

AST 301
Homework #3
Due Friday Sep. 26

1. Newton's version of Kepler's 3rd law doesn't just apply to the planets orbiting the Sun. It also applies to moons orbiting a planet. If we use the year as the unit of time, the AU as the unit of distance, and the Sun's mass as the unit of mass, Kepler's 3rd law becomes: $P^2(\text{in years}) = a^3(\text{in AU})/M(\text{in solar masses})$, where P is the orbital period, a is the semimajor axis of the orbit, and M is the sum of the masses of the two objects.

a) Find a table in your book that gives the orbital periods and distances from Jupiter of the moons of Jupiter. Pick one of Jupiter's moons (say which one). Convert the moon's orbital period into years and its distance from Jupiter into AU and put them into this formula to calculate M, which is approximately the mass of Jupiter (in solar masses).

From table A-11, for Io, $a = 422 \times 10^3 \text{ km}$ and $P = 1.769 \text{ days}$. In AU and yr, $a = 2.8 \times 10^{-3} \text{ AU}$ and $P = 4.84 \times 10^{-3} \text{ yr}$. So $M = a^3 / P^2 = 9.5 \times 10^{-4} M_{\text{Sun}}$

b) Multiply your answer to part a by the mass of the Sun to get the mass of Jupiter in kg. Compare your answer to the mass of Jupiter given in the book.

$9.5 \times 10^{-4} \times 1.99 \times 10^{30} \text{ kg} = 1.89 \times 10^{27} \text{ kg}$

The book says $318 M_{\text{Earth}} = 318 \times 5.98 \times 10^{24} \text{ kg} = 1.90 \times 10^{27} \text{ kg}$

c) The mass in Newton's version of Kepler's 3rd law is actually the sum of the masses of the two bodies (the Sun plus a planet orbiting it, or Jupiter plus a moon). But your answer should have come out very close to the mass of Jupiter given in the book. Why didn't the mass of the moon matter?

We could subtract the mass of Io from our answer, but it is so small compared to the mass of Jupiter, the answer would hardly change.

2. a) An FM radio station broadcasts radio waves with a frequency of 100 MHz.

What is the wavelength of those radio waves? $\lambda = c / f = 3 \times 10^8 \text{ m/s} / 100 \times 10^6 \text{ Hz} = 3 \text{ m}$

b) The wavelength of green light is 500 nm. What is the frequency of a green light wave? $f = c / \lambda = 3 \times 10^8 \text{ m/s} / 500 \times 10^{-9} \text{ m} = 6.0 \times 10^{14} \text{ Hz}$

c) What is the ratio of the energy of a green light photon to the energy of a 100 MHz radio photon? Since $E = hf$, $E_{\text{green}} / E_{\text{radio}} = f_{\text{green}} / f_{\text{radio}} = 6.0 \times 10^{14} / 1.0 \times 10^8 = 6.0 \times 10^6$

d) The radio station broadcasts a power of 10,000 Watts, but my light bulb radiates only 100 Watts. How can this be, when visible light photons have so much more energy than FM radio photons? Each visible photon has more energy than each radio photon, but the radio station emits many more radio photons than the light emits visible photons.