Kepler, Newton, and laws of motion

The only history in this course:

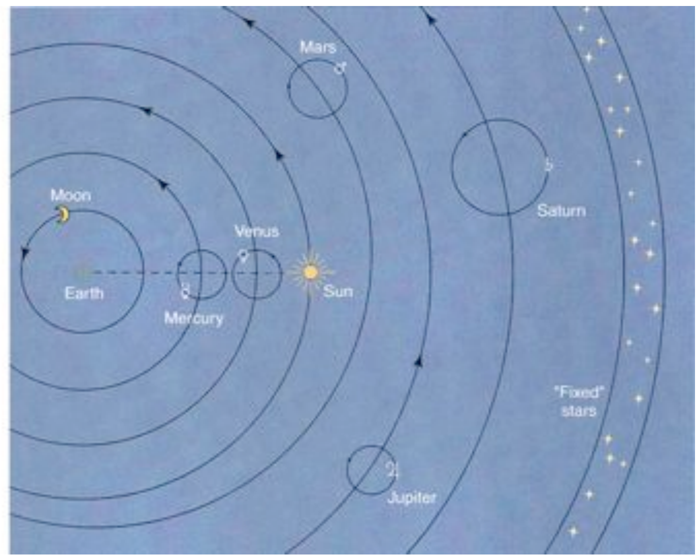
Geocentric vs. heliocentric model (sec. 2.2-2.4)

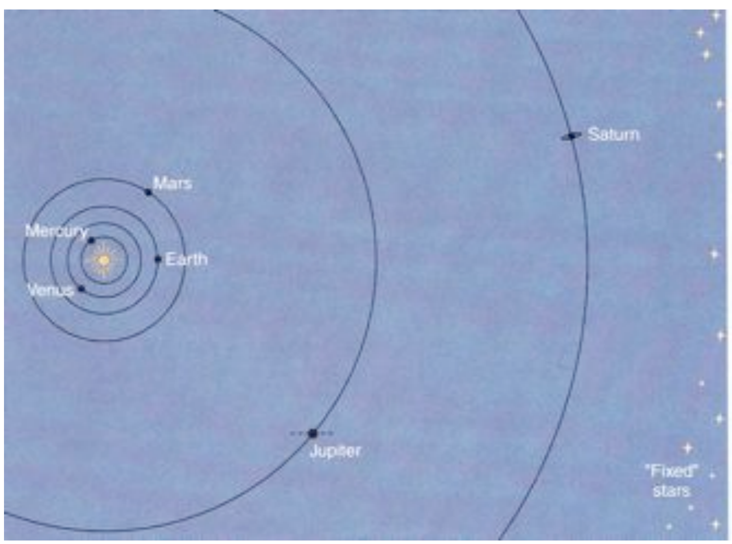
The important historical progression is the following:
➞ Ptolemy (~140 AD) … Copernicus (~1500 AD), Galileo (~1600), Tycho Brahe, **Kepler** (sec. 2.5), **Newton** (sec. 2.6).

It is important to recognize the change in world view brought about by this transition:
Geocentric model (Ptolemy, epicycles, planets and Sun orbit the Earth)

→ *Heliocentric model* (Cop

The solar system as conceived by Ptolemaic astronomers between about 100 and 1500. Ptolemaic astronomers made rough predictions of planetary motions, using a theory in which the Sun and other planets moved in circular orbits around a stationary, central Earth. Superimposed on the larger orbits were smaller circular motions called epicycles, introduced to try to make the theory more accurate.





The solar system as it might have been conceived around 1700, at the end of the Copernican revolution. The diagram shows the orbits of the 10 known planets to true scale. The view is correct except that the outermost planets (Uranus, Neptune, and Pluto) and the asteroids had not yet been discovered. Compare with Figure 3–2 to see the change from the Ptolemaic view.

Kepler's Laws

Empirical, based on observations; NOT a theory (in the sense of Newton's laws). So they are "laws" in the sense of *formulas that express some regularity or correlation*, but they don't *explain* the observed phenomena in terms of something more basic (e.g. laws of motion, gravity--that waited for Newton)

Kepler's 1st law:

➞ **1. *Orbits of planets are ellipses* (not circles), with Sun at one focus.**

Must get used to terms *period* (time for one orbit), *semimajor axis* ("size" of orbit), *eccentricity* (how "elongated" the orbit is), *perihelion* (position of smallest distance to Sun), *aphelion* (position of greatest distance to Sun)

Examples: comets, planets:

Why do you think these have such different eccentricities ?

