AST 376 Cosmology — Lecture Notes

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(Dated: April 8, 2014)

THE VERY EARLY UNIVERSE: INFLATION

Years ago inflation would not have been a major topic but there are now empirical tests that prove its importance. Inflation represents a significant paradigm shift from previous Big Bang models. It beautifully asks us to re-examine what shaped the Universe and more closely approach the very instant of our cosmic origins. This theory of inflation has high implications, but is supported by observations so we would ignore it at our peril.

Cosmological Puzzles

The standard cosmological model we have studied so far is incomplete and needs (extreme) fine-tuning of the initial conditions. We now explore three puzzles ultimately solved by inflation:

(i) **Flatness problem** – Why is the Universe so close to $\Omega_{\rm tot}(z) = 1$ early on? Recall that $\Omega_{\rm tot} = \Omega_m + \Omega_r + \Omega_{\rm vac} + \ldots$ and any difference from one is attributed to geometric curvature. However, this situation is incredibly unstable. Unless the initial conditions are fine-tuned then we expect the effects of curvature to be apparent withing 1 ns of the Big Bang!

(ii) **Horizon problem** – Why is the CMB so uniform on large scales? In other words, why is the temperature anisotropy on opposite sides of the sky only one part in a hundred thousand, i.e. $\delta T/T \lesssim 10^{-5}$? These are regions of the Universe which were never in causal contact! Therefore, something or someone has once again conspired to fine-tune the cosmic initial conditions. In the standard picture there is no room for a causal explanation.

(iii) **Magnetic monopole problem** – Alan Guth originally conceived of inflation to rectify aspects of Grand Unified Theories (GUTs). Why are there no singular magnetic sources – why are magnetic fields divergenceless (purely solenoidal), i.e. why does $\boxed{\nabla \cdot \mathbf{B} = 0}$? This is not the case for electric charges, where $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. Monopoles are a (0-dimensional) "topological defect" resulting from phase transitions in the early Universe.

Note: 'Cosmic strings' and 'domain walls' are 1- and 2-dimensional topological defects.

Analogy: The freezing of liquid water does not occur uniformly but rather in localized regions. Defects are produced when the regions coalesce throughout the phase transition.

Idea: Consider the symmetry breaking of a Grand Unified Theory (GUT). Physics before the Plank epoch $t_{\rm Pl} \sim 10^{-43}$ s (10^{19} GeV) necessarily involves a Theory of Everything (TOE), where the four fundamental forces become manifestations of the same physical phenomenon. Although the physics after $t_{\rm Pl}$ is still quite uncertain we believe the Strong, Weak, and Electromagnetic forces remain unified until the GUT time $t_{\rm GUT} \sim 10^{-36}$ s (10^{15} GeV). After this the Strong force acts independently from the Electroweak force. Finally, all four forces act as separate entities beginning with the Electroweak epoch $t_{\rm EW} \sim 10^{-12}$ s (10^3 GeV) and continuing through the lifetime of the Universe $\sim t_{\rm H} \sim 10^{17}$ s ($\sim 10^{-13}$ GeV). [Diagram.]

Ultimately, we believe inflation was triggered as a result of GUT symmetry breaking for reasons we do not yet know! However, because phase transitions are not perfect they leave behind defects and we expect magnetic monopoles to be created prior to the onset of inflation.

Q: How many monopoles are left after GUT symmetry breaking?
A: If the volume surrounding each monopole is set causally, we have a typical separation of $r_{\rm mono} \sim ct_{\rm GUT}$. Therefore, early on we expect a number density of

$$n_{\rm mono}(t_{\rm GUT}) \approx \frac{1}{(ct_{\rm GUT})^3} \sim 10^{76}\ {\rm cm}^{-3}\,.$$

To judge whether this is big or small we need to compare it to more familiar present-day quantities. However, the density of monopoles has also been diluted by cosmic expansion, i.e. $n_{\rm mono} \propto a^3$. The GUT redshift $z_{\rm GUT}$ can be found by considering the radiation temperature:

$$T_{\rm CMB}(t_{\rm GUT}) \approx 2.7\ {\rm K}\ (1+z_{\rm GUT}) \quad \text{and} \quad T_{\rm GUT} \approx \frac{\epsilon_{\rm GUT}}{k_{\rm B}} \sim 10^{28}\ K \quad \Rightarrow \quad z_{\rm GUT} \sim 10^{28}\,.$$

Taking this into account we obtain a present monopole density of

$$n_{\rm mono}(t_0) = \frac{n_{\rm mono}(t_{\rm GUT})}{(1+z_{\rm GUT})^3} \approx \frac{10^{76}\ {\rm cm}^{-3}}{(10^{28})^3} \approx 10^{-8}\ {\rm cm}^{-3}\,.$$

This is actually quite large! We expect $\sim$ 10 monopoles in this room, something that experiments would easily detect. The mass density is even more revealing:

$$\rho_{\rm mono}(t_0) \simeq n_{\rm mono}(t_0)\frac{\epsilon_{\rm GUT}}{c^2} \sim 10^{-17}\ {\rm g\ cm}^{-3} \gg \rho_{\rm crit,0} \sim 10^{-29}\ {\rm g\ cm}^{-3}\,.$$

Without inflation we would definitely notice the effect of monopoles on the cosmic scales!

Disclaimer: There is a way out of the monopole problem. We could explain it away by saying we don't know much about GUT symmetry breaking. Perhaps there are fewer topological defects than expected.

Summary: We have explored three cosmological puzzles, the biggest of which is the cosmic horizon conspiracy because it violates causality. All of the paradoxes are solved by inflation.