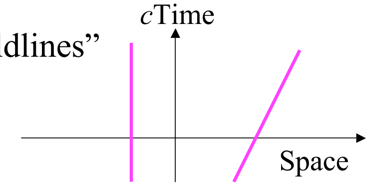
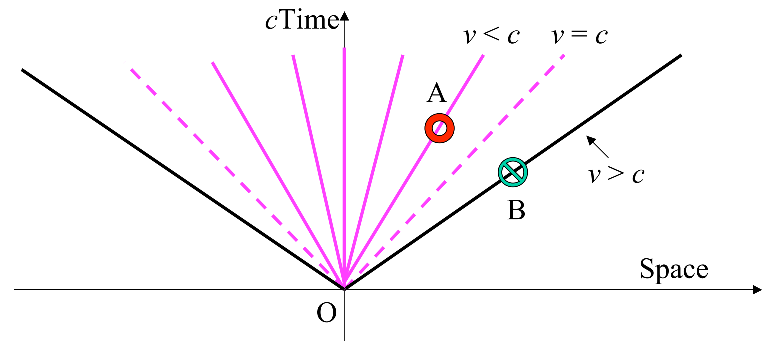
Four Dimensions

- Space (Cartesian) coordinate : (x,y,z)
  - Distance^2 = (x interval)^2+(y interval)^2+(z interval)^2
- Fourth dimension is **time**
  - More general coordinate = **spacetime** : (x,y,z,**t**)
  - (**Spacetime distance**)^2
    = c^2(**t** interval)^2 – [(x interval)^2+(y interval)^2+(z interval)^2]
- Spacetime Diagram
  - "Events" connected by "Worldlines"

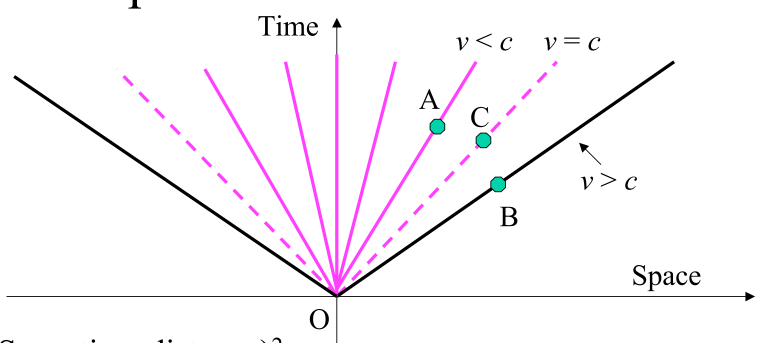


Velocity and Light cone



- The steeper the slope is, the smaller the velocity is.
- The lines with 45 degrees tilt represent the "light cone"
- Since nothing can travel faster than light…
  - O and A can communicate, but O and B cannot communicate
  - This diagram shows a "causal structure"

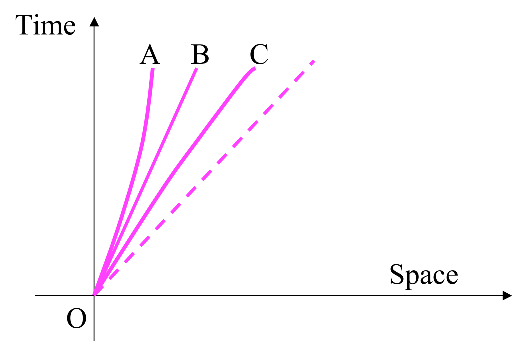
Spacetime Distances



(Spacetime distance)^2
= c^2(**time** interval)^2 – (space interval)^2

OA 1. "Timelike" worldline : (Spacetime distance)^2 > 0
OC 2. "Null" worldline : (Spacetime distance)^2 = 0
OB 3. "Spacelike" worldline : (Spacetime distance)^2 < 0