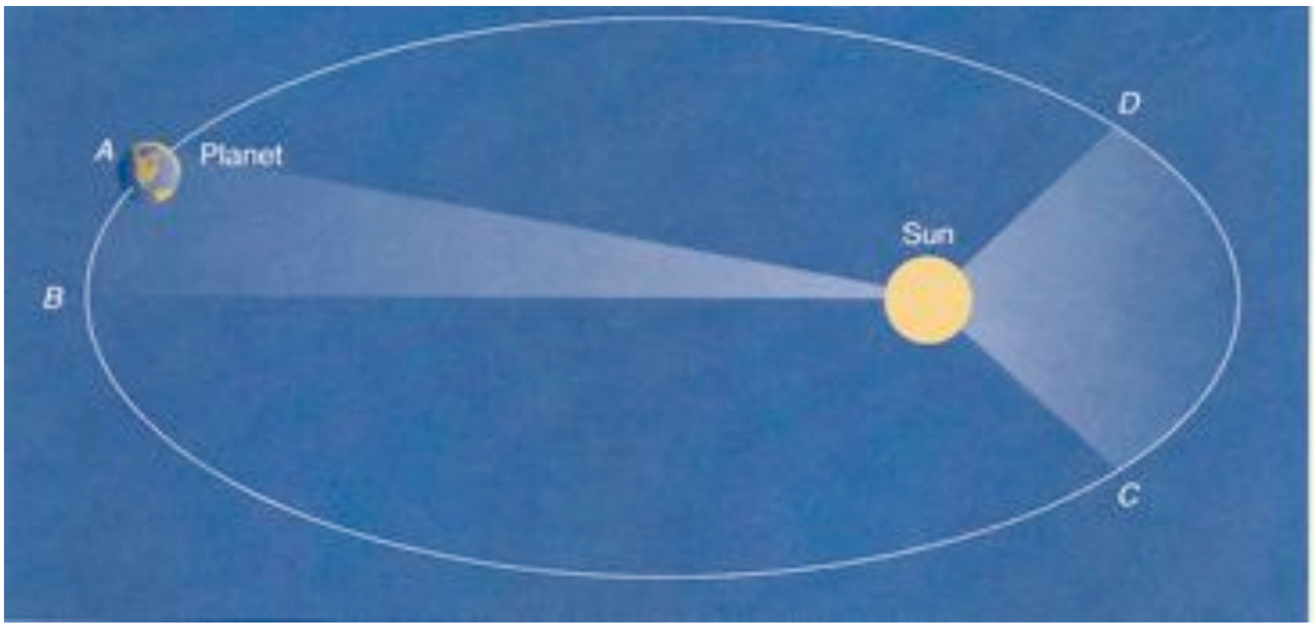
(Don't expect to be able to answer this, just find whether you understand the question.)

➞ *Escaping from the assumption of perfect circles for orbits was a major leap, that even Copernicus was unwilling to take.*

Kepler’s 2nd law:

→ 2. Equal areas swept out in equal times
more simple: planet moves faster when closer to the sun.

Good example: comets (very eccentric orbits, explained in class).
Once you know the slightest thing about the force of gravity, this law is obvious.



