AST376 (Spring 2014)

COSMOLOGY

Problem Set 3

Due in class: Thursday, April 10, 2014

(worth 10/100)

1. Dark Energy Equation of State

Suppose dark energy has an equation of state $P_{\rm vac} = w\rho_{\rm vac}c^2$, where here w is supposed to be time dependent, i.e., $w = w(z)$. Show that the Hubble expansion rate, well *after* the radiation-dominated epoch, can be written as:

$$\frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_{\rm DE}\exp\left[3\int_0^z [1+w(x)]d\ln(1+x)\right] , \qquad (1)$$

where $\Omega_{\rm DE}$ is the fraction of critical density contributed by dark energy (DE) today. For simplicity, let's pick the following cosmological parameters: $\Omega_m = 0.3, \Omega_{\rm DE} = 0.7, H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

Many experiments in the next decade, including UT's Hobby-Eberly Telescope Dark Energy Experiment (HETDEX), aim at constraining $w(z)$ to infer clues on the nature of dark energy. To understand how sensitive these measurements have to be, plot the:

(a) age of the universe

(b) luminosity distance,

as a function of redshift for 4 different models: $w = -1$ (Einstein's cosmological constant), $w = -1/3, w = -0.5 + 0.1z, w = -0.5 - 0.05z$, for $0 < z < 5$.

If you have trouble evaluating the expression in equ. (1), proceed numerically. Pick the scaling, and units, of your axes appropriately.

2. From Cosmic Deceleration to Acceleration

Early on in cosmic history, dark energy was not important, and normal matter caused cosmic expansion to decelerate ($\ddot{a} < 0$). At some time, however, below a critical redshift, $z_{\rm crit}$, dark energy takes over, causing an accelerated expansion ($\ddot{a} > 0$).

a. Using current cosmological parameters, calculate the value of $z_{\rm crit}$ (where $\ddot{a} = 0$).

b. How long ago was that (in units of Gyr)? I.e., calculate the *lookback time* to redshift $z_{\rm crit}$.