AST 376 Cosmology — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith
(Dated: January 14, 2014)

INTRODUCTION

The Cosmological Principle

One of the basic realizations of the universe is that it is the same everywhere. This is called the **Cosmological Principle** which asserts that we live in a universe that is both *homogeneous* and *isotropic*, meaning that it is both uniform along paths and is also the same in every direction.
Q: How do we know the Cosmological Principle represents reality?
A: By combining the following arguments:

(i) We observe isotropy around us! When we look out at the night sky each direction is almost indistinguishable from any other. Historically, the cosmic microwave background (CMB) provides the clearest picture of just how isotropic the universe began. With fluctuations on the order of only a part in a hundred thousand the CMB radio waves were serendipitously discovered by Penzias and Wilson in 1965 as noise in their equipment. Still, many groundbreaking phenomena (and even Nobel Prizes) have been discovered as a result of CMB observations. The main space-based probes are COBE, WMAP, and PLANCK. Other examples of observational evidence for isotropy include galaxy surveys and gamma-ray bursts (GRBs).
Note: The local distribution of stars and galaxies is not isotropic! It is only when we look at large scales that this becomes as elegant as the Cosmological Principle suggests.

(ii) Assume isotropy *everywhere*! This is an application of the Copernican principle, i.e. we do not occupy a special place in the universe.

(iii) If we buy the idea of isotropy everywhere then logically we must have homogeneity, or uniformity. In other words, isotropy is given to us by observations but we need to appeal to philosophical arguments to get homogeneity.

How can we prove this? Pick two arbitrary points in space. Draw circles around each point that intersect one another. Because of isotropy the distribution of matter is uniform along circle. Therefore, it is also uniform in the annulus formed by the intersection. An infinite number of these points may be chosen so that the entire volume of the universe is uniform. Thus, isotropy implies homogeneity. (However, homogeneity does not require isotropy.)

The need for General Relativity in Cosmology

On large scales, the universe is described by an Einsteinian theory of gravity. By this we mean it is governed by the principles of relativity and (if we are required to be specific about it) we will employ the General Theory of Relativity to build our models.
Q: How do we know we have to use general relativity (GR) in cosmology?
A: There are of course flaws with the Newtonian picture of gravity because of action at a distance and inconsistencies with observations, but this is not what we are getting at here. Perhaps what we mean is 'under what conditions are we required to consider GR in cosmology?' To motivate

this question we recall the condition for deciding to use special relativity (SR):

$$\boxed{\beta \equiv \frac{v}{c} \ll 1 \qquad \text{(Condition to choose Newtonian physics over SR)}\,.} \tag{1}$$

For GR we must compare the size of the object R with its Schwarzschild radius R_{S}. This is a very important concept so we recall a heuristic derivation of R_{S} by considering the escape velocity v_{esc} in Newtonian physics. Energy conservation dictates that potential and kinetic energies be balanced:

$$E_{\mathrm{pot}} + E_{\mathrm{kin}} = -\frac{GM}{R}m + \frac{1}{2}mv_{\mathrm{esc}}^2 = 0\,. \tag{2}$$

Solving for v_{esc} in Eq. 2 gives

$$v_{\mathrm{esc}}^2 = \frac{2GM}{R}\,. \tag{3}$$

In the limit that the escape velocity approaches the speed of light a relativistic black hole is formed from which nothing can escape. Therefore, in the Newtonian view the Schwarzschild radius is defined as the point where $v_{\mathrm{esc}} \to c$. Formally,

$$\boxed{R_{\mathrm{S}} \equiv \frac{2GM}{c^2} \qquad \text{(Schwarzschild Radius)}\,.} \tag{4}$$

If $R \gg R_{\mathrm{S}}$ gravity is weak and GR is not needed. We can show the relative importance of relativity by comparing the radius of the sun to its Schwarzschild radius and likewise for neutron stars:

$$* \ \textbf{Sun}: \quad \frac{R_{S,\odot}}{R_\odot} \sim \frac{3\ \text{km}}{10^6\ \text{km}} \sim 10^{-6} \qquad \text{Small but measurable effect.}$$

$$* \ \textbf{NS}: \quad \frac{R_{S,NS}}{R_{NS}} \sim \frac{6\ \text{km}}{10\ \text{km}} \sim 0.6 \qquad \text{GR is crucial for neutron stars.}$$

$$\boxed{R \gg R_{\mathrm{S}} \qquad \text{(Condition to choose Newtonian gravity over GR)}\,.} \tag{5}$$

Q: What about the universe? How does its size compare to its Schwarzschild radius?
A: We must first determine the size of the universe but it could (and likely is) infinite so this is not very meaningful! To get around this trick question we introduce the concept of a "cosmic horizon." This is the volume enclosed by a light cone going back to the Big Bang. With no empirical determination of the size of the universe we define the Hubble radius R_{H} as the distance light has traveled in the Hubble time $t_{\mathrm{H}} \sim 14$ Gyr, or roughly the age of the universe. Thus,

$$R_{\mathrm{H}} \sim ct_{\mathrm{H}} \sim 5\ \text{Gpc} \sim 10^{28}\ \text{cm}\,. \tag{6}$$

We next determine the mass contained in the universe. To do this we assume the density of the Milky Way (MW) is representative of the average density of the universe $\bar{\rho}$ or roughly

$$\bar{\rho} \sim \frac{M_{\mathrm{MW}}}{R_{\mathrm{MW}}} \sim \frac{10^{12}\ \mathrm{M}_\odot}{\text{Mpc}^3} \sim \frac{10^{12}\ \left(2 \times 10^{33}\ \text{g}\right)}{(3 \times 10^{24}\ \text{cm})^3} \sim 10^{-30}\ \text{g cm}^{-3}\,. \tag{7}$$

The mass of the universe is its average density multiplied by the volume of the cosmic horizon

$$M_{\mathrm{H}} \sim \frac{4\pi}{3}R_{\mathrm{H}}^3\bar{\rho} \sim \frac{4\pi}{3}\left(10^{28}\ \text{cm}\right)^3\left(10^{-30}\ \text{g cm}^{-3}\right) \sim 10^{55}\ \text{g} \sim 10^{22}\ \mathrm{M}_\odot\,. \tag{8}$$

The linear relation of Eq. 4 allows us to scale up from the solar Schwarzschild radius:

$$R_{\mathrm{S,universe}} \sim 3\ \text{km}\ \left(\frac{M_H}{\mathrm{M}_\odot}\right) \sim 10^{28}\ \text{cm}\,. \tag{9}$$

The punchline is summarized by

$$\boxed{R_{\mathrm{S}} \sim R_{\mathrm{H}} \qquad \Rightarrow \qquad \text{GR is necessary in cosmology!}} \tag{10}$$