AST 376 Cosmology — Lecture Notes

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DARK MATTER

When we talk about 'high precision cosmology' we usually mean the advancements of anisotropy measurements of the cosmic microwave background (CMB) to pin down the parameters of fiducial cosmological models. An experiment such as the Wilkinson Microwave Anisotropy Probe (WMAP) observes small temperature deviations ($\Delta T \lesssim 10\ \mu$K) and decomposes the map into Fourier modes to get the power spectrum, which is sensitive to parameters like Ω_m. To build up to this we continue our discussion of the constituents of the Universe. In particular, we know dark matter exists but we have yet to detect it in particle experiments, e.g. at the Large Hadron Collider (LHC). The algebra will not be too difficult but we will borrow concepts from many different fields.

Introduction to Dark Matter

The most popular dark matter candidates are weakly interacting massive particles (WIMPs), predicted by supersymmetry (SUSY). In the Standard Model (SM), which was completed with the discovery of the Higgs boson, every particle has a counterpart – *fermions* $\longleftrightarrow$ *bosons* and vice versa, e.g. electron $\longleftrightarrow$ selectron and photon $\longleftrightarrow$ photino. This introduces a new quantum number called R-parity that can either be $+1$ or -1. The WIMP is stable, i.e. survives the duration of the Universe, because further decay is rare. The ordinary matter we are used to only weakly interacts with this shadow world via gravity. The lightest SUSY particle (LSP) is the most stable neutral fermion so we call it the *neutralino*.

Production in the Early Universe

There are many channels to make dark matter particles, but the main mechanism is that they are **thermally produced** in the early Universe when $k_\mathrm{B}T \gtrsim m_X c^2$. (**Note:** The WIMPs need a name so we denote them by X subscripts.) This is because the thermal energy of a particle in equilibrium is $\epsilon_\mathrm{th} \approx k_\mathrm{B}T$ where $k_\mathrm{B} \equiv$ "Boltzmann constant" $= 1.38 \times 10^{-16}$ erg K^{-1}. It turns out that there is enough energy early on to create dark matter, i.e. for $t \lesssim 10,000$ yr. During this time we are radiation dominated so neglecting Ω_m and Ω_Λ yields

$$H \equiv \frac{\dot{a}}{a} \approx H_0 \sqrt{\Omega_r}(1+z)^2 \,,$$

where the present day fraction of radiation is $\Omega_r \approx 9 \times 10^{-5}$. This Friedmann equation gives

$$\boxed{a \propto t^{1/2} \qquad \text{and} H \approx \frac{1}{2t} \,.} \tag{1}$$

The temperature as a function of redshift is

$$\boxed{T(z) \equiv T_\mathrm{CMB} = 2.725\ \mathrm{K}\ (1+z) \sim 235\ \mathrm{GeV}\ k_\mathrm{B}^{-1} \left(\frac{1+z}{10^{15}}\right) \qquad (\text{“Radiation Temperature”})\,.} \tag{2}$$

Early on the cosmic plasma is hot enough for dark matter production from quark pairs. When $k_\mathrm{B}T \gtrsim m_\mathrm{X}c^2$ we have the schematic relation of $X\overline{X} \longleftrightarrow q\bar{q}$. The arrow goes both directions because the reactions are in thermal equilibrium. The equilibrium abundance $n_{X,\mathrm{EQ}}$ of the dark matter composed of fermions has a Fermi-Dirac distribution:

$$n_{X,\mathrm{EQ}} = \frac{\text{(Statistical Weight)}}{e^{\epsilon_X/k_\mathrm{B}T}+1} \qquad \text{where} \qquad \epsilon_X = \sqrt{m_X c^2 + p^2c^2}\,.$$

Note: Early on $k_\mathrm{B}T \gg m_X c^2$ so the $n_{X,\mathrm{EQ}} \approx$ constant.