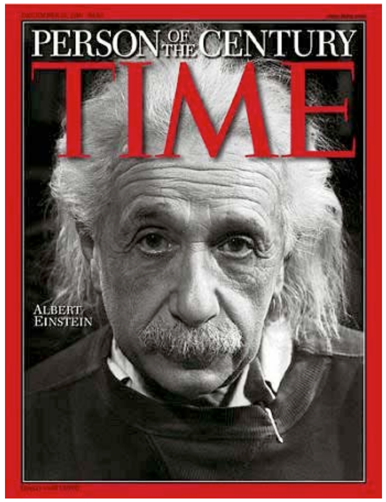
Acceleration and Deceleration



- OA : Decelerated
- OB : Constant velocity
- OC : Accelerated

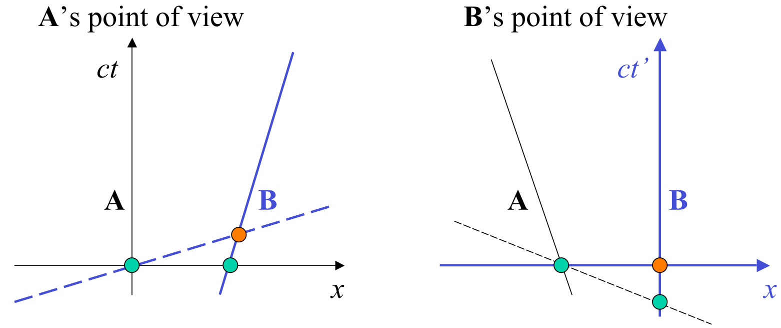
Special Relativity (1905)

- Two Invariants
  - Speed of light, *c*
  - Spacetime distance ($ds^2=c^2dt^2-dx^2$)
- Unification of space and time
  - No absolute space or time exists: **Relativity**
- Special relativity does not include gravity



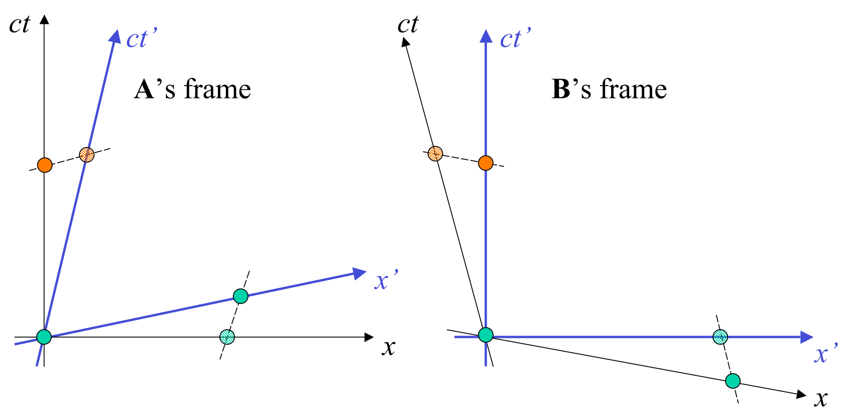
Albert Einstein (1879-

Relativity of Space and Time

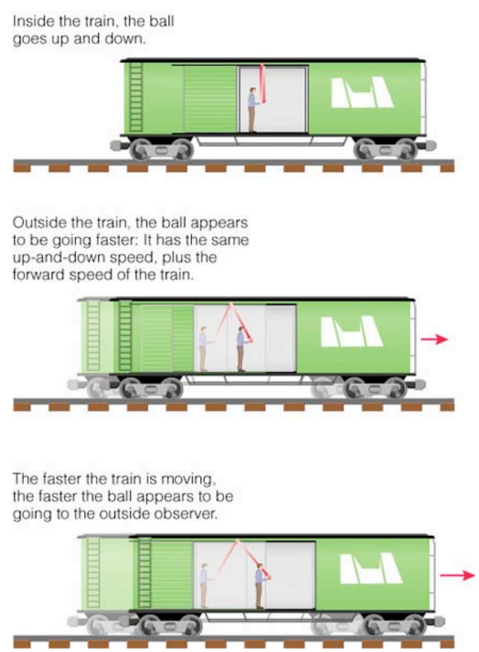


- A's space coordinate, ***x***, does not coincide with B's, ***x'***. Rather, ***x*** is a **combination of *x'* and *ct'*.**
- The same is true for time coordinate.
  - This means that simultaneous events in A's coordinate would not appear simultaneous in B's coordinate.
- But, spacetime distance remains unchanged.

Time Dilation and Length Contraction



- When A sees B moving, B's time interval appears to be longer (clock ticks more slowly; *time dilation*) ed and B's length appears to be shorter (*length contraction*). And vice versa.



Intuitive way to understand it

- From your point of view, the ball appears to move faster; however, light cannot travel faster!
- Therefore, it must take light **more time** to come back down to the laser
  - Time Dilation

Relativistic Gamma Factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

- γ is always greater than 1.
- As v approaches c, γ becomes large.
- When $v=c$, γ is infinite.

