Brightness, Magnitudes, and Luminosity: A Tutorial
(Prof. Harriet Dinerstein, Ast 307, 4/21/14)

1. **Motivation:** Many students find the magnitude scale confusing. Why, then, do we teach it? *Because it is the common currency:* the brightnesses of astronomical objects are almost always stated in magnitudes in catalogs and databases used by both amateur and professional astronomers. Therefore it is necessary to know how to translate between apparent brightness (also called "flux," this is the power received per unit surface area of your telescope) and magnitudes. This involves basically a single formula, although it takes on a variety of forms under different circumstances.

2. **Formulas:** The magnitude scale expresses a given ***ratio*** of brightness (say, between two stars) as a ***difference*** in magnitudes. This involves taking logarithms, with a base of 10 but an additional scaling factor chosen to make the scale fit a previously-existing star catalog; the modern magnitude scale was "reverse-engineered." The defining equation is:

$$\frac{b_1}{b_2} = \frac{f_1}{f_2} = 10^{0.4(m_2-m_1)} = 10^{-0.4(m_1-m_2)} \qquad (1)$$

where m_1 and m_2 are the apparent magnitudes and the b's and f's are power per unit area, for example, W m^{-2}. Notice the sign change in the exponents between the third and fourth expressions, which is introduced when we flip the order of m_1 and m_2.

At first glance, the formula given in your textbook (Math Insight 15.3, p. 491) appears to be different from the above expression but in fact it is equivalent, as shown below:

$$\frac{b_1}{b_2} = (100^{1/5})^{m_2-m_1} = (10^{2(1/5)})^{m_2-m_1} = (10^{2/5})^{m_2-m_1} = 10^{0.4(m_2-m_1)} \qquad (2)$$

This tells us how to convert from a magnitude difference to a ratio of brightnesses. To go in the other direction, we take the logarithms (base 10) of both sides, then divide by the constant, 0.4. Swapping the right and left-hand sides of the equation:

$$m_2 - m_1 = \frac{1}{0.4} \log_{10}\left(\frac{b_1}{b_2}\right) = 2.5 \log_{10}\left(\frac{b_1}{b_2}\right) = -2.5 \log_{10}\left(\frac{b_2}{b_1}\right) \qquad (3)$$

Now we have a prescription for converting brightness ratios to magnitudes.

You may have noticed that these calculations do not yield or use physical fluxes, in units of W m^{-2} or erg cm^{-2} s^{-1}. To do so, one needs to know or set the "zero point," the brightness corresponding to a magnitude of 0. We haven't given you a value for this quantity because it depends on wavelength λ as well as how wide an interval of wavelength Δλ is considered. Some time ago, the bright star Vega was chosen as the zero point: V (Vega) = 0.00. However, later more accurate measurements indicated that a small correction was required, so today we use V (Vega) = +0.03. The apparent magnitude of Vega is close to, but not exactly, zero for most other photometric bands (see SIMBAD if you're curious).

3. Exercise 1: From magnitudes to brightness ratio. Consider Sirius, with apparent magnitude in $V = m_V = -1.46$, and Spica, with $V = +0.91$. Calculate the brightness ratio b_{Sirius}/b_{Spica}.

$$\frac{b_{Sirus}}{b_{Spica}} = 10^{0.4(V_{Spica}-V_{Sirius})} = 10^{0.4[0.91-(-1.46)]} = 10^{0.4[2.37)]} = 8.87$$

4. Exercise 2: From apparent magnitude to absolute magnitude. If we want to compare the luminosities of two stars, their intrinsic light energy output, we need to mentally shift them to the same distances, so that inverse-square law (brightness declines as $1/d^2$) does not affect the ratio. Again we need a zero point: in this case the reference distance is 10 parsecs (pc). We define the ***absolute magnitude*** as the apparent magnitude the star would have if it were at a distance of 10 pc. Absolute magnitude in V is written M_V. Using the inverse square law, one can derive the following quantity, which is called the ***distance modulus***. It is really nothing more than the inverse-square law converted to magnitudes. (You will find the derivation in the slides from Lecture 17 on March 25.)

