AST 353 Astrophysics — Lecture Notes

Prof. Volker Bromm — TA: Aaron Smith*

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WHITE DWARF STARS (CONT.)

Review

The condition that

$$P_{\rm UR} \gtrsim -\frac{1}{3}\frac{E_{\rm pot}}{V} \approx \frac{GM^2}{R^4}$$

gives a constraint on the mass of White Dwarf (WD) stars:

$$\boxed{M_{\rm WD} \lesssim \left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_{\rm H}^2} = \frac{m_{\rm Pl}^3}{m_{\rm H}^2} \approx 1.4\ M_\odot\,,}$$

where the Planck mass is the smallest mass of a classical black hole (BH) because at this point the quantum wavelength can no longer be contained in the event horizon

$$\boxed{M_{\rm BH} \gtrsim m_{\rm Pl} = \sqrt{\frac{hc}{G}} \sim 10^{-5}\ {\rm g}\,.}$$

Re-phrase $M_{\rm Ch}$ in terms of the fundamental strength of gravity

We do this by comparison with the strength of the electro-magnetic (em) interaction. Recall one derivation for the "fine-structure constant" α. Place two protons a distance r away from each other. Each proton has a mass $m_{\rm H}$ and a charge $+e$. The potential energy is

$$U_{\rm em} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}\,,$$

but in the conversion from SI to cgs units we have $\epsilon_0 = 1/4\pi$ and $e = 4.8\times10^{-10}$ e.s.u. (electrostatic units) instead of the familiar $e = 1.6\times10^{-19}$ C from SI units. We want to measure the strength of electromagnetism on quantum scales so we evaluate $U_{\rm em}$ at the compton wavelength

$$r = \lambda_{\rm C} = \frac{h}{m_{\rm H}c} \approx \frac{\hbar}{m_{\rm H}c}\,,$$

where it is conventional to use $\hbar = h/2\pi$ ("h-bar") rather than our regular Planck's constant h. Thus, we define the e-m "fine-structure constant" as

$$\boxed{\alpha \equiv \frac{U_{\rm em}(r=\lambda_{\rm C})}{m_{\rm H}c^2} = \frac{e^2}{(\hbar/m_{\rm H}c)m_{\rm H}c^2} = \frac{e^2}{\hbar c} \approx \frac{1}{137}\,.} \tag{1}$$

*asmith@astro.as.utexas.edu

By analogy we may define a "gravitational fine-structure constant"

$$\boxed{\alpha_{\rm G} \equiv \frac{U_{\rm G}(r=\lambda_{\rm C})}{m_{\rm H}c^2} = \frac{Gm_{\rm H}^2}{\hbar c} \approx 10^{-38}\,.} \tag{2}$$

This says that we have discovered a force (electromagnetism) which is 36 orders of magnitude stronger than gravity! Furthermore the Chandrasekhar mass is fundamental:

$$\boxed{M_{\rm Ch} \sim \alpha_{\rm G}^{-3/2} m_{\rm H}\,.} \tag{3}$$

Note: Since gravity is so weak we need about 10^{57} nucleons for gravity to overcome the e-m force! This is the reason our sun is the size it is. In fact, $M_{\rm Ch} \sim 1M_\odot$ provides the TYPICAL mass of any star in the Universe!

Another way of comparing the force of gravity and electro-magnetism is to consider the effect of global charge neutrality. Gravity can have a head start if all of the long-scale e-m forces cancel out. Thus, we compare the same $U_{\rm em}$ with a different $U_{\rm G}$ where we use the macroscopic quantities M and R for the mass and radius of the system. We note that in our strange NR objects the number density and the radius are related to give

$$r \sim n^{-1/3} \quad \text{and} \quad n = \frac{N}{R^3} \quad \Rightarrow \quad R \sim N^{1/3} r\,,$$

where r is the typical separation of protons (a scale for which we cannot impose charge neutrality). Finally, if the mass of the object is $M \approx N m_{\rm H}$ then the number of particles can be found:

$$\begin{aligned} U_{\rm em} = U_{\rm G} \qquad &\Rightarrow \qquad \frac{e^2}{r} = \frac{Gm_{\rm H}M}{R} \approx \frac{Gm_{\rm H}Nm_{\rm H}}{N^{1/3}r} \\ \Rightarrow \qquad N \approx \left(\frac{e^2}{Gm_{\rm H}^2}\right)^{3/2} &\equiv \left(\frac{\alpha}{\alpha_{\rm G}}\right)^{3/2} \sim \left(10^{36}\right)^{3/2} \sim 10^{54}\,. \end{aligned} \tag{4}$$

Note: This is the TYPICAL mass of a planet! How can you have objects smaller than this? It is possible (e.g. potato-shaped comets or dust grains!) they just wouldn't be dominated by gravity.

Cooling of White Dwarfs

White Dwarfs are dying stars! They have no internal energy source to regulate their temperature and in time will radiate away their energy. If we assume WDs emit as a blackbody then we can take advantage of the Stefan-Boltzmann law:

$$L = 4\pi R^2 \sigma_{\rm SB} T_{\rm eff}^4\,,$$

where the Stefan-Boltzmann constant is $\sigma_{\rm SB} = 5.67 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}. Recall that the mass-radius relationship for NR WDs is

$$\boxed{R = \frac{R_\odot}{100}\left(\frac{M}{M_\odot}\right)^{-1/3}.} \tag{5}$$

WDs usually start out with a temperature of $T_{\rm eff} \sim 20,000$ K. Therefore the initial luminosity is

$$\boxed{L = \frac{L_\odot}{100}\left(\frac{M}{M_\odot}\right)^{-2/3}\left(\frac{T_{\rm eff}}{20,000\ {\rm K}}\right)^4.} \tag{6}$$

We find a typical WD has a luminosity of about $10^{-2}\ L_\odot$, which is small because although WDs are hot, they have much less surface area to radiate out their heat.

As WDs cool, their temperature drops very slowly. Their cooling time is longer than the age of the Universe. WD observations reveal two mass limits–an upper bound set by the Chandrasekhar limit of 1.4 $M_\odot$ and a lower bound set by stellar lifetimes because stars with masses below 0.25 $M_\odot$ take more than the Hubble time ($\sim$ 14 Gy) to become WDs.

Note: Some astrophysical situations (e.g. strong magnetic fields) may produce additional pressure contributions and therefore increase the Chandrasekhar mass! Recent studies claim to have observed 2 $M_\odot$ WDs!