AST353 (Spring 2013)

ASTROPHYSICS

Project 2: Cosmic Entropy

15-min presentations: individually scheduled during week of April 15

Written report on puzzle part (one per group is enough), due in class:

Thursday, April 18

(worth 10/100)

You should work on all components of this project in close collaboration with your team. You are also expected to get stuck occasionally, at which point you should not hesitate to ask the TA or the professor for help.

1. Presentation: Time and the Big Bang

Prepare a brief (10-15 min) presentation (PPT, or white- blackboard, whatever you prefer) on the following (possibly related) questions:

a. Since all cosmic evolution, according to the Second Law of Thermodynamics, increases entropy, the initial Big Bang must have represented a state of minimal entropy. The smaller the entropy, the less likely a given state. The Big Bang must therefore have been *extremely* unlikely. How could such an unlikely event ever have occurred?

b. How is the entropy concept related to the fact that time is unidirectional? Put differently: Why do we remember the past, but not the future?

You might not immediately know how to attack the problem. Have some brainstorm sessions within your group, and do some background reading. Feel free to ask the TA or the professor for pointers. Each team will meet with the professor in individually scheduled appointments during the week of April 15 (for a 15-min time slot).

2. The puzzle

Again, you are expected to work on this part together with your teammates. At the end of it, each group will hand in a brief written report (due in class: April 18). One report per group suffices.

2.1. Boltzmann entropy

On Boltzmann's gravestone in Vienna, there is written the famous formula (in modern notation):

$$S = k_{\rm B} \ln W \ \ ,$$

where $k_{\rm B} = 1.38 \times 10^{-16}$erg K^{-1} is Boltzmann's constant, S is the entropy, and W gives the number of microstates that give rise to the same macrostate.

Now, consider a gas consisting of N (classical) particles. The particles can diffuse freely over a volume that is partitioned into k equal subvolumes. (There are no walls between the subvolumes, so that the particles can freely move around.) A certain macrostate of this system is specified by a set of k numbers N_i, which give the number of particles that occupy subvolume i.

a. Show that the entropy of a given macrostate is approximately

$$S \simeq k_{\rm B}(N \ln N - \sum_{i=1}^{k} N_i \ln N_i) \ \ .$$

You will need to use Stirling's formula to evaluate the factorials of very large numbers ($\ln y! \approx y \ln y - y$ for $y \gg 1$).

b. Prove that the entropy is maximized when $N_i = N/k$ for all i. Write down an expression for this maximum (equilibrium) entropy, $S_{\rm max}$.

c. Now, assume $N = 50$ and $k = 6$. What is $S_{\rm max}$ in units of the Boltzmann constant? That is, write your answer in the form: $S_{\rm max} = x k_{\rm B}$, and your job is to find x.

2.2. Entropy and the Universe

Here, you might not immediately see how to proceed (or which formulae to use). Give it a first try, and then ask the TA or the professor for further hints.

a. What is the entropy of the Sun? Express your result again in units of the Boltzmann constant: $S_{\odot} = x_1 k_{\rm B}$, and your job is to find x_1.

b. What is the entropy of the observable universe? Again, express your result as: $S_{\rm universe} = x_2 k_{\rm B}$, and your job is to find x_2.