$$m - M = V - M_V = 5 \log_{10} d[pc] - 5 = \text{"distance modulus"} \qquad (4)$$

Solving this expression for M_V, we have

$$M_V = V - 5 \log_{10} d[pc] + 5 \qquad (5)$$

and substituting the data for our two stars,

$$M_V(Sirius) = -1.46 - 5 \log_{10} 2.64[pc] + 5 = -1.46 - 5\,(0.42) + 5 = +1.43\,;\, M_V(Spica) = -3.61$$

5. Exercise 3: From absolute magnitudes to luminosity ratio. There is an expression parallel to equation (1) above, that relates absolute magnitudes to luminosities. This is given in the box on p. 491 as well. For two stars at the same distance, the ratio of luminosities must be the same as the ratio of apparent brightnesses, because they are at the same distance:

$$\frac{L_1}{L_2} = 10^{0.4(M_2-M_1)} = 10^{-0.4(M_1-M_2)} \qquad (6)$$

Since Spica is more luminous than Sirius, let 1 = Spica and 2 = Sirius:

$$\frac{L_{Spica}}{L_{Sirius}} = 10^{0.4(M_{Sirius}-M_{Spica})} = 10^{0.4[1.43-(-3.61)]} = 10^{0.4(5.04)} = 104$$

6. Exercise 4: Luminosities in units of solar luminosity ($L_\odot$). It is convenient to express the luminosities of stars relative to the luminosity of the Sun; furthermore, since we know the Sun's luminosity in physical units (3.8×10^{26} W), we can easily convert to them. We do need to know the absolute magnitude of the Sun: $M_V = +4.8$. Repeating the calculation in part (5), you should get $L_{Sirius} = 22.3\ L_\odot$ and $L_{Sirius} = 2{,}300\ L_\odot$. Do this yourself and check the answers against these values.

7. Exercise 5: Magnitudes of combined images. Sirius and Spica are far apart on the sky, so we will leave them and choose a different example. Suppose you are observing a pair of stars that are so close together, they look like one star. At higher magnification it is possible to see that there are actually two stars, with apparent magnitudes V_1 = +8.0 and V_2 = +10.0. You are asked to calculate the apparent magnitude of the combined image.

   One thing that you ***cannot*** do is to add the magnitudes. Remember that they are actually logarithms. When you add logarithms, it is the equivalent of multiplying their antilogs, and it clearly makes no sense to multiply brightnesses! One must convert from magnitudes to brightnesses before performing the addition. As pointed out earlier, you don't have enough information to calculate an actual physical brightness (flux); instead, you must work with brightness *ratios*. We apply equation (1) again:

$$\frac{b_1}{b_2} = 10^{0.4(V_2 - V_1)} = 10^{0.4(10-8)]} = 10^{0.8} = 6.31$$

   But now we consider the ratio of the combined light to that of one of the stars,

$$\frac{b_{1+2}}{b_2} = \frac{b_1}{b_2} + \frac{b_2}{b_2} = 6.31 + 1 = 7.31$$

   and converting back to magnitudes using equation (3):

$$V_{1+2} - V_2 = -2.5 \log_{10}\left(\frac{b_{1+2}}{b_1}\right) = -2.5 \times 0.86 = -2.16$$

   So, we conclude that $V_{1+2} = V_2 - 2.16 = 10.0 - 2.16 = 7.84.$ The second star is sufficiently faint that it doesn't affect the value by a large amount, so this number is close to the value for the brighter of the two stars.

8. Additional Examples. Although these may have gone by quickly during class, there are two similar calculations on the slides entitled "Working with the Magnitude Scale," from Lecture 17 (March 25). One of these slides is included twice and some of the slides look out of order, but try working the examples shown there for extra practice. Alternatively or in addition, make up your own examples, have a friend do them too, and compare your answers.