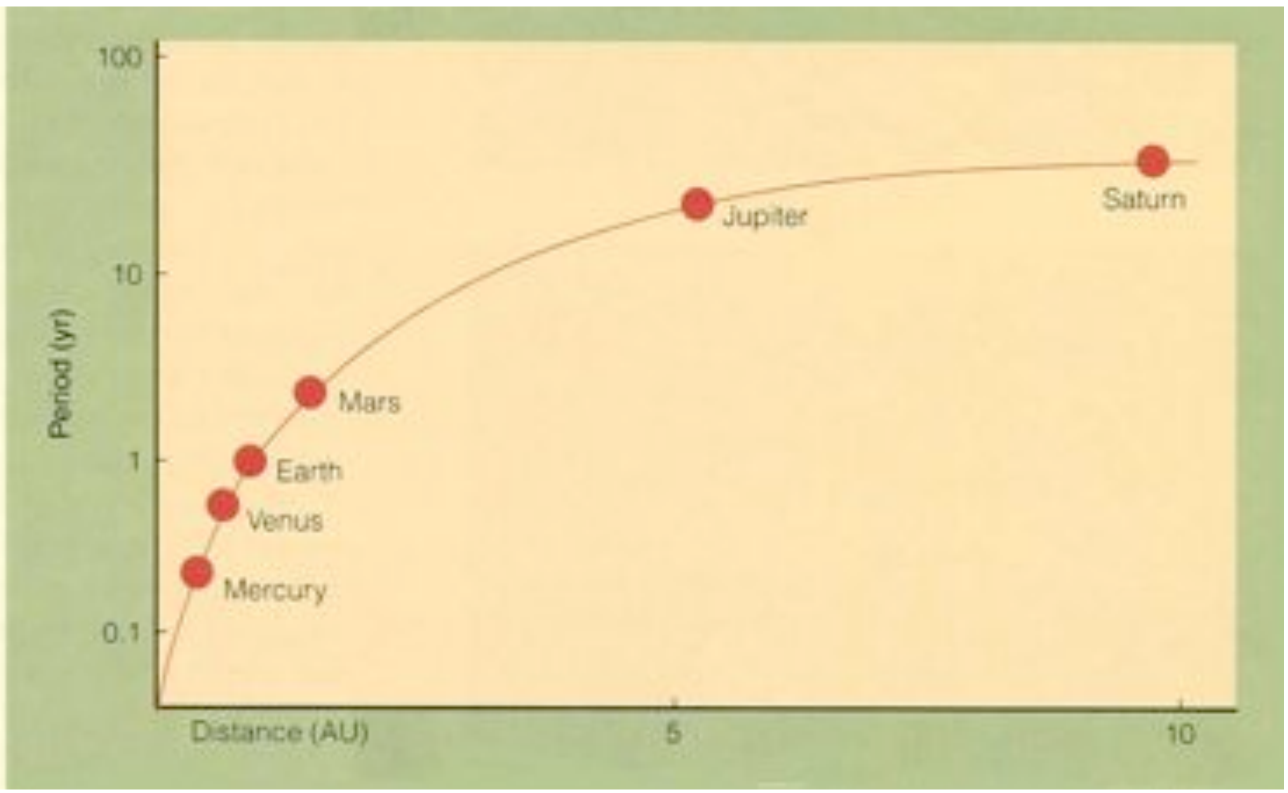
Kepler’s second law of planetary motion. The orbit sweeps out an ellipse where an imaginary line connecting the planet to the Sun sweeps out equal areas in equal time intervals. The time taken to move from *A* to *B* equals the time taken to move from *C* to *D*. In other words, planets travel faster when they are close to the Sun and more slowly when they are far from the Sun. The true planet orbits are much closer to circles, and the speed only changes by a small percentage along the orbit.

Kepler's 3rd law

→ *Square of the period "P" is proportional to the cube of the semimajor axis "a"*

→ $P^2 = a^3$

IF P is expressed in Earth years and "a" is in units of A.U. (astronomical unit; average distance from Earth to Sun). ***A graph of the periods vs. the distances from the sun (a) is shown below.*** (Absolute size of A.U. unit determined from radar observations of Venus and Mercury, and other methods--see textbook.)



Kepler's third law of planetary motion. The period of a planet's orbit increases with increasing distance from the Sun. Planets discovered after the invention of the telescope obey this law, too. Notice that according to Kepler's law, planets can have properties that place them *anywhere* on this curve. The planets of the solar system have a particular pattern in their spacing from the Sun.

