Radiative Processes & Radiation Transport

Homework #4
Due October 15, 2009

Curve of Growth analysis to determine column density

Column density and elemental abundance for a wide variety of astronomical objects are determined using the curve of growth we discussed in class. In this homework you are asked to estimate the neutral-Hydrogen column density for Lyman-α clouds using real data from a paper by Lu et al. (ApJ, 472, 509, 1996; astro-ph/9606033). Browse though this paper and see how the authors go about determining HI column density etc. (you will be doing a simplified version of their analysis). To help you in this work I have broken down the task in 5 smaller steps that are outlined below. **Each part carries 20 points**.

1. Cross-section calculation: we derived the following expression for the cross-section $\sigma(\omega)$ in class:

$$\sigma(\omega) = \left(\frac{m}{2\pi kT}\right)^{1/2} \omega_0^2 \sigma_T \int_{-\infty}^{\infty} dV_z \, \frac{\exp(-mV_z^2/2kT)}{4(\omega - \omega_0 - \omega_0 V_z/c)^2 + \Gamma^2},$$

where m is the mass of HI atom, k is Boltzmann constant, T is gas temperature, V_z is the thermal speed of atoms, ω_0 is the Lyman-alpha transition frequency in the rest frame of the atom, c is speed of light, and $\Gamma = A_{12}$ is the transition rate from $n = 2$ to $n = 1$ state.

Show that the above equation can be transformed into the equation below by a change of variable–

$$\sigma(\omega) = \frac{\sigma_T}{4\pi^{1/2}} \frac{c^2}{V_T^2} \int_{-\infty}^{\infty} d\beta \, \frac{\exp(-\beta^2)}{\left[\frac{\Delta\omega}{\Delta\omega_D} - \beta\right]^2 + \frac{\Gamma^2}{4(\Delta\omega_D)^2}},$$

where $\beta = V_z/V_T$, $V_T = \sqrt{2kT/m}$, $\Delta\omega = \omega - \omega_0$, and $\Delta\omega_D = V_T\omega_0/c$.

2. Integrate the above equation for $\sigma(\omega)$ numerically. Use the following constants for your numerical integration: $\Gamma/\omega_0 = 3 \times 10^{-8}$, $V_T/c = 4 \times 10^{-5}$, $\sigma_T = 6.65 \times 10^{-25}\text{cm}^2$. Plot $\sigma(\omega)$ in the frequency interval $-50\Delta\omega_D$ to $50\Delta\omega_D$ — using log-scale for the y-axis; it is convenient to choose the unit for frequency such that $\Delta\omega_D = 1$ in this unit. Identify the part of the curve that is due to the Doppler broadening and the part that comes from the Lorentzian profile associated with the finite lifetime for the excited state. *(Note: because of the* $\exp(-\beta^2)$ *factor in the integrand it is sufficient to carry out the* β *integral from -10 to +10, but make sure that the grid size you choose for* β *is sufficiently small so that the numerical error in your integral is less than 1%.)*

3. The **flux transmitted** by a Ly-α cloud, $I_\nu^{(T)}$, is related to the incident flux, I_ν, via

$$I_\nu^{(T)} = I_\nu \exp(-\tau_\nu),$$

where $\tau_\nu = N\sigma(\omega = 2\pi\nu)$, and N is the column density of neutral hydrogen atoms in the ground state.

Using your result for $\sigma(\omega)$*, calculate, and plot, the Ly-*α *absorption line profile for* $N = 10^{11}$, 10^{13}, 10^{15} *and* $N = 10^{17}$ cm^{-2}*. Use the units such that* $\Delta\omega_D = 1$*, and* $I_\nu = 1$*.*

4. The equivalent width is defined as

$$W \equiv \int d\lambda \, \frac{I_\nu - I_\nu^{(T)}}{I_\nu}.$$

Using the equation for $I_\nu^{(T)}$ this can be rewritten as

$$\frac{W}{\lambda} = \int \frac{d\omega}{\omega} \, \left[1 - e^{-N\sigma(\omega)}\right].$$

The curve-of-Growth is a relation between W/λ and N. *Calculate, and graph, the curve-of-growth for the Ly-*α *transition for* N *in the interval* $10^{10} cm^{-2}$ *and* $10^{22} cm^{-2}$*; it is best to display the curve-of-growth on a log-log plot. Identify and label the three regimes for the curve-of-growth on your graph* (these regimes are: linear, log, and square-root).

5. Use figure 2 in Lu et al. (1996) to estimate W/λ for a few of the lines in the Ly-α forest. This part of the homework is not meant to be very precise i.e., it is okay if you were to use the full-width at half maximum of an absorption line, using a ruler, as a rough estimate for W; (Lu et al. (1996) fit a Voigt profile to each of the absorption lines in the forest and thereby determine W and cloud temperature). *Using your estimated values for* W/λ*, and your graph of the curve-of-growth, estimate the HI column* *density.* Compare these values with the values given in Lu et al. (you should be within a factor of 10 or so of the published value).