AST 376 Cosmology — Lecture Notes

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EXPANSION OF THE UNIVERSE (CONT.)

Quiz

Q: Consider a region of the Universe with roughly the size of the local group ($R \sim 1$ Mpc). Determine whether Newtonian gravity is good enough.
A: Ultimately we must compare the Schwarzschild radius R_S to the size of the local group R:

$$\frac{R_S}{R} \sim \frac{3\text{ km}\left(M_{LG}/M_\odot\right)}{1\text{ Mpc}} \sim \frac{10^5\text{ cm }(10^{12}\text{ M}_\odot/\text{M}_\odot)}{10^{24}\text{ cm}} \sim 10^{-7} \ll 1\,. \tag{1}$$

What does this tell us? It is a justification for using Newtonian cosmology in the local group. Actually, we can use Newton's laws for any portion of the universe smaller than about 100 Mpc.

A closer look at Hubble's law

It is an observed fact that wavelengths are redshifted. Historically, people were convinced that this was evidence for a velocity induced doppler effect. However, this immediately leads to further questions about why the galaxies are moving away from each other.

Q: Does the Hubble law define a privileged observer, i.e. us?
A: No! Consider the view with respect to different observers.
Note: Hubble's law is a vector law.

We observe: $\mathbf{v} = H_0\mathbf{r}$ and $\mathbf{v}_B = H_0\mathbf{r}_B$

Observer B: $$\boxed{\mathbf{v}' = \mathbf{v} - \mathbf{v}_B = H_0\left(\mathbf{r} - \mathbf{r}_B\right) = H_0\mathbf{r}'\,.} \tag{2}$$

This means that all observers see the same law!
Note: This only works for linear relations.

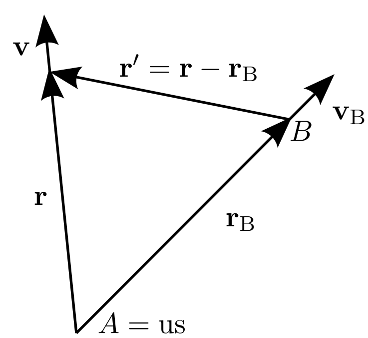

FIG. 1. Hubble's law is universal.

Q: What is the proper interpretation of the Hubble law?
A: The old (wrong) idea was that galaxies are in flight with respect to fixed (absolute) space. This resembles some kind of cosmic super-explosion, hence the name Big Bang. Everything would fly away from a special location, but WHY should there be such a special point in space?
The better (correct) idea is that space itself is expanding! When preaching to the converted relativist this seems like the obvious answer but in reality the mindset represents an enormous paradigm shift. In summary, all relative distances increase with time, however, galaxies are essentially fixed with respect to the expanding space.
Caveat: Galaxies also have a peculiar velocity component representing local forces.

Q: How do we describe expanding space?
A: We introduce a scale factor $a(t)$ which compares distances at different times. Then physical

distances $\mathbf{r}$ and comoving distances $\mathbf{x}$ are related through a as follows:

$$\boxed{\mathbf{r} = a(t)\mathbf{x}} \qquad \text{where} \qquad \begin{aligned} a &= \text{"cosmic scale factor"} \\ \mathbf{x} &= \text{"comoving coordinates"} \\ \mathbf{r} &= \text{"physical (or proper) coordinates"}\,. \end{aligned} \tag{3}$$

Note: $\mathbf{x}$ stays the same during expansion so the entire effect of the expanding space is encoded in $a(t)$. Furthermore, only $\mathbf{r}$ has physical meaning as comoving distances are not directly observable.

For an expanding universe the scale factor is an increasing function with time, i.e. $a(t_1) < a(t_2)$, so we often normalize the scale factor. The parametrization is chosen such that the present-day value of the scale factor is one:

$$a_0 \equiv a(t_0) \equiv 1 \qquad \text{where} \qquad t_0 \equiv \text{today.} \tag{4}$$

Now we have a completely different (and correct) interpretation of the redshift – light travels through expanding space – so the physical wavelength is changed according to $\boxed{\lambda \propto a\,.}$

$$\begin{aligned} \text{At emission:} \quad & t = t_{\rm em} \qquad \lambda_{\rm em} \propto a(t_{\rm em}) \\ \text{Observe today:} \quad & t = t_0 \qquad \lambda_{\rm obs} \propto a(t_0) \end{aligned} \tag{5}$$

Therefore, the redshift is

$$z = \frac{\Delta\lambda}{\lambda_0} = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0} = \frac{\lambda_{\rm obs}}{\lambda_0} - 1 = \frac{a_0}{a(t_{\rm em})} - 1\,, \tag{6}$$

providing the cosmological definition of redshift once we substitute $a(t_{\rm em}) \to a(t) \to a$ and $a_0 \to 1$

$$\boxed{1 + z = \frac{1}{a}\,.} \tag{7}$$

Note: The redshift is just another measurement of cosmic time with today corresponding to $z = 0$ and the Big Bang corresponding to the limit as $z \to \infty$. This is the first indications that something is not right with our physics. One infinity after another tell us "our understanding is not complete." However, even without a full quantum theory of gravity we will be able to discuss aspects of the early universe after $\sim 10^{-43}$ seconds.

Definition of the Hubble parameter

We now generalize the concept of the Hubble constant H_0. The idea is to describe expansion with a Hubble law that is valid for all times, not just today. Formally, the Hubble relation is $\mathbf{v} = H(t)\mathbf{r}$ but velocity is fundamentally defined by $\mathbf{v} = \dot{\mathbf{r}} = \frac{d}{dt}[a(t)\mathbf{x}] = \dot{a}\mathbf{x}$ so we obtain

$$\boxed{H(t) \equiv \frac{\dot{a}}{a} \quad (\text{"Hubble parameter"})} \quad \text{and} \quad \boxed{H_0 \equiv H(t_0) = \frac{\dot{a}_0}{a_0} \qquad (\text{"Hubble constant"})\,.} \tag{8}$$

Simple estimate for the age of the universe

The Hubble time $t_{\rm H}$ may now be defined in terms of the Hubble constant. An estimate gives

$$H_0 = \frac{\dot{a}_0}{a_0} = \dot{a}_0 \sim \frac{a_0}{t_{\rm H}} \sim \frac{1}{t_{\rm H}}\,. \tag{9}$$

Therefore, if $H_0 \sim 70$ km s^{-1} Mpc^{-1} the Hubble time $t_{\rm H}$ and Hubble radius $R_{\rm H}$ are roughly

$$\boxed{t_{\rm H} \equiv H_0^{-1} \sim 10 \text{ Gyr}} \qquad \text{and} \qquad \boxed{R_{\rm H} \sim ct_{\rm H} \sim \frac{c}{H_0} \sim 10^{28} \text{ cm} \sim \text{a few Gpc}\,.} \tag{10}$$