→ *Kepler's 3rd law, as modified by Newton (coming up), will be a cornerstone of much of this course, because it allows us to estimate masses of astronomical objects (e.g. masses of stars, galaxies, the existence of black holes and the mysterious "dark matter").*

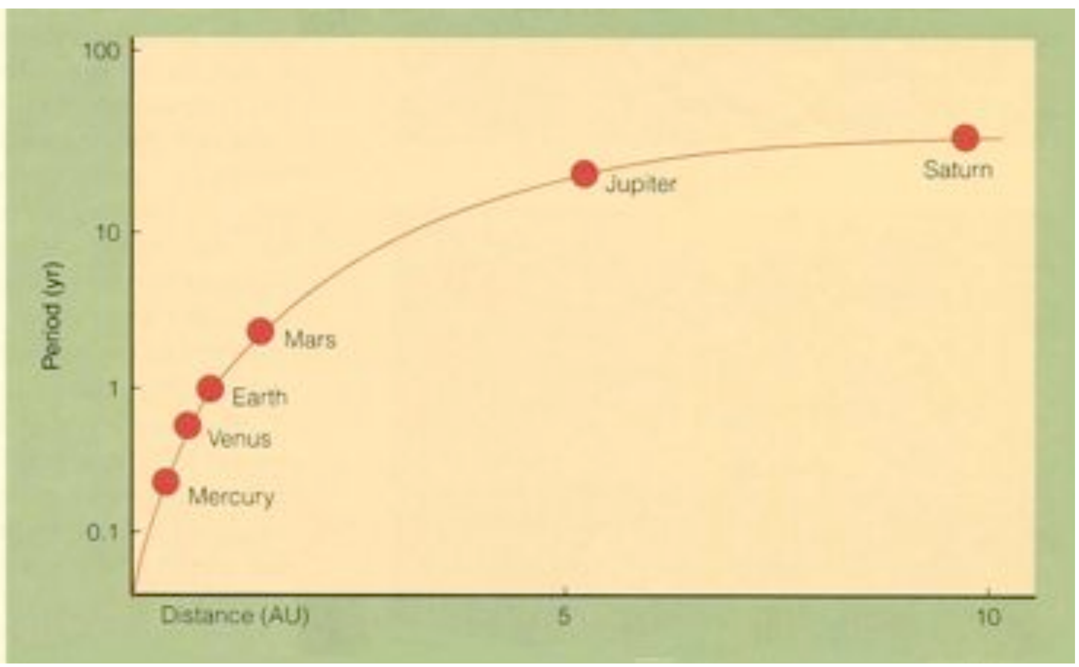
Example of use of Kepler's 3rd law:

The planet Saturn has a period of about 30 years; how far is it from the Sun?

Answer: Using $P^2 = a^3$, with P = 30 yr, $a = (30)^{2/3} = ((30)^2)^{1/3} = (900)^{1/3} \sim 10$AU.

Another example: An object is observed orbiting the Sun in an orbit of semimajor axis = 4 AU. How long is its year (period)?

[Note: *This is as tough as the math will get in this class.*]



Kepler's third law of planetary motion. The period of a planet's orbit increases with increasing distance from the Sun. Planets discovered after the invention of the telescope obey this law, too. Notice that according to Kepler's law, planets can have properties that place them *anywhere* on this curve. The planets of the solar system have a particular pattern in their spacing from the Sun.

Newton was able to propose more general laws that describe the motion of an object under the influence of any force, but in particular the force of gravity. Read about them by next class, but it may help if you keep in mind why you are reading about this:

Newton's laws will give us a way, basically our only way, to get the masses of objects, first stars that orbit each other (binary stars), then a technique to detect black holes, since 1995 the masses of extrasolar planets, and the evidence that there is some invisible mass called "dark matter."

Then try to answer this apparently boring question:

Gravity is what makes objects orbit around other objects, and gravity is a reflection of an object's mass.
So why doesn't the mass of the objects appear in Kepler's 3rd law?

Newton’s laws of motion and gravity

Newton's laws of motion

1. *Every body continues in a state of rest or uniform motion (constant velocity) in a straight line unless acted on by a force.*
(A deeper statement of this law is that momentum (mass x velocity) is a conserved quantity in our world, for unknown reasons.)
This tendency to keep moving or keep still is called "*inertia*."

2. *Acceleration* (change in speed or direction) of object is proportional to: applied force F divided by the mass of the object m

i.e. ⌘ $a = F/m$ or (more usual) $F = ma$

This law allows you to calculate the motion of an object, if you know the force acting on it. This is how we calculate the motions of objects in physics and astronomy.

