AST 376 Cosmology — Lecture Notes

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COSMIC MICROWAVE BACKGROUND (CMB) (CONTINUED)

Anisotropies

The idea last time was to decompose the 2D spherical view of the sky into spherical harmonics similar to the Fourier decomposition of 3D flat space with sines and cosines. This leads to the following definitions:

$$\frac{\Delta T}{T_0} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\varphi, \vartheta) \qquad (\text{“Spherical harmonic decomposition”}) \,, \tag{1}$$

and

$$C_\ell \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \qquad (\text{“Angular power spectrum”}) \,. \tag{2}$$

Recall that the angular scale of a given ℓ-mode is approximately $\theta \simeq \pi/\ell = 180\,^\circ/\pi$. Thus, we consider a square patch of the sky with sides of length $\Delta\varphi \sim 1/\bar{\ell}$. Notice that higher frequency modes than $\bar{\ell}$ don't contribute to the resolution element because their many peaks and troughs cancel out. At any rate, we want the mean squared deviation of the fluctuations, so we amplify the signal by the characteristic scale $\bar{\ell}$ and recount the number of modes:

$$\left(\frac{\Delta T}{T_0}\right)^2_{\bar{\ell}} \simeq \bar{\ell}(2\ell+1) C_{\bar{\ell}} \simeq \bar{\ell}(\bar{\ell}+1) C_{\bar{\ell}} \,.$$

In practice people just write (and plot) the following:

$$\boxed{\left(\frac{\Delta T}{T_0}\right)^2 \simeq \ell(\ell+1) C_\ell \qquad (\text{“Mean squared deviation of angular power spectrum”}) \,.} \tag{3}$$

However, inflation predicts no preferred scale for the $\Delta E_{\rm grav}/E_{\rm grav} \sim 10^{-5}$ fluctuations. COBE saw fluctuations down to its angular resolution of $7\,^\circ$ and WMAP measured the first and second peaks of the temperature power spectrum down to its resolution of $0.2\,^\circ$. Why would the amplitudes vary so much? The angular size of the fluctuations matters! In fact, super-horizon modes are frozen in and cannot grow or decay until the Universe expands to their characteristic size. So if (from Quiz 11) recombination corresponds to $1\,^\circ$ then these super-horizon fluctuations are essentially imprinted in the CMB for good! However, photons will engage in “sound waves” if they are “harmonics” of the sound horizon distance. In other words, the peaks are driven by ℓ-mode resonances of characteristic scales corresponding to sound horizon overtones.

The era of high precision cosmological parameters is then a product of performing a least-squares fit to the anisotropy data. Our confidence in parameters like Ω_m, etc. comes from trusting a best fit model of the temperature power spectrum. For example, the result of where the first peak is located ($1\,^\circ$) tells us about the geometric curvature of the Universe – it is flat! Other independent experiments (SNe and BAO) agree with the CMB determinations and give increasingly accurate values for the cosmological parameters. This is in contrast to the situation a few decades ago when very little was known.