AST353 (Spring 2013)

ASTROPHYSICS

Project 1: Calculus of variations

15-min discussion: Thursday, February 28, 2013, during class

Written report, due in class: same day

(worth 10/100)

You should work on all components of this project in close collaboration with your team (Groups A – F). You are also expected to get stuck occasionally, at which point you should not hesitate to ask the TA or the professor for help.

1. Euler-Lagrange equation(s)

Carefully go through the derivation of the Euler-Lagrange (E-L) equation (see handout). In particular, make sure that you understand the basic idea behind the calculus of variations. You are **not** expected to memorize the mathematics (with the exception of being able to write down the E-L equation itself). The goal here is insight and understanding, not rote memorization.

I will test your understanding in a short (15-min) discussion with the individual groups. These discussions will take place during class on Thursday, Feb. 28, where each group will have an appointment for a 15-min time slot.

2. The puzzle

Again, you are expected to work on this part together with your teammates. At the end of it, you will hand in a brief written report (due in class: Feb. 28). Although this report will reflect what you have done in your respective group, EVERY STUDENT should hand in his/her report. It is OK if these reports are identical within a given group, but the groups should work independently from each other.

a. Regarding the brachistochrone problem (see handout): Fill in the detailed steps that lead from the E-L equation to the ordinary differential equation (ODE) of the problem ($2yy'' + y'^2 + 1 = 0$). Show that the cycloid solves this ODE.

b. What is the time (in seconds) needed to fall along a brachistochrone path from the origin (point A) to point B, which is located 1.57 meters to the right and 1 meter down

($x_2 = 1.57\,\mathrm{m}, y_2 = 1\,\mathrm{m}$. Assume the usual acceleration of gravity on the surface of Earth ($g = 9.8$ m s^{-2}).

c. Consider light traveling in a ($x - y$) plane, through a medium where the index of refraction (and hence the speed of light) depends on position. According to *Fermat's Principle of Least Time*, light travels along a path for which the travel time is minimized. Now assume that the speed of light increases with height ($v \propto y$). What path is taken by light under these circumstances?