Physics of Compact Objects (cont)

Pressure inside stars:
know: \( n, T, \rho, \) etc. \( \Rightarrow \) Calculate \( P = P(n, T, ...) \)

Equation of state

Possible combinations of conditions of gas:

- Classical
  - (non-degenerate)
  - (ultra-relativistic (UR))
- Quantum
  - (degenerate)
  - (relativistic (NR))

\( E_{\text{kin}} \ll m_c^2 \quad E_{\text{kin}} \gg m_c^2 \)

Case 1: Classical + NR ("normal" \( \Rightarrow \) atmosphere of Earth)

\(- \text{gas is NR} \Rightarrow P = \frac{2}{3} U_{\text{kin}} \)

\( U_{\text{kin}} = \frac{N}{V} \langle E_{\text{kin}} \rangle = n \langle E_{\text{kin}} \rangle \)

\( = \frac{3}{2} n k_b T \)

\( \frac{1}{2} k_T \text{ per degree of freedom per particle} \)
\[
P = \frac{2}{3} \, U_{\text{kin}} = n \, k_B \, T \quad (\text{"ideal gas law"})
\]

**Classical/Quantum Boundary**

Q: When does quantum mechanics (QM) become important?

Define the average distance between particles:

\[
n = \frac{N}{V} = \frac{1}{L^3} \quad \Rightarrow \quad L = n^{-\frac{1}{3}}
\]

where \( L \) is the average density between particles.

Consider the QM wave associated with a particle:

\[
\lambda_{\text{deB}} = \frac{\hbar}{p}, \quad \text{when } \hbar \text{ is Planck's constant.}
\]

QM important when \( L \leq \lambda_{\text{deB}} \).

\[
\Rightarrow \quad n^{-\frac{1}{3}} \leq \frac{\hbar}{p} \quad \Rightarrow \quad p \leq \hbar n^{\frac{1}{3}}
\]

\( \Rightarrow \) Slow-moving particles, which are dense, can become quantum gases.
Now, slow-moving $\Rightarrow$ cold, as

$$\frac{3}{2} k_B T = \frac{p^2}{2m_0}$$

$$\Rightarrow k_B T = \frac{p^2}{m_0} \leq \frac{\hbar^2 n^{2/3}}{m_0}$$

So, the classical/quantum boundary exists along

$$T \propto n^{2/3}, \text{ or}$$

$$T = \text{constant} \times n^{2/3}.$$
Look at a slice of phase space:

\[ dx \, dy \, dz \, dp_x \, dp_y \, dp_z = \hbar^3 \]

**Quantum cells**

From Heisenberg’s Uncertainty Principle:  \( dx \cdot dp_x \geq \hbar \)