AST353 (Spring 2013)

ASTROPHYSICS

Problem Set 4

Due in class: Thursday, April 25, 2013

(worth 10/100)

1. Central Pressure in a Neutron Star

Consider a star with constant density, $\rho_0 =$ const., and in hydrostatic equilibrium. In class, we derived an expression for the central pressure in a star described by the Oppenheimer-Volkoff (OV) equation of General Relativity (GR):

$$P_c = \rho_0 c^2 \frac{1 - \sqrt{1 - R_S/R}}{3\sqrt{1 - R_S/R} - 1} \ ,$$

where R is the radius of the star, and R_S is the Schwarzschild radius.

a. Show that the relativistic expression for P_c can be simplified in the limit of weak gravitational fields ($R_S/R \ll 1$) to the Newtonian result that one would find for such a constant-density star:

$$P_c = \frac{2\pi G \rho_0^2}{3} R^2 \ .$$

(You will have to use Taylor expansions.)

b. Now, evaluate the GR expression for P_c, assuming that $R_S/R = 0.5$ (typical for a NS). What is P_c in this case? Express your result as $P_c = x \rho_0 c^2$, and your job is to find the numerical constant x. This is the central pressure *required* to keep the star in hydrostatic equilibrium. In the next part, you will compare this with the pressure that is actually *available*.

c. Assume that the pressure is due to *relativistically* (UR) degenerate neutrons. From your lecture notes, you find:

$$P_c = \frac{hc}{4} \left(\frac{3}{8\pi} \right)^{1/3} n_c^{4/3} \ ,$$

and $n_c \simeq \rho_c / m_{\rm H}$ (appropriate for a NS), where $\rho_c = \rho_0$ (constant density case). By equating this expression with the one in part b., find the density,ρ_0, in this particular NS! Express your result in g cm^{-3}.

d. Now, with the density in hand, find the mass and radius of the NS considered here! Express the mass in units of the solar mass ($M_\odot = 2 \times 10^{33}$ g), and the radius in km.

2. General Relativity

a. Assume that an atom, sitting on the surface of a WD, emits a spectral line with rest wavelength λ_0. The WD has a mass of one solar mass, and a radius of $\sim 10,000$ km. Now, a far-away observer measures a longer (redshifted) wavelength for this line of λ_∞. How large is the fractional shift in wavelength, $z \equiv (\lambda_\infty - \lambda_0)/\lambda_0$, due to the gravitational redshift?

b. The wavelength of a helium-neon laser is measured inside a spaceship freely floating far out in deep space, and is found to be 632.8 nm. What wavelength would an experimenter measure, if she and the laser fell freely together towards a neutron star (NS)? Assume that the NS has a mass of $1.5M_\odot$, and a radius of 10 km.

c. Consider the following spacetime metric, expressed in spherical coordinates (r, θ, ϕ):

$$ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ .$$

Use the transformation equations: $r = (x^2+y^2+z^2)^{1/2}$, $\theta = \arccos(z/r)$, $\phi = \arctan(y/x)$, to show that the metric above can be re-written as the usual Minkowski metric of flat spacetime:

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 \ .$$

Note, that in class we argued that we can discern curvature of spacetime when some of the metric coefficients depend on location. Surprisingly, this is the case, considered here, for *flat* spacetime expressed in non-Cartesian (spherical) coordinates. What is the *true* criterion for spacetime curvature?