AST 376 Cosmology — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith
(Dated: March 27, 2014)

DARK MATTER (CONTINUED)

Review

Last time we derived the freeze-out condition for WIMP dark matter. Over and over processes in the Universe are overcome by cosmic expansion so that particles simply don't run into each other anymore. The equation is as follows:

$$\boxed{\langle\sigma_A v\rangle n_X \simeq H \qquad (\text{“freeze-out condition”})\,.} \tag{1}$$

This is where "microphysics" meets "macrophysics."

The WIMP Miracle

The temperature at freeze-out T_f at the time t_f controls the DM abundance according to

$$n_{X,f} = n_{X,EQ}(t_f) \simeq \frac{(k_B T_f)^{3/2}\,(m_X c^2)^{3/2}}{\hbar^3 c^3} \exp\left(\frac{-m_X c^2}{k_B T_f}\right)\,.$$

Sometimes we have to make assumptions about the models and see where they lead. Our assumption is that WIMP dark matter is governed by the weak interaction so the mass is roughly $m_X c^2 \approx 100$ GeV. The Hubble parameter in the radiation dominated era ($t \lesssim 10,000$ yr) is

$$H = H(t_f) = H(z_f) \approx H_0\sqrt{\Omega_r}(1+z_f)^2\,,$$

and the corresponding temperature is

$$T = T_{CMB} = 2.725\text{ K }(1+z) \qquad (\text{“Radiation Temperature”})\,.$$

Putting this together gives one equation for one unknown:

$$\langle\sigma_A v\rangle \frac{(k_B T_f)^{3/2}\,(m_X c^2)^{3/2}}{\hbar^3 c^3} \exp\left(\frac{-m_X c^2}{k_B T_f}\right) \simeq H_0\sqrt{\Omega_r}\frac{(k_B T_f)^2}{(2.725\text{ K}k_B)^2}\,. \tag{2}$$

Q: How can we solve this?
A: Numerical root finding, i.e. "$F(x) = 0$, find x", resulting in a freeze-out at $k_B T_f \approx 5$ GeV.
Note: Since $k_B T_f \ll m_X c^2$ the WIMP DM is cold so this agrees with our previous discussion!

Q: What does this say about the current dark matter density?
A: We may recall the radiation temperature $T(z) \propto (1+z)$ to get

$$z_f \sim 2\times 10^{13} \quad\Rightarrow\quad t_f \sim 10\text{ ns} \quad\Rightarrow\quad n_{X,f} \sim 10^{32}\ cm^{-3} \quad\Rightarrow\quad \rho_{X,f} = n_{X,f} m_X \sim 10^{10}\text{ g cm}^{-3}$$

$$\rho_{X,0} \sim \rho_{X,f} z_f^{-3} \sim 10^{-30}\text{ g cm}^{-3} \quad\Rightarrow\quad \boxed{\Omega_X = \frac{\rho_{X,0}}{\rho_{crit,0}} \sim 0.1 \sim \Omega_{CDM} \qquad (\text{“WIMP miracle”})\,.} \tag{3}$$

Everyone wants this to be true, but the problem is the particles have not been detected yet. In the next five years we will either find the WIMPs in particle experiments or revise the theory. Additional candidates for DM contributors include axions, sterile neutrinos, Kaluza-Klein partner particles, etc. It cannot be the Higgs, however, because it is incredibly unstable!
Prediction: In our lifetimes we will see the resolution of DM. (Dark energy may still be ~ 100 yrs.)