AST 376 Cosmology — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith
(Dated: April 15, 2014)

THE VERY EARLY UNIVERSE: INFLATION (CONTINUED)

In Guth's original notebook he describes his eureka moment for inflation: "This kind of supercooling can explain why the Universe today is so incredibly flat." By 'supercooling' he refers to a rapid phase transition where the Universe has not yet had time to react to symmetry breaking.

Review

Recall the key figure – inflation brings a comoving patch which is in thermodynamic equilibrium well within the expanding light horizon. This motivates solutions to the horizon, flatness, and monopole problems. In order to motivate a reasonable mechanism for the beginning and end of inflation we introduced the inflaton field. The idea is that GUT symmetry breaking somehow triggered an exotic field which started off in a symmetric state but during 'supercooling' was stuck in a false vacuum state. The inflation turns itself off as the true vacuum state is slowly reached. In quantum field theory the energy density $\rho_\varphi c^2$ of a scalar field is closely related to the more familiar classical version and the pressure P_φ includes both kinetic and potential contributions:

$$\begin{aligned} \rho_\varphi c^2 &= \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \qquad &(\text{"Inflaton energy density"}) \\ P_\varphi &= \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \qquad &(\text{"Inflaton pressure"}) \,. \end{aligned} \tag{1}$$

Slow-roll Approximation

Recall the requirement for exponential expansion is that of the cosmological constant equation of state, i.e. a negative pressure of $P = -\rho c^2$.

Q: How can we get that?
A: The "slow-roll approximation" (SRA) gives a zeroth order equation of state for super dark energy that can drive inflation:

$$\dot{\varphi} \ll V(\varphi) \qquad \Rightarrow \qquad P_\varphi \approx -\rho_\varphi c^2 \qquad (\text{"Slow-roll approximation"}) \,. \tag{2}$$

Note: Inflation ends when the true vacuum is reached.

"Reheating" is the process when the potential dissipates large amounts of energy (sloshing at the bottom of the potential well) and creates all of the normal particles in the Universe. This also answers why we live in an interesting Universe where the seeds of inflation are responsible for the cosmic landscape.

Quantum Fluctuations

Inflation imprints small energy perturbations in the early Universe which are slowly but inextricably amplified by gravity. After billions of years the perturbations from inflation grow into macroscopic structures, including galaxies, galaxy clusters and the cosmic web. The initial conditions set by inflation are very noisy with overdensities/underdensities of varying amplitude and spatial position. However, the spectrum is well described by Gaussian fluctuations in Fourier space. Thus, what happens on microscopic (sub-horizon) scales is blown up to macroscopic (super-horizon) scales where normally there would not be enough time for causal processes. Super-horizon fluctuations are frozen in so they cannot grow or decay until the Universe expands to contain the modes within the cosmic horizon.

Heisenberg's Uncertainty Principle

For a small amount of time we can "borrow" energy to create particles (or overdensities) in the very early Universe according to the Heisenberg Uncertainty Principle:

$$\Delta E \Delta t \simeq \hbar \equiv \frac{h}{2\pi} \quad \text{where} \quad \Delta t \sim t_{\text{final}} \sim 10^{-34} \text{ s}\,. \tag{3}$$

For structure seeds we need to consider how far the fluctuation is relative to the mean, i.e. $\Delta\rho/\rho$. If $E = mc^2 = \rho V c^2$ then the uncertainty principle within a cosmic volume predicts typical fluctuations on the order of

$$\boxed{\frac{\Delta\rho}{\rho} \sim \frac{\Delta E}{E} \sim \frac{\hbar/t_{\text{final}}}{E_{\text{GUT}}} \sim \frac{(6.582\times10^{-16}\text{ eV s})}{(10^{-34}\text{ s})(10^{15}\text{ GeV})} \sim 10^{-5} \qquad (\text{“Cosmic overdensities”})\,.} \tag{4}$$

This is in perfect accord with CMB observations which precisely relate temperature fluctuations as $\langle \Delta T/T\rangle \sim 10^{-5}$. This means CMB experiments must be sensitive to $\Delta T \sim 10\ \mu$K fluctuations. **Note:** The Milky Way is about a million times more dense than the background so $\Delta\rho/\rho \sim 10^6$.