deduced from its wavelength

$$E_\gamma = \frac{2\pi\hbar c}{\lambda_\gamma}, \quad (1.23)$$

so we need an accurate value of $\hbar c$. This can be found most simply by considering photons from transitions of atomic hydrogen whose energies can be calculated theoretically. Neglecting calculable fine-structure corrections, the energy of photons in a transition between states of principal quantum numbers n and m is

$$E_{nm} = (1/2)\alpha^2 m_e c^2 (n^{-2} - m^{-2}), \quad (1.24)$$

corresponding to a wavelength

$$\lambda_{nm} = \frac{2\pi\hbar c}{(1/2)\alpha^2 m_e c^2 (n^{-2} - m^{-2})} = \frac{1}{R_\infty (n^{-2} - m^{-2})}, \quad (1.25)$$

where $R_\infty = \alpha^2 m_e c^2 / 4\pi\hbar c$ is the Rydberg constant and $\alpha = e^2 / 4\pi\epsilon_0 \hbar c \sim 1/137$ is the fine-structure constant. This gives

$$2\pi\hbar c = \lambda_{mn} (n^{-2} - m^{-2}) (1/2)\alpha^2 m_e c^2 = R_\infty^{-1} (1/2)\alpha^2 m_e c^2. \quad (1.26)$$

The value of R_∞ can be found from any of the hydrogen lines by using (1.25) [16]. The currently recommended value is [17]

$$R_\infty = 10\,973\,731.568\,549(83)\text{m}^{-1}. \quad (1.27)$$

Substituting (1.26) into (1.23) we get a formula relating photon energies and wavelengths

$$\frac{E_\gamma}{m_e c^2} = \frac{R_\infty^{-1}}{\lambda_\gamma} (1/2)\alpha^2. \quad (1.28)$$

The fine-structure constant can be determined by a variety of methods, for example by comparing the electron cyclotron frequency with its spin-precession frequency.

The wavelength of the photon emitted in (1.20) was determined [18] by measuring the photon's diffraction angle (to a precision of 10^{-8} deg) on a silicon crystal whose interatomic spacing is known to a precision of 10^{-9} yielding

$$\lambda_\gamma = 5.576\,712\,99(99) \times 10^{-13}\text{m}. \quad (1.29)$$

Substituting this into (1.28), using the value of m_e (Table 1.2) and then using (1.21) we get

$$B(2,1)/c^2 = 2.388\,170\,07(42) \times 10^{-3}\text{u}. \quad (1.30)$$

Substituting this into (1.19) and using the deuteron and proton masses gives the neutron mass.

The eV scale. To relate the atomic-mass-unit scale to the electron-volt energy scale we can once again use the hydrogen spectrum

$$m_e c^2 = \frac{4\pi\hbar c R_\infty}{\alpha^2}. \quad (1.31)$$

The electron-volt is by definition the potential energy of a particle of charge e when placed a distance $r = 1$ m from a charge of $q = 4\pi\epsilon_0 r$, i.e.

$$1\text{eV} = \frac{eq}{4\pi\epsilon_0 r} \quad r = 1\text{m}, \quad q = 1.112 \times 10^{-10}\text{C}. \quad (1.32)$$

Dividing (1.31) by (1.32) we get

$$\frac{m_e c^2}{1\text{eV}} = \frac{4\pi}{\alpha^3} \frac{e}{1.112 \times 10^{-10}\text{C}} \frac{R_\infty}{1\text{m}^{-1}}. \quad (1.33)$$

We see that in order to give the electron rest-energy on the eV scale we need to measure the atomic hydrogen spectrum in meters, e in units of Coulombs, and the unit-independent value of the fine-structure constant. The currently accepted value is given by (1.3). This allows us to relate the atomic-mass-unit scale to the electron-volt scale by simply calculating the rest energy of the ^{12}C atom:

$$m c^2 (^{12}\text{C atom}) = m_e c^2 \frac{12\text{u}}{m_e} = 12 \times 931.494\,013(37)\text{MeV}, \quad (1.34)$$

or equivalently $1\text{u} = 931.494\,013\text{MeV}/c^2$.

The kg scale. Finally, we want to relate the kg scale to the atomic mass scale. Conceptually, the simplest way is to compare the mass of a known number of particles (of known mass on the atomic-mass scale) with the mass of the platinum-iridium bar (or one of its copies). One method [19] uses a crystal of ^{28}Si with the number of atoms in the crystal being determined from the ratio of the linear dimension of the crystal and the interatomic spacing. The interatomic spacing can be determined through laser interferometry. The method is currently limited to a precision of about 10^{-5} because of uncertainties in the isotopic purity of the ^{28}Si crystal and in uncertainties associated with crystal imperfections. It is anticipated that once these errors are reduced, it will be possible to define the kilogram as the mass of a certain number of ^{28}Si atoms. This would be equivalent to fixing the value of the Avogadro constant, N_A , which is defined to be the number of atoms in 12 g of ^{12}C .

1.3 Quantum states of nuclei

While (A, Z) is sufficient to denote a nuclear *species*, a given (A, Z) will generally have a large number of quantum states corresponding to different wavefunctions of the constituent nucleons. This is, of course, entirely analogous to the situation in atomic physics where an atom of atomic number Z will have a lowest energy state (ground state) and a spectrum of excited states. Some typical nuclear spectra are shown in Fig. 1.6.

In both atomic and nuclear physics, transitions from the higher energy states to the ground state occurs rapidly. The details of this process will be discussed in Sect. 4.2. For an isolated nucleus the transition occurs with the emission of photons to conserve energy. The photons emitted during the decay of excited nuclear states are called γ -rays. A excited nucleus surrounded by atomic electrons can also transfer its energy to an electron which is subsequently ejected. This process is called *internal conversion* and the ejected electrons are called *conversion electrons*. The energy spectrum of γ -rays and conversion electrons can be used to derive the spectrum of nuclear excited states.

Lifetimes of nuclear excited states are typically in the range 10^{-15} – 14^{-10} s. Because of the short lifetimes, with few exceptions only nuclei in the ground state are present on Earth. The rare excited states with lifetimes greater than, say, 1 s are called *isomers*. An extreme example is the first excited state of ^{180}Ta which has a lifetime of 10^{15} yr whereas the ground state β -decays with a lifetime of 8 hr. All ^{180}Ta present on Earth is therefore in the excited state.

Isomeric states are generally specified by placing a m after A , i.e.

$$^{180m}\text{Ta}. \quad (1.35)$$