AST 376 Cosmology — Lecture Notes

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COSMIC MICROWAVE BACKGROUND (CMB)

Creation and Basic Properties

Photons strongly interact with free electrons through Thompson scattering (also known as Compton scattering) but do not interact with bound electrons. Therefore, before recombination at $t_{\rm rec}$ there is constant photon-electron scattering and the cosmic plasma is in thermal equilibrium, i.e. $T = T_{\rm rad} = T_e = T_p$. However, cosmic expansion allows the gas to cool, recombine, and allow CMB photons to be released from the "last scattering surface" (LSS). Now the energy density of a blackbody follows a Planck distribution so that

$$\boxed{u_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_{\rm B}T}-1} \qquad (\text{“Energy density of a blackbody”})\,,} \tag{1}$$

where the present-day CMB temperature is $T_{\rm CMB}(z=0) = T_{\rm CMB,0} = 2.7$ K, or at arbitrary redshift

$$\boxed{T_{\rm CMB}(z) = 2.7\ {\rm K}\ (1+z) \qquad (\text{“CMB temperature”})\,.} \tag{2}$$

Derivation: Blackbodies have $\epsilon = hc/\lambda \simeq k_{\rm B}T$, which can only be true at all redshifts if $T \propto a^{-1}$.

Consider the energy density of a blackbody in expanding space:

$$u \propto \frac{1}{e^{h\nu/k_{\rm B}T_{\rm CMB}}-1} \qquad \text{but} \quad T \propto (1+z) \quad \text{and} \quad \nu \propto (1+z)$$

$$\Rightarrow \qquad \frac{h\nu}{k_{\rm B}T_{\rm CMB}} = \text{constant} \qquad \Rightarrow \qquad \text{Once a blckbody, always a blackbody.}$$

Historical Aside: In the 1940s Gamow predicted the cosmic background radiation but his work was not noticed until the serendipitous discovery of isotropic (non-terrestrial) radio waves by Penzias and Wilson in the 1960s. They asked Princeton cosmologists Dicke and Peebles about it, wrote a paper, and later won a Nobel prize.

Photon-to-baryon Ratio

In cosmology the term radiation is practically synonymous with the CMB. Indeed the number of CMB photons is much, much larger than the total number of photons produced from all stars in the entire Universe. But how does this compare to the number of baryons? Specifically, we want the present-day "photon-to-baryon ratio" defined as $\eta \equiv n_\gamma/n_{\rm b}$. The number density of baryons is

$$n_{\rm b} \simeq \frac{\Omega_{\rm b}\rho_{\rm crit,0}}{m_{\rm H}} \approx \frac{(0.04)(10^{-29}\ {\rm g\ cm^{-3}})}{1.67\times10^{-24}\ {\rm g}} \approx \text{few}\times10^{-7}\ {\rm cm^{-3}}\,.$$

Now for photons we use the Stefan-Boltzmann law, i.e. $u \propto T^4$, to arrive at

$$n_{\rm b} \simeq \frac{u_{\rm CMB}}{\bar\epsilon} \approx \frac{a_{\rm rad}T_{\rm CMB}^4}{k_{\rm B}T_{\rm CMB}} \approx \frac{(7.56\times10^{-15}\ {\rm ergs/K^4})(2.7\ {\rm K})^3}{1.38\times10^{-16}\ {\rm ergs/K}} \approx 150\ {\rm cm^{-3}}\,.$$

Therefore, the ratio is roughly

$$\boxed{\eta \equiv \frac{n_\gamma}{n_\mathrm{b}} \approx 10^9 \qquad (\text{“Photon-to-baryon ratio”})\,.} \tag{3}$$

This number does not change because the number of both photons and baryons are roughly conserved over the history of the Universe.

Note: This tells us about matter-antimatter asymmetry in the early Universe. We expect the number densities of baryons and antibaryons to be roughly equal, i.e. $n_\mathrm{B} \sim n_{\bar{\mathrm{B}}}$, because the CMB photons were originally created together via $\gamma + \gamma \rightleftarrows B + \bar{B}$ reactions. However, there is a slight overabundance of normal matter according to

$$\frac{n_\mathrm{B} - n_{\bar{\mathrm{B}}}}{n_\mathrm{B}} = \eta^{-1} \approx 10^{-9}\,.$$

First high energy creates B and $\bar{B}$ but an unknown process exposes an asymmetry that (luckily) favors normal matter. The reason presents a very difficult question which we presently postpone.

The Epoch of Recombination

The million dollar question is now, “when did recombination happen?” In other words, when did electrons and protons combine to form neutral hydrogen $e^- + p^+ \to H^0$? We have to somehow bombard the atom with enough energy to knock off an electron from the ground state. Recall the quantum energy levels go as $E_n \propto n^{-2}$ where the ground state energy is $E_0 = -13.6$ eV.

Tentative Idea: When is the thermal energy of the CMB *no longer* sufficient to ionize hydrogen?
A: We simply equate the thermal energy $k_\mathrm{B} T_\mathrm{CMB}(z)$ with the ionization energy Q_H to get

$$z_\mathrm{rec} \approx \frac{Q_\mathrm{H}}{k_\mathrm{B} T_\mathrm{CMB}} \approx \frac{13.6 \text{ eV}}{(8.617 \times 10^{-5} \text{ eV/K})(2.7 \text{ K})} \approx 58,000\,.$$

Note: This is a bit too crude! We need a better estimate because the CMB is a blackbody so even at lower temperatures the broad distribution has enough energetic photons to ionize hydrogen.

Q: How many of the photons need to have an energy ($\epsilon > Q_\mathrm{H}$) large enough to ionize hydrogen?
A: Systems in thermal equilibrium follow a Boltzmann dist. so the energetic photon density is

$$n_\gamma(\epsilon > Q_\mathrm{H}) \simeq n_\gamma \exp\left[\frac{-Q_\mathrm{H}}{k_\mathrm{B} T_\mathrm{CMB}}\right]\,.$$

Note: Before recombination this density is much higher than the number density of baryons or ionized hydrogen, i.e. $n_\gamma(\epsilon > Q_\mathrm{H}) \gg n_\mathrm{b} \sim n_{\mathrm{H}^+}$, but at recombination this is no longer the case:

$$n_\gamma(\epsilon > Q_\mathrm{H}) \simeq n_\mathrm{b} \qquad \Rightarrow \qquad \exp\left[\frac{-Q_\mathrm{H}}{k_\mathrm{B} T_\mathrm{CMB}(z)}\right] \approx \eta^{-1} \approx 10^{-9}\,. \tag{4}$$

Therefore, the redshift at recombination or the redshift at last scattering is

$$\boxed{z_\mathrm{rec} \simeq \frac{Q_\mathrm{H}/\ln\eta}{k_\mathrm{B} T_\mathrm{CMB}} \approx 2000 \quad (\text{precisely} \Rightarrow z_\mathrm{rec} \simeq 1100) \qquad (\text{“Recombination redshift”})\,.} \tag{5}$$

Finally, the epoch of recombination t_rec for a matter dominated universe is then

$$\boxed{t_\mathrm{rec} = t(z_\mathrm{rec}) \simeq \frac{2}{3H_0\sqrt{\Omega_m}}(1+z)^{-3/2} \approx 400,000 \text{ yr} \qquad (\text{“Epoch of Recombination”})\,.} \tag{6}$$