**Homework #8**

1. a). If all of the mass of the cloud is concentrated at the center, the acceleration on the outer edge would be $g_0 = F/m = \frac{GMm}{r_0^2}/m = \frac{GM}{r_0^2} = \frac{6.67\times10^{-11}N\ m^2\cdot kg^{-2}\times1.99\times10^{30}kg}{(0.05\times3.09\times10^{16}m)^2} = 5.56\times10^{-11}N\ kg^{-1} = 5.56\times10^{-11}m\cdot s^{-2}$. Notice the acceleration doesn't depend on the mass of the molecule. Galileo did a very famous experiment on the top of a tower to prove this.

b). If we keep the acceleration constant, the formula associated with the distance and the time needed to cover the distance is $\frac{1}{2}g_0\cdot t^2 = r_0$. Now $r_0 = 0.05pc = 1.545\times10^{15}m$ ($0.1pc$ is the diameter) and $g_0 = 5.56\times10^{-11}m\cdot s^{-2}$, we can calculate the time $t = \sqrt{\frac{2r_0}{g_0}} = \sqrt{2\times\frac{1.545\times10^{15}m}{5.56\times10^{-11}m\cdot s^{-2}}} = \sqrt{5.557\times10^{25}s^2} = 7.455\times10^{12}s = 236393\ yrs$.

c). The distance between the molecule and the center of the cloud changes during the course of the molecule's falling, the force exerted on the molecule by the center mass also changes, so the acceleration doesn't stay constant, it changes according to the force. We need to do an integration to get the time. The derivation is attached at the end of this document.

We assumed spherical configuration of the cloud at the beginning. If it's not spherical, we cannot use simple Newton's theorem to calculate the force of other particles on the molecule we are interested in.

We also assumed that the pressure (both gas pressure and magnetic) inside the cloud is negligible. The pressures will keep the cloud from contracting, slow down the in-falling process. Free-fall approximation is not valid anymore.

2. a). $flux = \frac{luminosity}{4\pi r^2}$. Two stars with a same luminosity will have fluxs proportional to the reciprocals of the square of their distances. So the closer star will have a flux 100 times bigger.

b).If star A is 10 times as distant as star B, $\frac{D_A}{D_B} = 10$, we have:

$$m_A - M = 5log_{10}(\frac{D_A}{10pc}) \quad (1)$$

$$m_B - M = 5log_{10}(\frac{D_B}{10pc}) \quad (2)$$

$$m_A - m_B = 5log_{10}(\frac{D_A}{D_B}) = 5log_{10}(10) = 5 \quad (3)$$

Star A is 5 magnitude fainter than star B.

c). Because the flux drops with the square of the distance, 16 times brighter in the flux means 4 times closer in the distance.

d). If we can find a way to get their distances, from the flux measured through the telescope we can calculate the luminosities they have.

3. We can observe molecules through emission or absorption lines associated with specific molecular transitions. They usually occur in the infrared(IR) and the radiowave bands.
We can detect the presence of interstellar dust (large size complex of molecules) through its obscuration of background starlight.
Reflection on gas and dust particles has the preferred wavelength, which gives the reflection nebula different colors.
Dust and large molecules also cause polarization of starlight when they pass through these materials.

**Appendix:**

1.c) Here is the trick how to do the integral in 1.c). You can imagine an extremely elongated elliptical orbit around a solar mass star. The star is at one focus of the ellipse, which is very close to one sharp edge of the ellipse. Well, a molecule was at the other size of the ellipse at the beginning. The time required to let the molecule fall onto the surface of the star is approximately equal to a half of the peried of objects on that orbit. The orbit has a major-axis $a = \frac{1}{2}r_0 = 0.5 \times 0.05pc = 5156.62\ AU$. Put it into the formula $P^2 = a^3$, we get the period $P = a^{\frac{3}{2}} = 5156.62^{\frac{3}{2}} = 370295\ yrs$. A half of it is $185147\ yrs$. It's very close to the exact answer from the following method.

A more formal way to do this is integrating the dynamical equation as the following. You can skip this if you want to. In the derivation, we use the following notations:$v = -\frac{dr}{dt} = -r'$, $g = \frac{dv}{dt} = v' = -\frac{dr'}{dt} = -\frac{d^2r}{dt^2} = -r''$ and $g(r) = \frac{GM}{r^2}$. The negative sign is because the acceleration increases when the distance decreases.

$$
\begin{aligned}
\frac{d^2r}{dt^2} &= -g(r) && (4)\\
\frac{dr}{dt}\cdot\frac{d^2r}{dt^2} &= -\frac{dr}{dt}\cdot g(r) && (5)\\
r'r'' &= -r'\frac{GM}{r^2} && (6)\\
\frac{1}{2}\frac{d}{dt}(r')^2 &= -\frac{GM}{r^2}\frac{dr}{dt} = \frac{d}{dt}(\frac{GM}{r}) && (7)\\
\frac{1}{2}(r'^2 - r'^2_{r=r_0}) &= \frac{GM}{r} - \frac{GM}{r_0} && (8)\\
\frac{1}{2}r'^2 &= (GM)(\frac{1}{r} - \frac{1}{r_0}) \qquad (r'_{r=r_0} = 0) && (9)\\
\frac{dr}{dt} &= -\sqrt{2GM}\sqrt{(\frac{1}{r} - \frac{1}{r_0})} && (10)\\
\frac{dr}{\sqrt{(\frac{1}{r} - \frac{1}{r_0})}} &= -\sqrt{2GM}dt && (11)\\
\int_0^{r_0}\sqrt{\frac{rr_0}{r_0 - r}}dr &= -\sqrt{2GM}(t_0 - t) && (12)\\
\int_0^{r_0}\sqrt{\frac{r}{1 - r/r_0}}dr &= -\sqrt{2GM}(-\Delta t) && (13)\\
\int_0^{1}\sqrt{\frac{xr_0}{1 - x}}d(xr_0) &= \sqrt{2GM}(\Delta t) \qquad (x = \frac{r}{r_0}) && (14)\\
r_0^{3/2}\int_0^{1}\sqrt{\frac{x}{1 - x}}d(x) &= \sqrt{2GM}(\Delta t) && (15)\\
r_0\int_0^{1}\sqrt{\frac{x}{1 - x}}d(x) &= (\Delta t)\sqrt{\frac{2GM}{r_0}} && (16)\\
Because \;\; v_{esc}(r_0) = \sqrt{\frac{2GM}{r_0}} &= 414.5\ m/s, \quad \int_0^{1}\sqrt{\frac{x}{1 - x}}d(x) = \frac{\pi}{2} && (17)\\
\frac{\pi}{2}r_0 &= \Delta t v_{esc}(r_0) && (18)
\end{aligned}
$$

$$\Delta t = \frac{\pi}{2} \cdot \frac{r_0}{v_{esc}(r_0)} = \frac{\pi}{2}\frac{1.545\times 10^{15} m}{414.5\ m/s} \tag{19}$$

$$= 5.855\times 10^{12} s = 185653\ yrs \tag{20}$$

There should be two mathematically possible formula after (9), we pick the one with a negative sign because we know the velocity also increases when the distance decreases.