AST 376 Cosmology — Lecture Notes

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INTRODUCTION (CONT.)

Olbers' Paradox and the Beginning of Time

The night sky is dark! This is an observation we have all made and one that most of us take for granted. However, a century ago this apparently simple realization puzzled some of the greatest minds of the time. Even Einstein believed in a static universe. Indeed, if one (incorrectly) assumes the universe is Euclidean – flat and infinite in extent – and eternal (i.e. with no Big Bang) then one runs into what is called *Olbers' paradox*. As we look deeper into the problem we will find it forces us to question these assumptions. These are ideas worth losing sleep over – the genesis of the universe and the beginning of time.

To get at the heart of Olbers' paradox we first calculate the total flux from all the stars in the sky. Assume the stars have a constant number density n_* and fill the entire universe. Each star has a luminosity L_* where luminosity, like power, has units of energy per time. Then consider the number of stars in a radial shell of radius r and thickness dr,

$$dN_* = (\text{volume}) \times (\text{number density}) = 4\pi r^2 dr n_* \,. \tag{1}$$

The flux f_* from a given star in the shell is diluted by its distance according to the "inverse square law," i.e.

$$f_* = \frac{\text{energy}}{\text{time} \times \text{area}} = \frac{L_*}{4\pi r^2} \,, \tag{2}$$

so the total flux from each shell is

$$df = f_* dN_* = 4\pi r^2 dr n_* \frac{L_*}{4\pi r^2} = n_* L_* dr \,. \tag{3}$$

Already we see a problem because each shell contributes the same (constant) flux and there are an infinite number of shells. Formally, the total flux from all shells diverges in the limit of infinite volume:

$$f_{\text{tot}} = \lim_{R\to\infty} \int_0^R df(r) = n_* L_* \lim_{R\to\infty} \int_0^R dr = n_* L_* \lim_{R\to\infty} R \,. \tag{4}$$

Q: What went wrong?
A: We only receive starlight from a finite distance! The exact radius R is on the order of the cosmic horizon, or the Hubble radius R_{H}, so $f_{\text{tot}} \sim n_* L_* R_{\text{H}}$. Now all we need is a simple estimate of the number density of stars and the luminosity of each star. Under the "order of magnitude" or "back of the envelope" paradigm it is usually a good approximation to use the values of the Sun, Milky Way, or local group as being typical. Therefore, each star has a luminosity of roughly $L_* \sim L_\odot \sim 3.8 \times 10^{33}$ erg s^{-1}. (Note: Astronomers use the **CGS** (centimeters-grams-seconds) unit system...NOT SI.) The value for n_* comes from the number of stars in the universe divided by the Hubble volume. If there are about 10^{11} stars in the Milky Way and about the same number of galaxies then the number density is on the order of

$$n_* \sim \frac{N_*}{V_{\text{H}}} \sim \frac{N_*}{(R_{\text{H}})^3} \sim \frac{10^{22}}{(10^{28}\ \text{cm})^3} \sim 10^{-62}\ \text{cm} \,, \tag{5}$$

so the total flux is

$$f_{\rm tot} \sim n_* L_\odot R_{\rm H} \sim \left(10^{-62}\ {\rm cm}\right)\left(10^{33}\ {\rm erg\ s^{-1}}\right)\left(10^{28}\ {\rm cm}\right) \sim 1\ {\rm erg\ s^{-1}\ cm^{-2}}\,. \tag{6}$$

To get a feel for this number we compare it with the solar flux $f_\odot$. The astronomical unit (AU) is the distance from the Earth to the Sun, or $d_\odot \equiv 1$ AU $\sim 150 \times 10^6$ km $\sim 10^{13}$ cm, making the daytime flux roughly

$$f_\odot \sim \frac{L_\odot}{4\pi d_\odot^2} \sim \frac{10^{33}\ {\rm erg\ s^{-1}}}{4\pi\left(10^{13}\ {\rm cm}\right)^2} \sim 10^6\ {\rm erg\ s^{-1}\ cm^{-2}} \gg f_{\rm tot}\,. \tag{7}$$

So why does this mean there is a beginning of time? If the universe had an infinite amount of time the light would eventually reach us making the sky infinitely bright. Even if the interstellar dust obscures this light too would eventually come into thermal equilibrium and shine with the stars. However, the night sky is dark! That simple observation led to a remarkable realization. It's unfortunate that when some student learns the sophisticated math behind the physics they stop to wonder about the underlying issues.

EXPANSION OF THE UNIVERSE

Basic Facts and Concepts

For all galaxies we find a "redshift" z which is proportional to the distance D light has traveled from the galaxy, i.e. $z \propto D$. The constant of proportionality is related to the Hubble constant H_0 which we discuss in greater detail later on. If λ_0 is the laboratory wavelength measurement of a quantum mechanical absorption/emission line and $\lambda_{\rm obs}$ is the observed wavelength of the redshifted line then $\Delta\lambda \equiv \lambda_{\rm obs} - \lambda_0$ is the difference between the two and the Hubble law is

$$\boxed{z \equiv \frac{\Delta\lambda}{\lambda_0} = \frac{H_0}{c} D \qquad (\text{"Hubble law" for redshifts})\,.} \tag{8}$$

Note: All measurements are stretched by a factor of $(1+z)$, i.e. $\lambda_{\rm obs} = (1+z)\lambda_0$. This is often incorrectly interpreted as a Doppler effect where $v/c = \Delta\lambda/\lambda_0$. Nonetheless, this idea lends itself to the more familiar version of the Hubble law:

$$\boxed{v = H_0 D \qquad (\text{"Hubble law" for velocities})\,.} \tag{9}$$

The Hubble constant is now known from precision cosmology experiments to be

$$\boxed{H_0 = 70 \pm 3\ {\rm km\ s^{-1}\ Mpc^{-1}}\,.} \tag{10}$$

Historically, the value of H_0 has been incredibly hard to measure. Two camps emerged led by Allan Sandage (a student of Edwin Hubble) and Gérard de Vaucouleurs (a professor at UT Austin) who respectively argued for values of 50 km s^{-1} Mpc^{-1} and 100 km s^{-1} Mpc^{-1}. The bimodality dissapeared in the era of precision cosmology. Ironically, the observed value forced the camps to converge and settle in the middle.