AST 376 Cosmology — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith
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COSMIC MICROWAVE BACKGROUND (CMB) (CONTINUED)

Review

The questions on the exam will be related to (1) Memorization – see below (2) PS 3 (3) PS 4 (4) Quiz 11 and (5) a surprise. The memorization part requires you to know the equation of energy conservation in an expanding universe:

$$\boxed{\dot{\rho} = -3H\left(\rho + \frac{P}{c^2}\right) \quad \text{where} \quad H \equiv \frac{\dot{a}}{a} \qquad (\text{“Energy conservation”})\,.} \tag{1}$$

As an example we consider cold matter or “dust” which has $P = 0$. This means

$$\dot{\rho} = -3H\rho = -3\frac{\dot{a}}{a}\rho \qquad \Rightarrow \qquad \frac{d\rho}{\rho} = -3\frac{da}{a} \qquad \Rightarrow \qquad \rho = \rho_0 a^{-3} = \rho_0(1+z)^3\,.$$

Note: You can also remember this as $d(a^3\rho) = -Pd(a^3)$.

Recap: Why is the CMB released 400,000 years after the Big Bang? Free electrons are strongly couple to photons but after this time the expansion has cooled enough for neutral hydrogen to recombine and produce a surface of last scattering. “Recombination” is somewhat of a misnomer because the atoms were never “combined” in the first place. Later the Universe becomes “reionized” but this is a good name because it started off ionized!

Anisotropies

The big questions have mostly been answered by precise measurements of the CMB. In the early 1990s the Cosmic Background Experiment (COBE) revealed the 2.7 K background temperature after the first year. Then after another year COBE measured the (∼mK) Doppler-induced dipole moment from the relative motion of the solar system compared to the Hubble flow. Finally, at the end of its mission COBE revealed the ($\sim 10\ \mu$K) fluctuations that we associate with CMB maps. A decade later the Wilkinson Microwave Anisotropy Probe (WMAP) captured the CMB signal with the same sensitivity but with much finer resolution. The Planck satellite is currently taking data with even greater sensitivity and resolution.

We now translate this qualitative picture of the CMB into the concrete mathematical framework describing anisotropies. Recall Fourier analysis in 3D flat space:

$$A(\vec{x}) = \sum_{\vec{k}} A_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \qquad \text{where} \qquad A_{\vec{k}} = \int A(\vec{x}) e^{-i\vec{k}\cdot\vec{x}} d^3x \qquad (\text{“Fourier coefficients”})\,, \tag{2}$$

and the wavenumber is described by $\vec{k} = (k_x, k_y, k_z)$ with the property $|\vec{k}| = \frac{2\pi}{\lambda}$. The power spectrum is given as the squared amplitude of the coefficients, i.e. $P(k) = A_k^* A_k = |A_k|^2$. However, there is one major difference between this and what we need for CMB anisotropies. We slightly tune this machinery up by decomposing the 2D sphere instead of 3D flat space. The new base functions are spherical harmonics, i.e. $e^{i\vec{k}\cdot\vec{x}} \mapsto Y_\ell^m(\varphi,\vartheta)$. One can read up on their many properties, but for our purposes they form an orthogonal set of functions to fully describe the 2D sphere line sines and cosines do for the 3D Fourier series. **Note:** The coefficients here will be like temperature overdensities, i.e. $A(\varphi,\vartheta) \approx \Delta T/T$. The exact decomposition is

$$A(\varphi,\vartheta) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_\ell^m(\varphi,\vartheta) \qquad (\text{“Spherical harmonic decomposition”})\,. \tag{3}$$

Note: For every ℓ we have $2\ell+1$ m-modes, just like the quantum mechanical orbitals of atomic hydrogen. The ℓ-mode is related to the angular scale via $\Delta\varphi \sim \pi/\ell$ so that higher resolution is equivalent to higher ℓ-modes.

Define: The angular power spectrum is taken to be the average of the harmonic coefficients:

$$\boxed{C_\ell \equiv \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \qquad (\text{“Angular power spectrum”})\,.} \tag{4}$$

Note: We could plot C_ℓ directly but we often plot this in the slightly different form of $\ell(\ell+1)C_\ell/2\pi$. The justification for the $(\ell+1)$ comes from wanting higher modes to have more weight, i.e. in a given resolution element there are $2\ell+1$ effective modes. The final factor of ℓ comes from considering a characteristic length scale where only larger modes provide small scale features.