AST 376 Cosmology — Lecture Notes

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THE VERY EARLY UNIVERSE: INFLATION (CONTINUED)

Dynamics of Inflation

The idea behind inflation is that a mysterious form of 'super dark energy' in the very early Universe, briefly after the GUT epoch, drives a period of exponential growth:

$$a(t) = a(t_{\rm GUT}) \exp\left[H_i \left(t - t_{\rm GUT}\right)\right] .$$

Q: What do we mean by 'super' dark energy?
A: To see that this name is truly justified we consider inflation under a 'super' cosmological constant model. The inflationary Hubble parameter and cosmological constant are

$$H_i = c\sqrt{\frac{\Lambda_i}{3}} \simeq \frac{1}{t_{\rm GUT}} \sim 10^{36}\ {\rm s}^{-1} \qquad \Rightarrow \qquad \Lambda_i \simeq 10^{51}\ {\rm cm}^{-2} .$$

This is inconcievably large compared to the current cosmological constant:

$$\Lambda = \frac{3H_0^2}{c^2}\Omega_\Lambda \sim 10^{-56}\ {\rm cm}^{-2} \sim 10^{-107}\ \Lambda_i \qquad \Rightarrow \qquad \Lambda_i \gg \Lambda .$$

Q: How does inflation solve the horizon problem?
A: The hyper-expansion of space moves the observable horizon well beyond the edge of the observable Universe. To put this in context we pick a comoving patch of space where everything is in thermal equilibrium. Originally at $t_{\rm init} \approx t_{\rm GUT} \approx 10^{-36}$ s the observable horizon is much larger than the comoving patch but by the time inflation ends at $t_{\rm final} \approx 10^{-34}$ s the exponentially increasing scale factor brings the horizon in to a microscopic fraction of the comoving patch. Eventually, even though the size of the observable Universe at $t_0 \approx 10^{17}$ s has grown outward at the speed of light, the Hubble radius $R_{\rm H}$ is still within the cosmic patch.
Note: Inflation also nicely solves the monopole and flatness problems.

The Inflaton Field

Idea: Inflation is driven by an unknown quantum field called the "inflaton" field denoted by $\varphi(t)$. During the GUT symmetry breaking there may have been a field transition to an unsymmetrical state. The potential energy $V(\varphi)$ of the field may have originally been symmetric with one vacuum minimum like a parabola. However, if $V(\varphi)$ is affected by symmetry breaking the inflaton field may get stuck in a nonzero "false vacuum" energy state. This scenario also provides a mechanism to end inflation as the field rolls to a "true vacuum" energy state.

In field theory the energy density of the inflaton field mimics what one might expect form classical mechanics with kinetic and potential energy terms:

$$\boxed{\rho_\varphi c^2 = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \qquad (\text{"Inflaton energy density"})\, .} \tag{1}$$

Recall that pressure can be thought of as momentum flux:

$$\boxed{P = \frac{\Delta F}{\Delta A} = \frac{\Delta p}{\Delta t \Delta A} \qquad (\text{“Pressure as momentum flux”})\,.} \tag{2}$$

There are two ways for the field to contribute to the overall pressure:

(i) Kinetic energy – the momentum-energy relation $\Delta p \simeq \Delta E_{\mathrm{kin}}/c$ leads to

$$P = \frac{\Delta p}{\Delta t \Delta A} = \frac{\Delta E_{\mathrm{kin}}}{c \Delta t \Delta A} = \frac{\Delta E_{\mathrm{kin}}}{\Delta V} \simeq \text{kinetic energy density.}$$

(ii) Potential energy – the force-energy relation $\Delta p/\Delta t = \Delta F = -\Delta E_{\mathrm{pot}}/\Delta x$ leads to

$$P = \frac{\Delta p}{\Delta t \Delta A} = -\frac{\Delta E_{\mathrm{pot}}}{\Delta x \Delta A} = -\frac{\Delta E_{\mathrm{pot}}}{\Delta V} \simeq -\text{potential energy density.}$$

The kinetic and potential energy contributions summarize our knowledge of the inflaton pressure:

$$\boxed{P_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \qquad (\text{“Inflaton pressure”})\,.} \tag{3}$$