

## How long would the Sun shine?

- The Sun needs fuel to shine.
  - The Sun shines by consuming the fuel -- it generates energy from the fuel.
- The lifetime of the Sun is determined by
  - How fast the Sun consumes the fuel, and
  - How much fuel the Sun contains.
- How fast does the Sun consume the fuel?
  - Energy radiated per second is called the “*luminosity*”, which is in units of watts.
  - The solar luminosity is about  $3.8 \times 10^{26}$  Watts.
    - Watts = Joules per second
    - Compare it with a light bulb!
- What is the fuel??

## Fuel = Gravitational Energy?

- The mass of the Sun is  $M = 2 \times 10^{30}$  kg.
  - The amount of the fuel should be related to the amount of mass.
- Gravity can generate energy.
  - A falling body acquires velocity from gravity.
  - Gravitational energy =  $(3/5)GM^2/R$
  - The radius of the Sun:  $R = 700$  million m
  - Gravitational energy of the Sun =  $2.3 \times 10^{41}$  Joules.
- How long could the Sun shine on gravitational energy?
  - Lifetime = (Amount of Fuel)/(How Fast the Fuel is Consumed)
  - Lifetime =  $(2.3 \times 10^{41} \text{ Joules}) / (3.8 \times 10^{26} \text{ Joules per second}) = 0.6 \times 10^{15}$  seconds.
  - Therefore, the Sun lasts for **20 million years** (Helmholtz in 1854; Kelvin in 1887), if gravity is the fuel.

## Fuel = Nuclear Energy

- Einstein’s Energy Formula:  $E=Mc^2$ 
  - The mass itself can be the source of energy.
- If the Sun could convert *all* of its mass into energy by  $E=Mc^2$ ...
  - Mass energy =  $1.8 \times 10^{47}$  Joules.
    - Compare it with the gravitational energy,  $2.3 \times 10^{41}$  Joules, which can keep the Sun shining for 20 million years.
  - The Sun would shine for about **15 trillion years** on mass energy!
- How can the mass be converted into energy?
  - Nuclear reaction
  - Nuclear reaction in the Sun can convert ~0.07% of the mass into energy.
- Therefore, the Sun shines for...
  - 15 trillion years x 0.0007 ~ **10 billion years**

## Burning Hydrogen: p-p chain

- $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$
- $^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma$
- $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$ 
  - $^1\text{H}$ : proton,  $^2\text{H}$ : deuteron
  - $^3\text{He}$ : Helium-3,  $^4\text{He}$ : Helium-4
  - $e^+$ : positron,  $\nu_e$ : neutrino,  $\gamma$ : gamma ray
- In total, four protons are fused into one helium-4 and produce energy:
  - $^1\text{H} + ^1\text{H} + ^1\text{H} + ^1\text{H} \rightarrow ^4\text{He} + \text{“binding energy”}$
  - One proton weighs  $1.6726 \times 10^{-27}$  kg
  - One helium-4 weighs  $6.643 \times 10^{-27}$  kg
  - Four protons minus one helium =  $4.7 \times 10^{-29}$  kg  $\rightarrow$  energy
    - This is 0.7% of the original mass of four protons.

## How Hot is the Sun?

- Hydrogen gas is pulled inward by gravity.
- **Hydrogen Gas is pushed outward by pressure.**
  - Nuclear energy heats up gas -> high pressure
  - $\text{Pressure} = k_B \times (\text{number density of hydrogen}) \times (\text{temperature})$
  - Pressure is the highest at the center and decreases at larger distances.
- “Hydrostatic equilibrium” = “Pressure force balances gravitational force”
  - Gravity =  $GM^2/R^2$
  - Pressure =  $k_B nT$
  - Calculations show that temperature should be **~20 million K**
  - High temperature is necessary to balance enormous gravity.

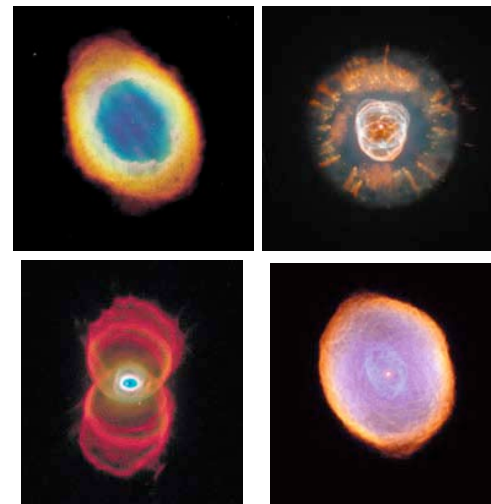
## Core is Hotter, Surface is Cooler

- **Core** (0-0.25  $R_{\text{solar}}$ ): ~15 million K
  - Energy is produced by hydrogen burning
- **Radiative zone** (0.25-0.70  $R_{\text{solar}}$ ): 2 to 8 million K
  - Energy is carried by radiation (photons) up to near the surface of the Sun
  - Photons frequently scattered by electrons (random walk)
- **Convective zone** (0.70-1  $R_{\text{solar}}$ ): < 2 million K
  - Photons are absorbed by atoms near the surface; unable to carry energy
  - Energy is then carried by “convection” (e.g., boiled water)
- **Surface**: 5,800 K
  - It takes about a million years from the core to the surface.

