AST 376 Cosmology — Problem Set 4

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I. INFLATING-AWAY THE MONOPOLE PROBLEM

We estimated that at the time of grand-unified (GUT) symmetry breaking, $t_{\rm GUT} \sim 10^{-36}$ s, the number density of magnetic monopoles should have been of order $n_{\rm mono} \sim 10^{76}$ cm^{-3}. We also argued the monopole mass should be of order the GUT mass-energy scale, $m_{\rm mono} \simeq 10^{15}$ GeV/c^2.

Current observational limits on the density of magnetic monopoles indicate that their (present-day) density parameter is: $\Omega_{\rm mono} < 10^{-6}$. If monopoles formed at $t_{\rm GUT}$, how many e-foldings (i.e., the number N in e^N) of inflation would be required to drive the current monopole density below the observational bound given above? Assume that inflation took place immediately after the creation of the monopoles.

Answer: The present-day monopole energy density is $\rho_{\rm mono,0} < \Omega_{\rm mono}\rho_{\rm crit,0} \approx 10^{-6} \cdot 10^{-29}$ g cm^{-3}. Now, if the monoplole mass corresponds to the GUT scale then the number density of monopoles at the GUT redshift (see problem 2, Eq. 5) is less than

$$\begin{aligned} n_{\rm mono,GUT} &\lesssim z_{\rm GUT}^3 \Omega_{\rm mono} \rho_{\rm crit,0}/m_{\rm H} \\ &\approx \left(\frac{\epsilon_{\rm GUT}}{k_{\rm B} T_{\rm CMB}}\right) \Omega_{\rm mono} \left(\frac{3H_0^2}{8\pi G}\right) / m_{\rm H} \\ &\approx \left(4.3 \times 10^{27}\right) \left(10^{-6}\right) \left(9.2 \times 10^{-29} \text{ g cm}^{-3}\right) / \left(1.67 \times 10^{-24} \text{ g}\right) \\ &\approx 4 \times 10^{56} \text{ cm}^{-3} . \end{aligned} \tag{1}$$

The density at the end of inflation is really about a factor of $\sim 100^{3/2} = 1000$ smaller than this because inflation is non-instantaneous and the redshift has been overestimated by a factor of $z \propto t^{1/2}$, i.e. $n \propto t^{-3/2}$. Therefore the number density at the end of inflation is closer to $n_{\rm mono,end} \approx 4 \times 10^{53}$ cm^{-3}.

In class we calculated an expected value for the monopole number density to be $n_{\rm mono,GUT\ (expected)} \approx 10^{76}$ cm^{-3} at the start of inflation, i.e. $t = 10^{-36}$ s. Because the monopole density scales as $n_{\rm mono} \propto a^{-3}$ the number of e-foldings is the natural log of the cubic root of expected and observed densities:

$$N \approx \frac{1}{3} \log \left[\frac{n_{\rm mono,GUT\ (expected)}}{n_{\rm mono,end\ (observed)}}\right] \approx \frac{1}{3} \log \left[\frac{10^{76}}{4 \times 10^{53}}\right] \approx 17.2 . \tag{2}$$

This is not quite the expected value of $N \sim 60$ because the monopole observations are still not sensitive enough to constrain the monopole density within the theoretical range.

II. SIZE OF POST-INFLATION UNIVERSE

Consider our present-day observable universe (all the space inside the current horizon), and figure out the size of this spherical region at the time just after inflation ended, $t_{\rm final} \simeq 10^{-34}$ s.

Answer: When we extrapolate back in time, cosmic expansion was most recently dominated by a cosmological-constant dark energy, before that by matter, and before that by radiation. Therefore, we use the following Hubble parameter:

$$H(t) \equiv \frac{\dot{a}}{a} = H_0\sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda}\,, \tag{3}$$

with $H_0 = 70$ km/s/Mpc, $\Omega_r = 9\times 10^{-5}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$. Therefore, the comoving radius of the observable universe is

$$r_0 = \frac{c}{H_0}\int_0^\infty \frac{dz'}{\sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda}} \approx 13.89 \text{ Gpc}\,. \tag{4}$$

From the notes, the redshift of inflation is roughly

$$z_{\rm GUT} \approx \frac{\epsilon_{\rm GUT}}{k_{\rm B}T_{\rm CMB}} \approx \frac{10^{15}\text{ GeV}}{(1.38\times 10^{-16}\text{ ergs/K})(2.7\text{ K})} \approx 4.3\times 10^{27}\,. \tag{5}$$

so the rough size of the observable Universe around inflation is

$$r_{\rm GUT} \approx \frac{r_0}{z_{\rm GUT}} \approx \frac{13.89\text{ Gpc}}{4.3\times 10^{27}} \approx 3.23\times 10^{-27}\text{ Gpc} \approx 10\text{ cm}\,. \tag{6}$$

The size of the observable universe after inflation is really about a factor of ~ 10 larger than this because inflation is non-instantaneous and the redshift has been overestimated by a factor of $z \propto t^{1/2}$.