AST353 (Spring 2013)

ASTROPHYSICS

Problem Set 1

Due in class: Thursday, February 7, 2013

(worth 10/100)

1. Simple stellar model

Assume a star has a radius of $R = 3R_\odot$ and a quadratic density profile:

$$\rho(r) = \rho_c \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad .$$

Here, $\rho_c = 20$ g cm^{-3} is the central density.

a. What is the mass M of the star (in units of the solar mass $M_\odot$)?

b. What is the average density, $< \rho >$, for this star? What is the free-fall time, $\tau_{\rm ff} \simeq 1/\sqrt{G < \rho >}$? Very briefly explain the meaning of $\tau_{\rm ff}$!

c. Solve the equation of hydrostatic equilibrium to find the pressure, $P(r)$, as a function of radius. When doing the integration, assume that the pressure drops to zero at the outer boundary, $P(R) = 0$ (so-called *zero boundary condition*). Express your answer in the form:

$$P(r) = K \left[1 + a_1 x + a_2 x^2 + a_3 x^3 + ... \right] \quad ,$$

where $x = r/R$. Here, $a_1, a_2, a_3, ...$ are numerical constants, and your job is to find them. Also, you need to determine the constant K in front of the polynomial.

d. Find an expression for the central pressure P_c, and evaluate it for this star (in units of Pa or dyn cm^{-2})! Compare this with the value you get using the approximate formula for P_c that we have derived in class!

e. Find an expression for the total gravitational potential energy, $E_{\rm pot}$, of such a star! Express your result first in a general form, as a function of stellar mass and radius. You should find $E_{\rm pot} = -$ const GM^2/R, and your job is to determine the numerical constant. Then evaluate this expression for the particular star in this problem (in units of erg or J)!

2. Particle kinetic energies

From Special Relativity (SR), we know that a particle's total energy, ϵ, is related to its momentum, p, and rest mass,m_0, as follows: $\epsilon^2 = p^2c^2 + m_0^2c^4$. We can then define the particle's kinetic energy: $\epsilon_{\rm kin} = \epsilon - m_0c^2$ (all of the total energy that is not in the form of rest mass).

a. Using a Taylor expansion, show that $\epsilon_{\rm kin} = p^2/(2m_0)$ for non-relativistic (NR) particles! Use the defining property for NR particles: $pc \ll m_0c^2$.