AST353 (Spring 2013)

ASTROPHYSICS

Exam 1 (March 7, 2013)

(worth 15/100)

Write your name on top of every sheet that you use to answer this exam!

1. Virial Equilibrium

(Total of 4 points)

Consider a gas cloud, having a mass of $M = 10^6 M_\odot$, a radius of $R = 50$ pc, and an average temperature of $T = 10$ K. For simplicity, assume that the cloud consists of hydrogen only.

Using order-of-magnitude arguments, assess whether the cloud is in virial equilibrium, or not. If it is not, what would happen to the cloud?

2. Simple Stellar Model

(Total of 8 points)

Assume a star has a radius of $R = 2R_\odot$ and a uniform (constant) density profile:

$$\rho(r) = \rho_0 = 50 \text{ g cm}^{-3} \ .$$

a. What is the mass M of the star (in units of the solar mass $M_\odot$)?

b. What is the free-fall time for this star, in suitable units (s, h, years,...)? Very briefly explain the meaning of $\tau_{\rm ff}$!

c. To find the pressure, $P(r)$, as a function of radius, solve the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\rho \frac{Gm(r)}{r^2} \ ,$$

where $m(r)$ is the mass enclosed within a spherical shell of radius r. When doing the integration, assume that the pressure drops to zero at the outer boundary, $P(R) = 0$ (so-called *zero boundary condition*). Express your answer in the form:

$$P(r) = K \left[1 + a_1 x + a_2 x^2 + a_3 x^3 + ...\right] \ ,$$

where $x = r/R$. Here, $a_1, a_2, a_3, ...$ are numerical constants, and your job is to find them. (NOTE: Because of the symmetry of the problem, many of those numerical constants will just be zero.) Also, you need to determine the constant K in front of the polynomial.

d. Using your result in part c., evaluate the pressure at the center of the star, $P_c = P(r = 0)$ (in units of $\mathrm{dyn\,cm^{-2}}$)! Compare this with the value you get using the approximate formula for P_c that we have derived in class! (NOTE: If you got stuck in part c. somewhere, still go ahead and calculate the simplified estimate, so that you can earn partial credit.)

3. Planck time

(Total of 2 points)

In class, we have introduced the concept of the *Planck mass*: $m_{\rm Pl} \simeq (hc/G)^{1/2}$, and in the last problem set that of the *Planck length*: $l_{\rm Pl} \simeq (hG/c^3)^{1/2}$.

Find a combination of the three fundamental constants (h, G, c) that has the dimensions of time, giving the *Planck time*. What is the value (in units of s)? What is its meaning (interpretation)?

4. Chandrasekhar Mass

(Total of 1 point)

What is the *Chandrasekhar mass*? Why is this such an important concept in astrophysics?

Here, you don't have to write down any equations, or do any calculations. Just answer (very briefly) in words.

Newton's constant:

$$G = 6.67 \times 10^{-8} \mathrm{cm}^3\ \mathrm{g}^{-1}\ \mathrm{s}^{-2}$$

Boltzmann constant:

$$k_{\mathrm{B}} = 1.38 \times 10^{-16} \mathrm{erg}\ \mathrm{K}^{-1}$$

Electron rest mass:

$$m_e = 9.1 \times 10^{-28}\ \ \mathrm{g}$$

Mass of hydrogen atom:

$$m_{\mathrm{H}} = 1.67 \times 10^{-24} \mathrm{g}$$

Planck constant:

$$h = 6.626 \times 10^{-27} \mathrm{erg\ s}$$

Speed of light:

$$c = 3 \times 10^{10} \mathrm{cm\ s}^{-1}$$

Solar mass:

$$M_{\odot} = 2 \times 10^{33} \mathrm{g}$$

Solar radius:

$$R_{\odot} = 6.96 \times 10^{10} \mathrm{cm}$$

Parsec:

$$1\ \ \mathrm{pc} = 3.09 \times 10^{18} \mathrm{cm}$$