- B's unit time in A's frame equals A's unit time in A's frame **multiplied** by γ. (Hence *time dilation*)
  - Be careful! **The time actually elapsed** in B's frame gets shorter because **the unit time** gets longer.
- B's unit length in A's frame equals B's unit length in B's frame **divided** by γ. (Hence *length contraction*)

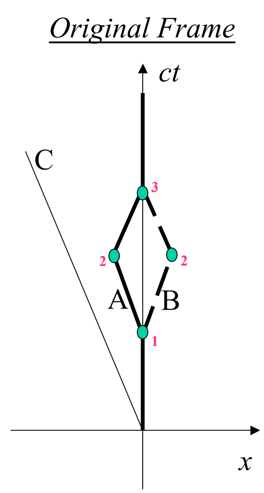
Mass Increase

- A pushes B (whose mass at rest is m) by applying a force F.
  - Acceleration is given by $a=F/m$.
  - Velocity acquired would be $v=a\ dt=Fdt/m$
- When B is moving, the clock ticks more slowly
  - B feels the force for a shorter time
  - $v'=a\ dt'=Fdt'/m=Fdt/(m\gamma)$
- Thus, the mass of B appears to be bigger by γ.
- Nothing can be accelerated to the speed of light, because the mass becomes infinite.

Twin Paradox

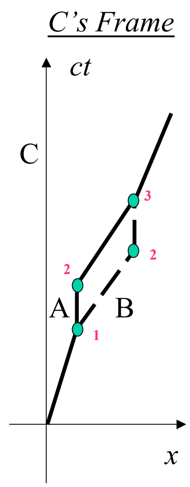
- There are twins, A and B
- B moves relative to A
  - A's point of view
    - B is moving at speed v
    - B's clock ticks more slowly by γ.
    - Therefore, B appears to be aging more slowly.
  - B's point of view
    - A is moving at speed v
    - A's clock ticks more slowly by γ.
    - Therefore, A appears to be aging more slowly.
- So, which one is older, when they meet?
  - **Twin Paradox**

Case 1



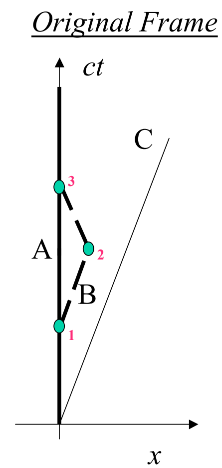
- A and B are at rest at the same place until event 1.
- Then A and B go on a trip on opposite directions.
- A and B turn around and come back at events 2.
- A and B finally meet at event 3.
- In this case, A's and B's worldlines are **symmetric**.
- A and B have traveled the **same spacetime distance**.
  - Therefore, A and B have aged the same years.

Case 1 (a different point of view)



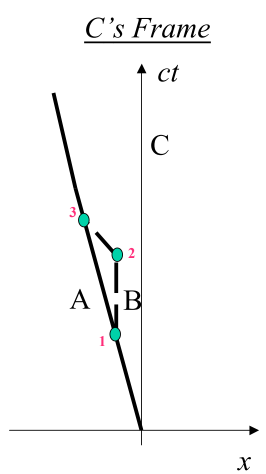
- C's point of view
  - C is moving to the left with respect to the original frame
- A and B are moving to the right together until event 1.
- Then A is at rest but B speeds up.
- A turns around earlier than B.
  - Then B is at rest but A moves to the right faster than before
- A and B finally meet at event 3.
- In this case, A's and B's worldlines are still **symmetric**.
  - A and B have traveled the same spacetime distance; thus, A and B have aged the same years

Case 2



- A remains at rest at all times.
- B leaves home at event 1, turns around at event 2, and finally meets A at event 3.
- In this case, A's and B's worldlines are **not symmetric**!
- What happens?
  - The answer is that A has aged more than B.
- Why?
  - B's spacetime distance is **shorter** than A's
  - Remember, $ds^2=c^2dt^2-dx^2$

Case 2 (a different point of view)



- In C's frame, A is moving to the left at all times. B is initially moving to the left, together with A.
- B becomes at rest at event 1, and then moves to the left faster than before.
- A and B meet at event 3.
- In this case, A's and B's worldlines are still **not symmetric**!
  - A has aged more than B.
  - B's spacetime distance is still shorter.

So, what was it?

- Motion of A and B remains completely relative **only when** both are moving at constant velocity.
  - Motion has to be inertial for two frames to be completely equivalent.
- However, for two people to know their initial ages and then meet later again, the motion cannot stay inertial → two frames are no longer equivalent.