AST 353 Astrophysics — Lecture Notes

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SPECIAL RELATIVITY

This material is important for understanding the astrophysics of massive, compact objects.

Concepts and motivation

In the late 1800's Maxwell's theory of electromagnetism gave a rigorous setting for a wave description of light. Therefore, it was believed that a medium (or "ether") was required for the waves to propagate in. To find the medium they built an interferometer (wikipedia). The differences in arm length can be taken into account by (i) rotating the apparatus and (ii) waiting in time for Earth's orbit to change. The result is that the speed of light is independent of the motion of the source and the observer! ... and

$$c \approx 300,000 \text{ km/s} \,. \tag{1}$$

Thought experiments: Travelers are moving toward each other and emit light at the same time in a central observer's frame. Each of the observers think their light flashes are in different places at later times. The end result is the non-uniqueness of **simultaneity**! The observers think that the light flashes were made at different times. However, all of this is consistent with the concept of **causality** (or cause and effect).

We now consider another experiment to make this concrete. This is a train experiment in which a light is sent from the ceiling to the floor. The train observer is the proper (or rest) observer. An observer at the train station sees the light moving to the left at the velocity of the train:

$$\begin{aligned} \Delta t_{\text{proper}} &= \frac{\ell}{c} \quad \text{and} \quad \Delta t = \frac{\sqrt{\Delta x^2 + \ell^2}}{c} \quad \text{where} \quad \Delta x = v \Delta t \,, \\ \Rightarrow \Delta t_{\text{proper}}^2 &= \frac{\ell^2}{c^2} = \Delta t^2 - \frac{\Delta x^2}{c^2} = \Delta t^2 \left(1 - \frac{v^2}{c^2} \right) \,, \\ \Rightarrow \Delta t &= \frac{\Delta t_{\text{proper}}}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_{\text{proper}} \,. \end{aligned} \tag{2}$$

What about changing both position and time? This brings us to the concept of "spacetime" which we keep track of by spacetime diagrams and a perverse Pythagorean Theorem where time and position are distinguished by a minus sign:

$$\begin{aligned} (ct')^2 - x'^2 - y'^2 - z'^2 &= (ct)^2 - x^2 - y^2 - z^2 \\ \text{and} \quad \Delta t_{\text{proper}}^2 &= -\Delta t^2 + \frac{1}{c^2} \left(\Delta x^2 + \Delta y^2 + \Delta z^2 \right) \,. \end{aligned} \tag{3}$$

There are many pedagogical resources on relativity. Please consult: wikipedia or a great textbook.

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