AST 376 Cosmology — Lecture Notes

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COSMOLOGICAL DISTANCES

The key to understanding the cosmological landscape is distances! We have used the term "proper distance" (or "comoving distance") to indicate the distance of an object whose light is received today. This has been shown to relate to redshift via the following equations:

$$\boxed{r(z) = \int_0^z \frac{c\,dz'}{H(z')} = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega(z')}} \qquad (\text{“Comoving distance”})\,.} \tag{1}$$

The idea is the following: Consider two different times, $t(z)$ and t_0 today. At some point in the past a photon was emitted but space has expanded during the time it took to reach us. Is the comoving distance an observable? Unfortunately it is not, however, it is easy to convert $r(z)$ into an observable. To do this we need two more cosmological distances (angular diameter distance D_A and luminosity distance D_L) that are directly tied to observations. **Note:** In cosmology we either observe angles or fluxes, i.e. how big does an object appear and/or how bright is it?

Angular diameter distance

Somewhere in the universe there is an object (galaxy, etc.) that has physical size ℓ. The angular diameter distance D_A is given by assuming the small angle approximation, where the observed angle (in radians) is $\varphi \approx$ (opposite)/(hypotenuse), or taken as a definition:

$$\boxed{\varphi = \frac{\ell}{D_\mathrm{A}}\,.} \tag{2}$$

Note: These are "unit less" angles. The definitions are in terms of radians, degrees ($^\circ$), arc minutes ($'$), and arc seconds ($''$). The conversion is 1 rad $= \frac{180}{\pi}^\circ = \frac{180}{\pi} \cdot 60 \cdot 60$ arcsec ≈ 206265 arcsec.

Q: How does this work in an expanding space?
A: Remember all lengths are stretched by a factor of $(1 + z)$. So the same analysis gives

$$\boxed{\varphi = \frac{\ell(1+z)}{r(z)}\,.} \tag{3}$$

This is also a definition because we chose to treat the distances based on the current observation. The photons travel on geodesics in expanding space so although the galaxy does not increase size, the photon appears to come from an objects that is larger by a factor of $(1 + z)$. The light creates an optical illusion! By comparing Equations 2 and 3 we have the result (in flat space) that

$$\boxed{D_\mathrm{A} \equiv \frac{r(z)}{1+z} \qquad (\text{“Angular diameter distance”})\,.} \tag{4}$$

Luminosity distance

To compare how bright sources appear at different redshifts we need the luminosity distance $D_{\rm L}$. We assume an isotropic source and apply the inverse square law for the observed flux (power/area)

$$\boxed{f \equiv \frac{\Delta E}{\Delta t \Delta A} = \frac{L}{4\pi D_{\rm L}^2} \qquad (\text{“Flux-luminosity relation”})\,.} \tag{5}$$

But remember that space is stretched and photons lose energy from the changing potential. Recall that $\epsilon = h\nu = hc/\lambda$ so the observed energy changes according to $\Delta E_{\rm obs} = \Delta E_{\rm em}/(1+z)$. Also the “cosmological time dilation” produces an observed time of $\Delta t_{\rm obs} = \Delta t_{\rm em}(1+z)$. Finally, the proper area over which the photons arrive is $\Delta A_{\rm obs} = 4\pi r^2(z)$. Thus, the observed flux is

$$f_{\rm obs} = \frac{\Delta E_{\rm obs}}{\Delta t_{\rm obs}\Delta A_{\rm obs}} = \frac{\Delta E_{\rm em}}{(1+z)^2 \Delta t_{\rm em} 4\pi r^2(z)} = \frac{L_{\rm em}}{(1+z)^2 4\pi r^2(z)}\,,$$

and the luminosity distance is

$$\boxed{D_{\rm L} = r(z)(1+z) \qquad (\text{“Luminosity distance”})\,.} \tag{6}$$

I. WORKED EXAMPLE

For the high-z QSO we want to calculate the emitted and observed luminosity. This is done by first measuring the redshift as $z \approx 7$ and flux as $f_{\rm obs} \approx 10^{-13}$ erg s^{-1} cm^{-2}. From this we calculate $L_{\rm em} \approx f_{\rm obs} 4\pi D_{\rm L}^2 \approx f_{\rm obs} 4\pi (1+z)^2 r(z)^2$. We may assume the quasar is at the edge of the Universe so $r(z) \sim R_{\rm H} \sim c/H_0$, which gives an emitted luminosity of

$$\begin{aligned} L_{\rm em} &\approx f_{\rm obs} 4\pi (1+z)^2 r(z)^2 \\ &\sim \left(10^{-13}\ \text{erg s}^{-1}\ \text{cm}^{-2}\right)(10)\left[(10)(10\ \text{Gpc})\right]^2 \\ &\sim 10^{46}\ \text{erg s}^{-1} \\ &\sim 10^{13}\ L_\odot \\ &\sim 10^{2}\ L_{\rm MW}\,. \end{aligned}$$

Note: The solar luminosity is $L_\odot \sim 3 \times 10^{33}$ erg s^{-1}.