AST 376 Cosmology — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith
(Dated: April 3, 2014)

DARK ENERGY (CONTINUED)

Review

We know the current value of w is about -1 indicative of the cosmological constant model; however, we don't know how/if this evolves in time, i.e. $w(z)$. UT Austin's Hobby-Eberly Telescope Dark Energy Experiment (HETDEX) is currently trying to measure this. The dynamics for dark energy with the w e.o.s. is summarized by

$$\boxed{P_{\text{vac}} = w \rho_{\text{vac}} c^2 \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{\text{eff}} \qquad \rho_{\text{eff}} = \rho + \frac{3P}{c^2} \qquad a \propto \exp[Ht]\,.}$$

Historical Aside

Einstein originally believed in a static universe and attempted to make the correct cosmological model. When he solved his equation for this universe he realized he needed positive geometric curvature, i.e. a closed universe. Analogous to the surface of a two-dimensional sphere light travels onward, around, and back to the same point after ~ 100 Gyr. A static universe requires $\ddot{a} = 0$.

Q: How do we accomplish this if the universe is full of matter?
A: Two components! This is curious, Einstein introduced a cosmological constant to avoid cosmic deceleration from normal cold matter. In particular, the density and pressure components are

$$\textit{(i)}\ \text{Normal matter:} \begin{cases} \rho_m \\ P_m \sim 0 \end{cases} \qquad \textit{(ii)}\ \text{Vacuum energy:} \begin{cases} \rho_{\text{vac}} \\ P_m = -\rho_{\text{vac}} c^2 \end{cases} .$$

Therefore, a static universe requires that

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m + \rho_{\text{vac}} + \frac{3P_{\text{vac}}}{c^2}\right) = 0 \qquad \Rightarrow \qquad \boxed{\rho_m = 2\rho_{\text{vac}} = \frac{\Lambda_{\text{E}} c^2}{4\pi G} \qquad (\text{“Static universe”}).}$$

Note: This is not true for our universe, only Einstein's static universe. The "radius of Einstein's universe" is $R_{\text{E}} = 1/\sqrt{\Lambda_{\text{E}}}$. This is a beautiful model, a universe of finite volume with no boundary. It is philosophically appealing but completely wrong.

The Quantum Nature of the Vacuum

Disclaimer: This subject is way above the pay grade of undergrads. Even Steven Weinberg doesn't know the answer to the nature of dark energy.

In quantum field theory (QFT) the vacuum is teaming with energy. It is full of particle-antiparticle pairs and vacuum fluctuations. We can describe this scenario as an ensemble of simple harmonic oscillators (SHOs). Visually, this is a set of vibrating springs. The quantum mechanical energy of an oscillator in the n^{th} excited state is

$$\boxed{E_n = \left(n + \frac{1}{2}\right) h\nu \qquad (\text{“Energy of QM SHO”})} \tag{1}$$

Idea: The vacuum energy is due to zero-point energy $E_0 = \frac{1}{2}h\nu$. In this picture our job is to estimate the energy density of the vacuum. We first find the density of states as a function of frequency based on the volume of quantum cells (λ^3):

$$\begin{pmatrix}\text{density}\\ \text{of states}\end{pmatrix} \simeq \frac{\Delta V}{\lambda^3} = \frac{\Delta V}{c^3}\nu^3 \,.$$

The total energy is taken as the sum of all contributions to the zero-point energy:

$$\Delta E \simeq \sum_{\nu} \begin{pmatrix}\text{density}\\ \text{of states}\end{pmatrix} \frac{h\nu}{2} \simeq \frac{\Delta V h}{c^3} \sum_{\nu=0}^{\infty} \nu^4 \,.$$

The energy density clearly diverges with increasing frequency, i.e. $\rho_{\text{vac}}c^2 = \Delta E/\Delta V \mapsto \infty$. However, if we impose an upper energy cutoff of $\epsilon_{\text{max}} = h\nu_{\text{max}}$ then we get the correct quantum field theoretical calculation for the energy density (barring a factor of $1/16\pi^2$)

$$\boxed{\rho_{\text{vac}}c^2 \simeq \frac{\epsilon_{\text{max}}^4}{c^3h^3} \qquad (\text{“Zero-point vacuum energy density”})\,.} \tag{2}$$

Q: How do we choose ϵ_{max}?
A: The energy cutoff corresponds to the scale of quantum gravity (QG) where quantum mechanics breaks down and has to be unified with gravity.

The Planck scale: The idea is to find the scale where the quantum mechanical wavelength overcomes the Schwarzschild radius, i.e. $\lambda_{\text{QM}} \sim R_{\text{S}}$. Remember the QM wavelength smears out position uncertainties so if it is on the order of the Schwarzschild radius then classical black holes don't make sense anymore. Thus, using the deBroglie and Compton wavelengths we have

$$R_{\text{S}} \equiv \frac{2Gm}{c^2} \qquad \text{and} \qquad \lambda_{\text{QM}} \simeq \frac{h}{p} \simeq \frac{h}{mc}\,,$$

so the Planck mass is

$$m_{\text{Pl}} \equiv \sqrt{\frac{hc}{G}} \approx 5 \times 10^{-5}\ \text{g} \approx 10^{19}\ \text{GeV/c}^2 \qquad (\text{“Planck mass”})\,. \tag{3}$$

This is the ultimate energy scale! For reference, the most extreme energies we can probe with particle experiments is $\sim 10^4$ GeV and the energy scale of Grand Unified Theories (GUT) is $\sim 10^{15}$ GeV. Similarly, the extreme Planck length is $\ell_{\text{Pl}} = h/m_{\text{Pl}}c \sim 10^{-33}$ cm and the Planck time is $t_{\text{Pl}} = \ell_{\text{Pl}}/c \sim 10^{-43}$ s, which corresponds to the earliest cosmic moment we can discuss after the Big Bang. Finally, if the Planck energy is $\epsilon_{\text{Pl}} = m_{\text{Pl}}c^2$ then the zero-point vacuum density is

$$\rho_{\text{vac}} \simeq \frac{\left(m_{\text{Pl}}c^2\right)^4}{c^5h^3} \simeq \frac{c^5}{G^2h} \sim 10^{93}\ \text{g cm}^{-3}\,.$$

We can compare this with the measured value assuming a cosmological constant:

$$\boxed{\rho_\Lambda = \Omega_\Lambda \rho_{\text{crit},0} \sim 10^{-29}\ \text{g cm}^{-3} \sim 10^{-122}\rho_{\text{vac}} \qquad (\text{“Vacuum density discrepancy”})\,.} \tag{4}$$

This is the single largest discrepancy between theory and observation in all of physics! In the past people have tried to remedy this by looking for symmetries, e.g. the local gauge symmetry of the electromagnetic interaction shows up to force the photon mass to be exactly zero. Supersymmetry could explain the very low vacuum density because of bosonic/fermionic partners. Although the bosonic oscillator only has one ground state $E_0 = \frac{1}{2}h\nu$ the fermionic ground state is split so that $E_{f,0} = \pm\frac{1}{2}h\nu$. If the supersymmetric partner is in the negative energy ground state then the zero-point energy cancels out exactly!

Problems: This picture still does not explain dark energy. It is easy to explain away the vacuum energy but a (small!) nonzero cosmological constant is not so easy. In other words, we do not know whether the measured Ω_Λ is due to vacuum energy or not. The other problem is that supersymmetry in the present-day Universe is (badly) broken. In summary, the problem of dark energy remains completely unsolved!