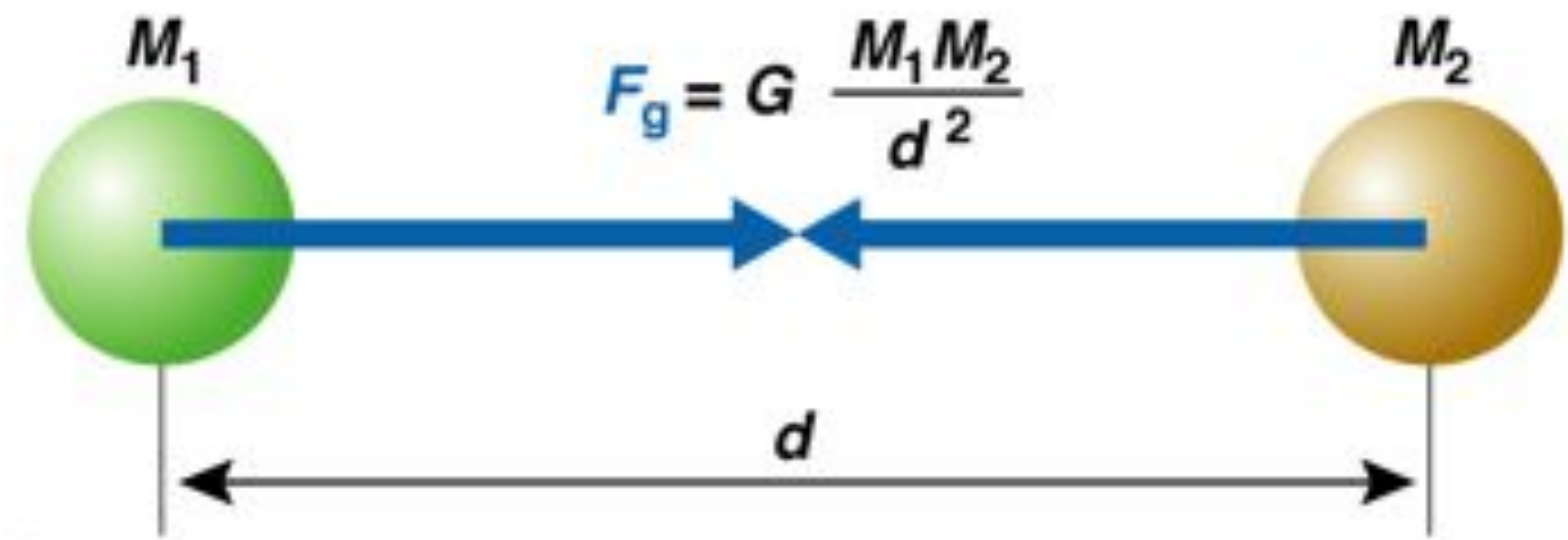
⌘ You can see that if you know the mass of something, and the force that is acting on it, you can calculate its *rate of change of velocity*, **so you can find its velocity, and hence position, as a function of time.**

3. *To every action, there is an equal and opposite reaction*, i.e. forces are mutual. A more useful equivalent statement is that *interacting objects exchange momentum through equal and opposite forces.*

What determines the strength of gravity?

The Universal Law of Gravitation (Newton's law of gravity):

1. Every mass attracts every other mass.
2. Attraction is directly proportional to the product of their masses.
3. Attraction is inversely proportional to the square of the distance between their centers.



Newton's Law of Gravity (cont'd):

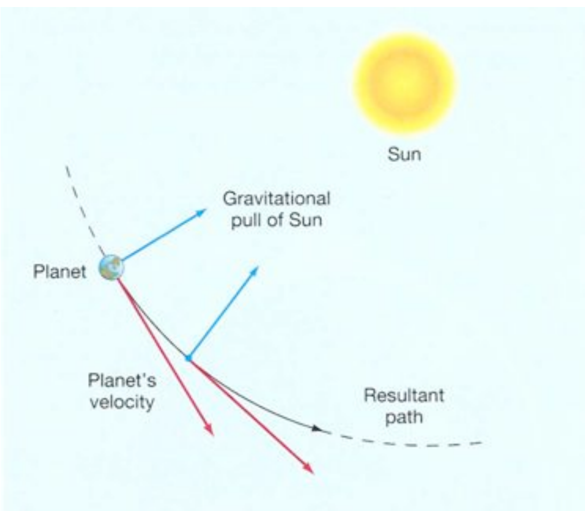
Every object attracts every other object with a force

F (gravity) = (mass 1) x (mass 2) / R^2 (distance squared)

Notice this is an "inverse square law" (right illus.).

