For light emitted from $r = R_s$:

$$\omega = 0 \quad \text{whence the emitted frequency } \nu = 0.$$

But: photon will emit frequency \(\nu = \frac{1}{2} \), or \(c = \nu = 0\) in vacuo.

Cases to exist (become invalid):

$$= 1 < \text{ Schwarzschild BH if } R < R_s$$

- Also: clocks at $r = R_s$, as seen from far away, move:

$$\Delta \tau \rightarrow \infty \quad \text{"infinite time dilation"}$$

- Notice: there are 2 pathological reason ("singularities") in

- \text{Schwarzschild metric}:

1) $t = 0 \rightarrow \text{physical singularity ("real")}$

where GR breaks down, but quantum gravity:

2) $t = R_s \rightarrow \text{coordinate singularity (not real)}$

- can be transformed away by choosing suitable coordinates.
- An study geometry near \( r = R_s \), introduce new \( \tilde{r} \) coordinate:

\[
\tilde{t} = t + \frac{R_s}{c} \ln \left| \frac{t}{R_s} - 1 \right|
\]

\[
d\tilde{t} = \frac{2\tilde{t}}{dt} dt + \frac{\partial \tilde{t}}{\partial t} dt
\]

\[
\frac{\partial \tilde{t}}{\partial t} = 1
\]

\[
\frac{\partial \tilde{t}}{\partial t} = \frac{R_s}{c} \frac{\frac{R_s}{c} - \frac{1}{c}}{\frac{1 - R_s}{t}} = \frac{1}{c} \frac{R_s}{t} \frac{1}{1 - \frac{R_s}{t}}
\]

- Eliminate with \( \tilde{r} \) coordinate:

\[
d\tilde{t} = d\tilde{t} - \frac{\partial \tilde{t}}{\partial r} dr
\]

\[
ds^2 = -c^2 \left( 1 - \frac{R_s}{r} \right) d\tilde{t}^2 + 2c \frac{R_s}{r} d\tilde{t} dr + \left( 1 + \frac{R_s}{r} \right) dr^2 + r^2 d\Omega^2
\]

- Note: non-singular at \( r = R_s \)

- Now consider radial light rays:

- Radial:

\[
\frac{dr}{dt} = \frac{d\tilde{t}}{dt} = 0 \quad (d\Omega^2 = 0)
\]

- Light rays: Around an unilluminated \( ds^2 < 0 \)

- Light cones:

- E.g.: in SR

\[
ds^2 = -c^2 dt^2 + dx^2 = 0
\]

\[
\frac{dx}{dt} = \pm c
\]

- Note: particles with \( \frac{dx}{dt} = c \) move with \( \frac{dx}{dt} = v < c \).
Light comes in Schwarzschild geometry:

\[
(1 + \frac{R_s}{r}) \left( \frac{dr}{dt} \right)^2 + 2C \frac{R_s}{r} \left( \frac{dr}{dt} \right) - C^2 \left( 1 - \frac{R_s}{r} \right) = 0
\]

\[
\Rightarrow \quad \text{quadratic equation for } \frac{dr}{dt}
\]

\[
\Rightarrow \quad 2 \text{ solutions:}
\]

\[
\left( \frac{dr}{dt} \right)_1 = -C
\]

\[
\left( \frac{dr}{dt} \right)_2 = -C \frac{1 - \frac{R_s}{r}}{1 + \frac{R_s}{r}}
\]

**N.H.:** \( r = R_s \) \( \Rightarrow \) no photon can escape to outside

(\"event horizon\")

for \( r < R_s \): all radiation is drawn to \( r = 0 \) singularity !

\( \Rightarrow \) no curve for all matter particles !

\( \Rightarrow \) no anti solution (\( dr = dq = dq = 0 \)) possible

**Conjecture of Cosmic Censorship (Penrose 1969):**

"The shall not have naked singularities!"

\( \Rightarrow \) every (real) singularity is surrounded by an event horizon