AST376 (Spring 2014)

COSMOLOGY

Problem Set 2

Due in class: Thursday, February 27, 2014

(worth 10/100)

1. Cosmic Expansion History

In class, we derived the Friedmann equation from the Einstein Field Equation.

a. Show that the solution to the Friedmann equation can be written as:

$$t(z) = \frac{1}{H_0} \int_z^\infty \frac{dz}{(1+z)\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \ , \tag{1}$$

where $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$, $H_0 = 70\,\mathrm{km\ s^{-1}\,Mpc^{-1}}$, and $t(z)$ is the age of the universe at redshift z.

b. Now, evaluate equ. (1) numerically, and plot the result with $a = 1/(1+z)$ on the y-axis, and time, t, on the x-axis. Use a log-log scaling, and choose the time units appropriately (please ask Aaron or me for help if needed). You can use a program like MATHEMATICA or MAPLE to do the numerical integration and the plotting; or whatever program(s) you are already familiar with.

c. For the matter dominated era ($1,000 \gtrsim z \gtrsim 1$), equ. (1) can be solved analytically. Derive an approximate solution, $t(z) \simeq ...$ for this situation.

2. Cosmological Distances

The most distant gamma-ray burst (GRB) was recently detected at $z \simeq 9.4$.

a. How old was the universe at the time of the GRB (use units of Myr)?

b. What is the proper distance to this GRB, evaluated at the present time, where $a_0 = 1$? Give your answer in Gpc.