Orbits of planets (and everything else) are a balance between the moving object's tendency to move in a straight line at constant speed (Newton's 1st law) and the gravitational pull of the other object (see below).

Now we'll see how all this can be combined to calculate the motion of any object moving under any force (gravity or otherwise--like a magnetic force, or friction, or anything.



▲ **FIGURE 2.24 Solar Gravity** The Sun's inward pull of gravity on a planet competes with the planet's tendency to continue moving in a straight line. These two effects combine, causing the planet to move smoothly along an intermediate path, which continually "falls around" the Sun. This unending "tug-of-war" between the Sun's gravity and the planet's inertia results in a stable orbit.

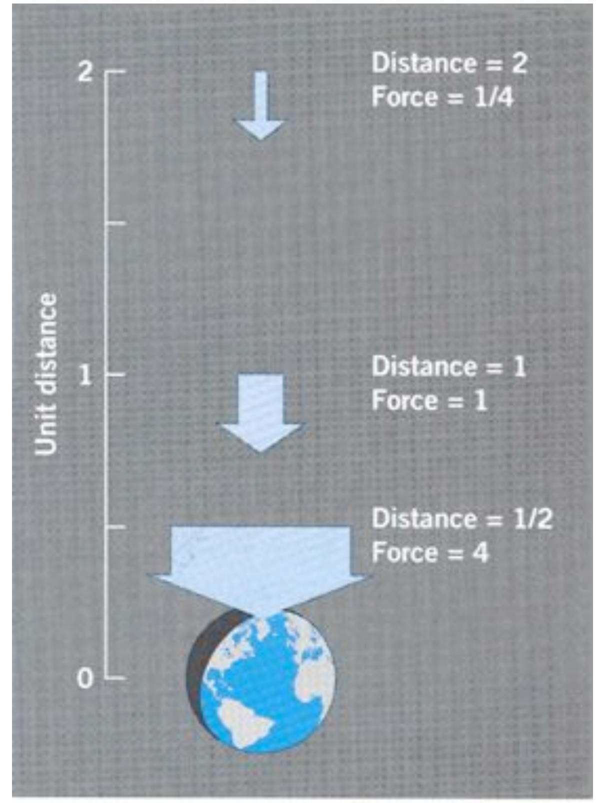


FIGURE 4.17 Moving the same mass at three different relative distances from the earth. For each distance, the thickness of the arrow indicates the relative amount of the gravitational force between the mass and the earth.

Using Newton's laws, continued…

Applying this procedure (Newton's 2nd law with the law of gravity) you (or at least someone) can derive Kepler's laws, **if** you know the form of the gravitational force. For gravity we have Newton's formula

→ $F_{grav} = G\, m_1 m_2 / d$

where G is Newton's gravitational constant (you don't have to know it's value), m_1 and m_2 are the two masses, and d is their separation (distance from each other).

From this "it can be shown" that all closed orbits are ellipses, that the orbital motion is faster when the two objects are closer to each other (Kepler's 2nd law), and Kepler's 3rd law, the most important result. Kepler's third law now contains a new term:

⇨ $P^2 = a^3 / (m_1 + m_2)$ → Newton's form of Kepler's 3rd law.

(Masses expressed in units of solar masses; period in years, a in AU, as before).

This is basically what is used (in various forms) to get masses of ALL cosmic objects! Another way to word it: if you know how fast two objects are orbiting each other, and their separation (notice you need the distance to get this), you can solve for the sum of their masses. We will use this over and over--it is the only way we have to get masses directly.

The most important thing about Newton's laws is that they are *general:* you can calculate the motion of *any* object (or any number of objects) acting under *any force* can be calculated, in principle*, if the force can be specified* (e.g. gravitational force as a function of mass and distance; but it could be frictional force, magnetic force, electrostatic repulsion, ….)