## Fate of the Sun

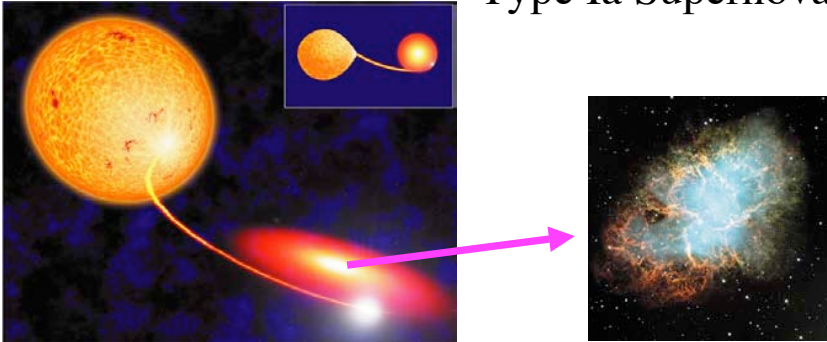
- The Sun will eventually run out of fuel...
  - The Sun is about 5 billion years old.
  - Hydrogen burning can keep the Sun shining for about 10 billion years.
- What would happen after that?
  - The Sun will find another source of fuel: **Helium**
- When hydrogen nuclei are exhausted, the Sun begins to contract, getting hotter.
- When temperature in the core increases to ~100 million K, helium begins to burn, generating nuclear energy again. The surface of the star expands.
  - $4\text{He} + 4\text{He} + 4\text{He} + 4\text{He} \rightarrow {}^{12}\text{C} + (\text{binding energy})$
- When helium nuclei are exhausted, the star wants to burn carbon, but a low-mass star can't burn it...
  - ${}^{12}\text{C} + 4\text{He} \rightarrow {}^{16}\text{O} + (\text{binding energy})$ ; this reaction requires ~600 million K!  
So, a low-mass star becomes a carbon star.

## White Dwarfs and Planetary Nebulae



- Expanding gas gets **ionized** by radiation from the central core
- The carbon core became a “White Dwarf”
  - Supported by “*degeneracy pressure*”
- The expanding gas
  - Planetary nebula

## Type Ia Supernovae



- If the W.D. has a companion star, mass from the companion accretes on the W.D., increasing mass.
- At some point ( $M > 1.4M_{\text{solar}}$ ), carbon begins to burn!
  - The “carbon bomb” disrupts the W.D. completely
  - This type of supernova is called the “Type Ia”, and plays a very important role in cosmology

## Importance of Type Ia in Cosmology



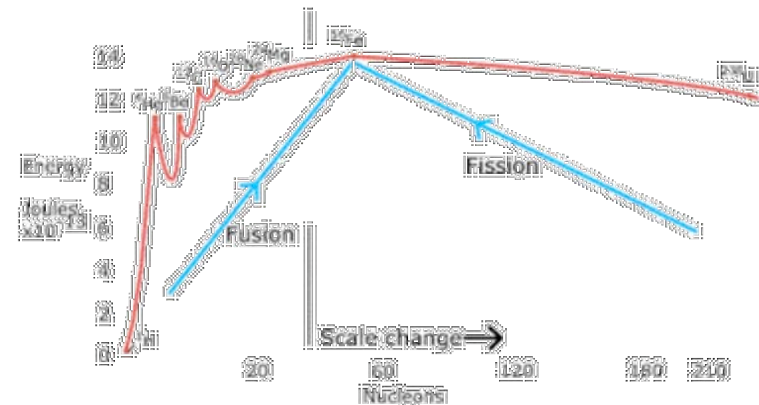
Type Ia supernova

- We can estimate distances to Type Ia supernovae fairly accurately.
  - Because they all explode at the same mass ( $\sim 1.44M_{\text{solar}}$ ), their luminosity is roughly the same for all Type Ia.
    - We measure their brightness.
    - We know their luminosity.
    - Luminosity-brightness relation gives distances.
- **Distance-redshift relation** is one of the fundamental cosmological probes.
- How do we find Type Ia?
  - No hydrogen line should be seen for Type Ia

## What about Type II?

- $M > 2M_{\text{solar}}$ 
  - More mass, more gravity  $\rightarrow$  More pressure, higher temperature
  - Hydrogen and helium are much more rapidly consumed ( $\sim$ a few 100,000 years or less vs billions of years)
  - Then, **carbon does fuse!!** (E.g.,  $^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O}$ )
  - Heavier elements are also burned one after another.
    - E.g.,  $^{16}\text{O} + ^4\text{He} \rightarrow ^{20}\text{Ne}$ ,  $^{20}\text{Ne} + ^4\text{He} \rightarrow ^{24}\text{Mg}$
    - E.g.,  $^{12}\text{C} + ^{16}\text{O} \rightarrow ^{28}\text{Si}$ ,  $^{28}\text{Si} + ^4\text{He} \rightarrow ^{32}\text{S}$
  - **Iron ( $^{56}\text{Fe}$ ) is the terminal:** no more energy gain by fusion.
  - The core keeps shrinking... Gravitational force is not balanced by thermal pressure... Where would the gravitational energy go...
    - **Type II Supernova!!** (Hydrogen lines should be seen.)
- Intermediate mass stars ( $8M_{\text{solar}} > M > 2M_{\text{solar}}$ )
  - The core becomes a *neutron star* ( $\sim 10\text{km}$  across; rapidly rotating)
- Very high mass stars ( $M > 8M_{\text{solar}}$ )
  - The core collapses into a *black hole*

## Binding Energy Diagram



- Fusion generates energy until it reaches the “iron peak”.
- Fission generates energy by destroying nuclei heavier than iron.

## *Life* and Low- and High-mass Stars

- Low-mass stars are necessary for life because...
  - Planets can form around low mass stars
  - Stars live long enough (~billions of years) for complex form of life to emerge
- High-mass stars are necessary for life because...
  - Low-mass stars alone cannot produce important heavy elements such as carbon, oxygen, nitrogen, etc.
  - High-mass stars can create heavy elements by fusion, and eject the created elements into space by Type II supernova explosion.
- Life is not possible without both!