AST353 (Spring 2013)
ASTROPHYSICS
Problem Set 2

Due in class: Thursday, March 7, 2013

(worth 10/100)

1. Mass and density in White Dwarfs

Assume that a white dwarf (WD) has sufficiently low mass to be supported by non-relativistic (NR), degenerate electrons. The mass-radius relation can then be written, when we scale all variables to the solar values:

$$R \simeq 0.01 R_{\odot} \left(\frac{M}{M_{\odot}}\right)^{-1/3} \quad .$$

a. Using the mass-radius relation from above, find an expression for the central pressure. Use the approximate formula derived in class: $P_c \simeq GM^2/R^4$. Your result should look like:

$$P_c \simeq K_1 \left(\frac{M}{M_{\odot}}\right)^{x_1} \quad ,$$

and your job is to find the constants K_1 (in units of dyn cm−2), and x_1 (a pure number).

b. Now, use the formula for NR degenerate electrons to connect the central pressure with the central density: $P_c = K_{\rm NR}(\rho_c/2m_{\rm H})^{5/3}$. In evaluating the numerical value of $K_{\rm NR}$, what should you use for the particle rest mass, m_0, in this case? Equate this expression for P_c with the one found in part a. This will allow you to solve for ρ_c as a function of WD mass. Your result should look like:

$$\rho_c \simeq K_2 \left(\frac{M}{M_{\odot}}\right)^{x_2} \quad ,$$

and your job again is to find the constants K_2 (in g cm^{-3}), and x_2 (a pure number).

c. Assume that relativistic effects become important for densities $\rho_{\rm crit} \simeq 10^5$ g cm^{-3}. Using your result from part b., find the approximate WD mass, $M_{\rm crit}$, for which relativistic degeneracy effects become important. Express your result in units of solar masses ($M_{\odot}$)!

2. Planck length

In class, we have introduced the concept of the *Planck mass* ($m_{\rm Pl}$). Now, define the *Planck length* ($l_{\rm Pl}$) as the Compton wavelength corresponding to $m_{\rm Pl}$. What is the *Planck length* (in cm)? What is its meaning (or interpretation)?