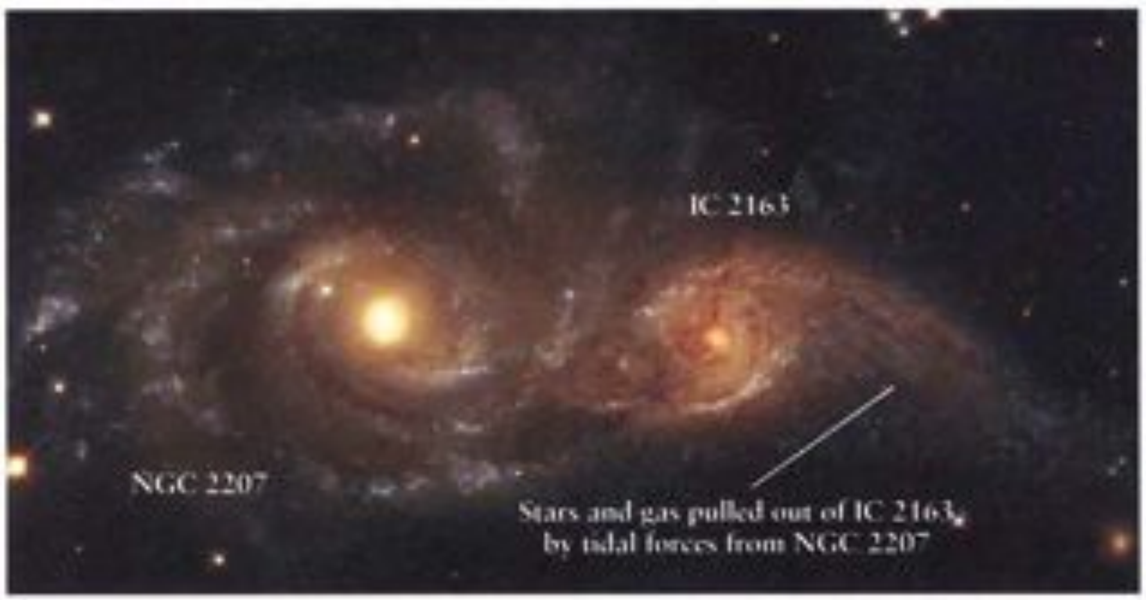
We calculate the evolution of clusters of stars, of millions of galaxies in an expanding universe, of a hot gas in a magnetic field, and almost everything else, although in general this is so difficult that you can only get computer solutions. Example shown on next page.

Examples:

- Earth's orbital period (1 year) and average distance (1 AU) tell us the Sun's mass (think: why don't you need to know the Earth's mass for this purpose?
- Orbital period and distance of a satellite from Earth tell us Earth's mass.
- Orbital period and distance of a moon of Jupiter tell us Jupiter's mass.
- This is how "black holes" were discovered to actually exist (later in course), and how the masses of planets orbiting other stars are determined.
- Motion of stars in galaxies reveals the existence of invisible mass, or "dark matter," whose nature remains unknown.

A complex example of the use of Newton's laws: Illustration below shows effect of gravitational forces between two galaxies that are in the early stages of collisional *merging*. Solving Newton's laws for *millions* of stars and for the gas within these galaxies, we can actually make models for such phenomena that show how tidal forces are distorting these galaxies. This example shows you that some orbits can decay, leading to merging of objects.

We will see this again when we discuss the cannibalism of planets by their parent stars.



Tidal Forces on a Galaxy For millions of years the galaxies NGC 2207 and IC 2163 have been moving ponderously past each other. The larger galaxy's tremendous tidal forces have drawn a streamer of material a hundred thousand light-years long out of IC 2163. If you lived on a planet orbiting a star within this streamer, you would have a magnificent view of both galaxies. NGC 2207 and IC 2163 are respectively 143,000 light-years and 101,000 light-years in diameter. Both galaxies are 114 million light-years away in the constellation Canis Major. (NASA and the Hubble Heritage Team, AURA/STScI)

End of material on orbits under gravity, Kepler's laws, Newton's laws and the way Newton's form of Kepler's third law can give us the *masses* of astronomical objects. In fact it is just about the *only* way.

How else can you learn about astronomical objects?
All you get from them is their *light*, so there are two chapters entirely concerned with how we can analyze light.

These are chapters 3 and 4.
We will only cover chapter 3 for the first exam.