Radiative Processes & Radiation Transport

Homework #7
Due November 24, 2009

This home work carres twice the weight of the other home works.

In this homework you will apply your understanding of synchrotron and inverse-Compton processes to figure out some basic properties of a spectacular gamma-ray burst.

On March 19th, 2008, a gamma-ray burst was detected by the Swift satellite; this satellite has γ-ray, x-ray and optical/UV telescopes on board. It was amongst the brightest bursts ever detected. The γ-ray emission from the burst lasted for about 50s. It turns out that a number of ground based telescopes started following this burst within a few seconds of the start of explosion. The optical flux peaked at 5.2-mag in the R-band (or 20 Jansky at $4\text{x}10^{14}$ Hz); this burst located at a redshift of 0.94 could have been detected with an unaided eye, and therefore it was dubbed "naked eye burst" in the popular press. The peak optical luminosity corresponds to $4\text{x}10^{50}\text{erg s}^{-1}$ if the explosion were to be isotropic (that is more than a million times brighter than the brightest supernova). A summary of some other relevant properties of this burst is summarized in the paragraph below. [1 Jansky = 10^{-23} erg cm^{-2} s^{-1} Hz^{-1}]

The burst redshift from the optical data was determined to be 0.94 and the luminosity distance $d_L = 1.9 \times 10^{28}$cm (Vreeswijk et al. 2008). The burst was also detected by the Konus satellite, which reported that the time-averaged spectrum during the burst was $F_\nu \propto \nu^{0.18\pm0.01}$ between 20 keV and 650 keV, and $F_\nu \propto \nu^{-2.87\pm0.44}$ above 650 keV (the highest energy bin for the γ-ray detector on Konus is about 7 MeV). The flux at the peak (650 keV) was 5 milli-Jansky, and the total energy release (for an isotropic explosion) was $1.3\text{x}10^{54}$erg (Golenetskii et al. 2008). You can find more detailed observational properties of this burst in Racusin et al. (2008), and theoretical interpretation of the data can be found in Kumar & Narayan (2008) and Racusin et al. (2008).

Let us try to understand a few of the basic properties of this spectacular explosion.

1. Were γ-rays produced via a thermal or a non-thermal mechanism? Using the spectral properties in gamma-ray band identify the emission mechanism as best you can. (*Explain your answer in a few sentences.*)

2. **Optical source properties**

   (a) The optical flux varied on a timescale δt of about 5 seconds. What is the upper limit to the physical size of source if its velocity with respect to us were much less than the speed of light?

   (b) What is the source radius (R) if its velocity is close to the speed of light? Take the source Lorentz factor to be $\Gamma \gg 1$, and express R in terms of δt & Γ.

   (c) We unfortunately don't have spectral information in the optical band during the burst[1], but you can still show that the optical emission we received from this burst **could not** have been produced by a thermal source. You will do this in four steps.

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[1] We do have optical spectral measurement during the "afterglow phase", but the spectrum during the burst could have been very different.

(d) The flux observed from a thermal source can be calculated from its temperature, size, Lorentz factor, and distance. Using the source radius you estimated in (**2b**), and the distance to the GRB provided above, estimate the temperature of the optical source, in its comoving frame, so that the flux in the optical band, at 4×10^{14}Hz, is equal to the observed value of 20 Jansky. (*The temperature depends on the unknown value of* Γ*, so express the temperature in terms of* Γ*. And remember that you receive radiation from only that part of a highly relativistic spherical source that is moving in a direction that lies within an angle* Γ^{-1} *of your line of sight.*)

(e) For the temperature you calculated in part (2d) determine the frequency at which the thermal radiation peaks and the bolometric luminosity (both in the observer frame). What should be Γ if you want the bolometric luminosity to be not much larger than the total luminosity of all the stars in the universe (about 10^{54}erg/s)?

(f) Estimate the source distance from the center of explosion (CoE) using the value of Γ you estimated in part (2e).

(g) Can you think of any problem for the large Γ and source distance from the CoE you have obtained? Is the assumption of a thermal source viable? (Clarify your answer in a few sentences).

3. **Let us consider synchrotron radiation mechanism for the observed optical emission and IC for the γ-rays.**

Assume that there are N_e electrons in the source which have an average energy of $m_e c^2 \gamma_e$ each, and that they are moving in a magnetic field of strength B; all quantities are measured in the comoving frame of the source which is moving with a Lorentz factor of Γ wrt the center of explosion (m_e is electron rest mass). Moreover, let us assume that the source is spherically symmetric; nothing other than the total energy estimate is affected by this assumption, and that is straightforward to correct for a beamed explosion.

(a) Take the peak of the synchrotron spectrum (ν_p) to be in the optical; $\nu_p \approx 4\text{x}10^{14}$ as seen by an Earth bound observer. Show that B, γ_e and Γ are related by the following equation for this ν_p

$$B\gamma_e^2\Gamma = 2.7\times10^8 \quad \text{Gauss} \tag{1}$$

(b) Show that the synchrotron flux in the observer frame at ν_p is

$$f_{\nu_p} \approx \frac{\sigma_T m_e c^2 B}{3q}\frac{(1+z)N_e\Gamma}{4\pi d_L^2} \tag{2}$$

Hint: use the Lorentz invariance of I_ν/ν^3.

(c) Use the result in (3b) and the observed optical flux of 20 Jansky to obtain a constraint on N_e, B and Γ.

$$B\Gamma N_{e55} \approx 130 \quad \text{Gauss}, \tag{3}$$

where $N_{e55} \equiv N_e/10^{55}$.

(d) Let us consider the possibility that optical photons undergo IC scattering that results in the observed γ-ray emission. In this case the peak of the IC spectrum is at:

$$\nu_{ic} \approx 3\gamma_e^2 \nu_p. \tag{4}$$

Use the observed value of ν_{ic} for GRB 080319B, and ν_p, to determine γ_e.

(e) Now you can determine N_e. (*Use* γ_e *from* **3d** *and eqs. 1 & 2.*)

(f) Using the relation between source distance (R), observed variability time (δt), and Γ you found in (**1b**), determine Γ and B in terms of R; *take* $\delta t = 5s$.

(g) Derive the optical depth of the source to Thomson scattering (τ_e), and calculate the observed IC flux at ν_{ic} in terms of unknown source distance.

(h) For what value of R is the IC peak flux equal to the observed value?

(i) Use the value of R you obtained above to estimate B.

(j) Estimate the total energy in the magnetic field and electrons in the CoE frame.

(k) Estimate the Compton-Y parameter.

4. **IC spectrum**

(a) Show that the synchrotron self-absorption frequency, ν_a, in the observer frame is given by

$$\nu_a \approx \left[\frac{\sigma_T c^2}{24\pi^2 q}\frac{B N_e \Gamma^2}{R^2 \gamma_e (1+z)^2 \nu_p^{1/3}}\right]^{3/5} \tag{5}$$

(b) Estimate ν_a using the values of B, γ_e etc. you have obtained.

(c) What is the IC spectrum between 20 keV and 650 keV?

A number of assumptions were made to simplify the problem and obtain some properties of GRB 080319B to within a factor of a few. If you are interested in a self consistent solution without these assumptions, you can find that in Kumar and Narayan (2008).

References

[1] Golenetskii S. et al., 2008, GCN 7482*

[2] Kumar, P. & Narayan, R. 2008, arXiv:0812.0021

[3] Racusin, J. L. et al. 2008, Nature 455, 183

[4] Vreeswijk P. et al., 2008, GCN 7444*