AST 353 Astrophysics — Lecture Notes

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NEUTRON STARS

The relativistic field equation (i.e. Einstein's equivalent of $\nabla^2\varphi = 4\pi G\rho$) is summarized as

$$\begin{pmatrix}\text{curvature}\\ \text{of spacetime}\end{pmatrix} = \frac{8\pi G}{c^4}\begin{pmatrix}\text{energy}\\ \text{density}\end{pmatrix}. \tag{1}$$

This relates the fact that matter tells space how to curve, and space tells matter how to move. The full Einstein equation in all its glory is

$$\boxed{G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \qquad \text{(Relativistic field equation)},} \tag{2}$$

which has second derivatives on the left and physics, $T^{\mu}_{\nu} = \text{diag}(\rho c^2, P, P, P)$, on the right. Schematically, we can think of the Ricci tensor as being

$$R_{\mu\nu} = \text{“Ricci”} = f\left(\frac{\partial^2 g}{\partial x^2}, \frac{\partial g}{\partial x}, g\right) \qquad \text{and} \qquad R = \text{“Ricci scalar,”}$$

with dimensions $[R] = [\partial^2/\partial x^2] = \text{length}^{-2} = \text{cm}^{-2}$ and $[g_{\mu\nu}] =$ pure number. **Note:** We expect 2^{nd} derivatives because (i) curvature requires it and (ii) they appear in the Newtonian field equation.

Last time we also derived the Oppenheimer–Volkoff (OV) equation

$$\frac{dP}{dr} = -\rho\frac{Gm}{r^2}\frac{(1+x_1)(1+x_2)}{(1+x_3)} \tag{3}$$

which is expressed here as small deviations (i.e. x_1, x_2, x_3)from the Newtonian equation of hydrostatic equilibrium. We did not quite justify the $r^2 \to r^2(1-2m/r)$ but we will come back to this when we derive the metric for black holes. For now, we can use the OV equation for neutron stars.

Neutron stars (NS) were first postulated by theory (Baade & Zwicky 1934; Landau 1938). Zwicky was a genius but was known for his strong personality. (People say a “spherical bastard” because it doesn't matter which way you look!) The main idea is that a NS is born in a supernova (SN) explosion at the death of a massive star ($> 8M_\odot$). However, because there were no observations to back up the idea nobody worked on NSs for 30 years when pulsars were serendipitously discovered with radio astronomy (Bell & Hewish 1967). Pulsars are rotating NSs with a period of about 33 ms. They are surrounded by synchrotron radiation, which is produced when fast (relativistic) particles accelerate or change direction s in magnetic fields. They cannot be White Dwarf (WD) stars because the rotation would tear the WD apart. Also, they cannot be WD radial oscillations because the free-fall time ($\tau_{\text{ff,WD}} \sim 1$ s), which tells how quickly it would take to notice deviations from HSE, is too large.

Basic properties of Neutron Stars

From pulsar observations we can infer the basic properties of NSs. From binary sources we find

$$\boxed{M_{\text{NS}} \sim 2M_\odot\,.} \tag{4}$$

We estimate the radius from the observed rotation period $P_{\rm NS}$. The equatorial (rot.) velocity is

$$v_{\rm rot} = \frac{2\pi R_{\rm NS}}{P_{\rm NS}}\,.$$

Therefore, we constrain the radius by requiring that the centrifugal force does not exceed gravity:

$$\frac{v_{\rm rot}^2}{R_{\rm NS}} < \frac{GM_{\rm NS}}{R_{\rm NS}^2} \qquad \Rightarrow \qquad R_{\rm NS} < \left(\frac{GM_{\rm NS}P_{\rm NS}^2}{4\pi^2}\right)^{1/3} \approx 100 \text{ km}\,.$$

However, we now know the radius is even smaller than this. For our calculations we will use

$$\boxed{R_{\rm NS} \approx 10 \text{ km}\,.} \tag{5}$$

The average density is calculated to be

$$\boxed{\langle\rho\rangle_{\rm NS} \approx \frac{M_{\rm NS}}{\frac{4\pi}{3}R_{\rm NS}^3} \sim 10^{14} \text{ g/cm}^3\,.} \tag{6}$$

Compared to the density of the sun $\langle\rho\rangle_\odot \sim 1$ g/cm^3 and the previous record holder $\langle\rho\rangle_{\rm WD} \sim 10^6$ g/cm^3 this is an extreme density! In fact, it is comparable to nuclear density. If the effective nuclear radius is given by $R \approx r_0 A^{1/3}$, where $r_0 = 1$ fermi $= 1$ fm $= 10^{-15}$ m $= 10^{-13}$ cm, then

$$\langle\rho\rangle_{\rm Nuclear} \approx \frac{m_H}{\frac{4\pi}{3}r_0^3} \sim 4\times 10^{14} \text{ g/cm}^3\,.$$

Note: NSs are like giant atomic nuclei, but not quite. A NS is held together by gravity whereas a nucleus is held together by the strong force.

Nuetronization

Why are NSs composed mostly of neutrons? This is not a trivial question because under normal conditions free electrons are unstable and decay through β-decay:

$$\boxed{n \to p^+ + e^- + \bar{\nu}_e \qquad (\beta\text{-decay}),} \tag{7}$$

where $\bar{\nu}_e$ is the anti-electron neutrino required to conserve lepton number. The decay process happens on a time scale of about $\tau_{1/2} \sim 10$ min.
Q: Why do neutrons NOT decay in Neutron Stars? **A:** Because it is energetically disfavored!
During the collapse of a massive star's core (en route to NS) neutrons are bathed in a sea of degenerate (UR) electrons. Recall the Fermi momentum $p_{\rm F} \sim h n_e^{1/3}$ which tells about the distribution of the degenerate particles. We can get an energy corresponding to the "surface of the Fermi sea" if we use the relativistic energy $\epsilon^2 = p^2c^2 + m_0^2c^4 \approx p^2c^2$ for UR particles. This energy is

$$\boxed{\epsilon_{\rm F} \equiv p_{\rm F}c \qquad \text{(Fermi energy).}} \tag{8}$$

What is important here is that $\epsilon_{\rm F} > 1.3$ MeV $= (m_n - m_p)c^2$. Thus, neutron decay is prohibitted because there is "no space in the Fermi sea." We do not obtain enough energy from a β-decay to put the electron in the lowest available energy state. Furthermore, it is favorable to get rid of electrons! Matter becomes increasingly neutron-rich via inverse β-decay, or "neutronization," as shown by the following reaction:

$$\boxed{e^- + p^+ \to n + \nu_e \qquad (\text{Inverse } \beta\text{-decay}).} \tag{9}$$