Now, recall that WIMPs are cold dark matter (CDM) so although they were produced when the Universe was "hot" they remained in equilibrium until after they were very nonrelativistic, i.e. $m_X c^2 > k_\mathrm{B}T$. In order to have the bottom-up model of structure formation the dark matter must have been slowly moving for most of cosmic history, otherwise relativistic free-streaming would have inhibited the direct formation of small-scale structures.

The statistical weight is found by dividing space into quantum cells of size Δx. The Heisenberg Uncertainty Principle relates the size of these cells to momentum according to $\Delta x \Delta p \gtrsim \hbar/2$. Therefore, the cell size is on the order of the deBroglie wavelength $\Delta x \sim \lambda \sim \hbar/p$, where p is the classical momentum derived by equating the kinetic energy $\epsilon_\mathrm{kin} \approx \frac{1}{2}mv^2 = p^2/2m$ with the thermal energy $\epsilon_\mathrm{th} \approx k_B T$. The momentum and quantum wavelength are respectively

$$p \approx (k_\mathrm{B}Tm_X)^{1/2} \qquad \text{and} \qquad \lambda \approx \hbar(k_\mathrm{B}Tm_X)^{-1/2}\,.$$

The final step in obtaining the weight is to recall the number of particles within a given volume: $N_X = V/V_\mathrm{cell} = V/\lambda^3$. It is clear from this expression that the number density is $n_X \propto \lambda^{-3}$, or

$$n_{X,\mathrm{EQ}} \approx \frac{\left(k_\mathrm{B}Tm_X/\hbar^2\right)^{3/2}}{e^{m_Xc^2/k_\mathrm{B}T}+1} \approx \frac{\left(k_\mathrm{B}Tm_Xc^2\right)^{3/2}}{\hbar^3c^3}\exp\left(\frac{-m_Xc^2}{k_\mathrm{B}T}\right)\,. \tag{3}$$

The last equality is a result of being in the cold regime so $m_Xc^2 \gg k_\mathrm{B}T$.

Freeze-out

Q: If WIMPs are exponentially suppressed then why are there any left today?
A: Freeze-out! There is something left when we expect nothing because the *expansion of the Universe* makes WIMP annihilation improbable. In short, particles are created in the heat of the Big Bang but this stops as the Universe cools. Then dark matter annihilation is the only channel until expansion facilitates freeze-out.

We now calculate the rate of WIMP annihilation $\dot{N}_\mathrm{A}$. We get this by considering the volume swept out by a particle with interaction cross section σ_A and velocity v:

$$\dot{N}_\mathrm{A} \approx \frac{\Delta N_\mathrm{A}}{\Delta t} \approx \frac{\text{(Volume)}}{\Delta t} \times \frac{\text{(\# of targets)}}{\text{(Volume)}} \approx \frac{(\sigma_\mathrm{A} v \Delta t)}{\Delta t} \times n_X = \langle \sigma_\mathrm{A} v\rangle n_X\,.$$

Now, the freeze-out condition is that the rate of expansion H overcomes the annihilation rate $\dot{N}_\mathrm{A}$:

$$\boxed{\boxed{\langle\sigma_\mathrm{A}v\rangle n_X \simeq H \qquad \text{(``freeze-out condition'')}\,.}} \tag{4}$$

If the annihilation cross section is on the same order as that of the weak interaction then $\sigma_\mathrm{A} \sim \sigma_\mathrm{weak} \sim 10^{-36}$ cm^2 then the typical weak-scale, or 'dark', particle mass is

$$\boxed{m_Xc^2 \approx m_\mathrm{weak}c^2 \approx 100\ \mathrm{GeV}\,.} \tag{5}$$

Q: What is the typical thermal velocity of particles created in the radiation dominated era?
A: It is of order the sound speed, i.e. $v \sim c_\mathrm{s} \sim c/3$, so $\langle\sigma_\mathrm{A}v\rangle \sim 3\times10^{-26}\mathrm{cm}^3\ \mathrm{s}^{-1}$.