AST 353 Astrophysics — Lecture Notes

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(Dated: April 9, 2013)

NEUTRON STARS (CONT.)

Upper mass limit

Last time we derived a rough upper mass limit for neutron stars:

$$\boxed{M_{\rm OV} = \frac{8}{27}\left(\frac{c^2}{G}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2} \qquad (\text{“Oppenheimer–Volkoff Limit”}).} \tag{1}$$

This is the equivalent of the Chandrasekhar limit of White Dwarf stars except the calculation is based on **GR**, not quantum mechanics. We used the most “extreme” equation of state possible

$$\rho = \rho_0 = \text{constant.} \tag{2}$$

This is a “stiff” equation of state because it models an incompressible fluid. In general one would write a polytropic equation of state

$$P = K\rho^\gamma\,,$$

where the constants K and γ (the adiabatic index) characterize the “resistance to compression.” The equation to substitute into the OV equation is then

$$\rho = \left(\frac{P}{K}\right)^{1/\gamma}\,. \tag{3}$$

For our stiff equation of state $\rho =$ constant $\gamma \to \infty$. A realistic e.o.s. is “softer” (less stiff) because gas can be compressed. However, because there is a great deal of uncertainty in the true e.o.s. for ultra-high density neutron stars we can only give an approximate range for the mass upper limit:

$$\boxed{M_{\rm OV} \approx 1.5 - 3\ M_\odot \qquad (\text{“Oppenheimer–Volkoff Limit”}).} \tag{4}$$

Note: All currently observed NS (e.g. pulsars in binary systems) have masses just above the Chandrasekhar limit $M_{\rm NS} \approx 1.45\ M_\odot$.

Structure of cold, dead stars

We may run a constant e.o.s. for matter in its ‘ground state.’ This is a zero energy state where **all** of the nuclear fuel is entirely spent. This end state is cold, dead matter or “cold, catalyzed matter.” From this we construct many equilibrium stellar models like snapshots of the various states. Obtaining initial conditions amounts to choosing a central density $\rho_{\rm c}$ and using the e.o.s. to calculate the central pressure $P_{\rm c}$. The pressure gradient is negative which means there is a point in this model where the pressure falls to zero. Thus, we integrate the OV equation (dP/dr) from the center $r = 0$ to the surface $r = R$ such that $P(R) = 0$. At this point we summarize the stellar snapshots by the total mass

$$M = M(\rho_{\rm c}) = m(R) \qquad \text{where} \quad R = R(\rho_{\rm c})\,.$$

The mass M and radius R are functions of the central density $\rho_{\rm c}$. At this point we may plot the mass-radius relation. Stability requires $dM/d\rho_{\rm c} > 0$ because otherwise as pressure is added the star can support less and less mass and collapse continues.