

Homework #7

1. First, let's convert the distance and the size of orbits into similar units. From the definition of parallax: if a baseline of length 1 AU causes 1 arcsec (1'') angular shift in a star's position, the distance between the star and the observer must be 1 parsec (1pc). The greater the distance, the smaller the shift. The relationship is linear. Now we have a parallax 5 times smaller (0.2''), the star should be 5 times more distant than 1pc, so the stars have a distance of 5pc.

By measuring the size of two orbits, we know the semi-major axes are $a_1 = 1''$ and $a_2 = 2''$. Based on the same reasoning, 1 AU baseline causes an angle of 0.2'' at a distance of 5pc, 5 AU baseline causes an angle of 1'' at the same distance and 10 AU makes 2''. Now we know, the distance of stars is $d = 5pc$. The semi-major axes are $a_1 = 5 AU$ and $a_2 = 10 AU$, respectively.

Another slower but conceptually easier way to get the size of the orbits is: converting the arcsec into radian, then times the distance, so $a_1 = 1'' \frac{2\pi}{360 \times 3600} \times d = \frac{1}{206265} \times 5pc = \frac{1}{206265} \times 5pc \times \frac{206265 AU}{1pc} = 5 AU$, and $a_2 = 1'' \frac{2\pi}{360 \times 3600} \times d = 10 AU$. Now, you can see these two methods are equivalent. Referring your math textbook to find out why you need to convert arcsec into radian before multiplying length.

According to Kepler's law, we now have

$$P^2 = \frac{(a_1 + a_2)^3}{m_1 + m_2} \quad (1)$$

$$m_1 \times a_1 = m_2 \times a_2 \quad (2)$$

where P is the period of the system, a_1 and a_2 are semimajor axes of two orbits. m_1 and m_2 are masses of two stars. Remember we have to use Earth year, AU and Solar mass as the units in the formula in order to get per answer. With $P = 25 yrs$, $a_1 + a_2 = 5AU + 10AU = 15AU$, we can get $m_T = m_1 + m_2 = \frac{15^3}{25^2} = 5.4M_\odot$ and $\frac{m_1}{m_2} = \frac{a_2}{a_1} = 2$. So the star with the smaller orbit has twice amount of mass as the star with the bigger orbit or it has two third of total mass. $m_1 = \frac{2}{3} \times 5.4 = 3.6M_\odot$ and $m_2 = \frac{1}{3} \times 5.4M_\odot = 1.8M_\odot$.

$$2. a). \delta E = \delta M \cdot c^2 = 4.8 \times 10^{-29} kg \times (3.0 \times 10^8 m \cdot s^{-1})^2 = 4.32 \times 10^{-12} kg \cdot m^2 \cdot s^{-2} = 4.32 \times 10^{-12} \text{ Joules}$$

b). $L_{\odot} = 3.90 \times 10^{26} \text{ Watts} = 3.90 \times 10^{26} \text{ Joules} \cdot \text{s}^{-1} = \Delta E \cdot \text{s}^{-1}$. To generate this amount of energy, a certain amount of mass has to be destroyed, because we are assuming that all energy comes from the nuclear fusion. $\Delta E = \Delta M \cdot c^2$. It's easy to see the fact that $\frac{\Delta M}{\delta M} = \frac{\Delta E}{\delta E} = \frac{3.9 \times 10^{26} \text{ Joules}}{4.32 \times 10^{-12} \text{ Joules}} = 9.03 \times 10^{37}$. If you noticed the relationship between δE and four protons, you can find the number of protons destroyed in one second is $\frac{\Delta E}{\delta E} \times 4 = 9.03 \times 10^{37} \times 4 = 3.612 \times 10^{38}$.

c). $\text{Total number of protons} = \frac{0.7 \times M_{\odot}}{m_p} = \frac{0.7 \times 2.0 \times 10^{30} \text{ kg}}{1.67 \times 10^{-27} \text{ kg}} = 8.38 \times 10^{56}$

d). The number of seconds needed to destroy all protons inside Sun is $\frac{\text{total number of protons}}{\text{number of protons destroyed in one second}} = \frac{8.38 \times 10^{56}}{3.612 \times 10^{38} \text{ s}^{-1}} = 2.3 \times 10^{18} \text{ sec} = 7.36 \times 10^{10} \text{ yrs}$.

3. a). Since Sirius's mass is twice of Sun's mass, the number of the protons it contains is also twice of that the Sun contains.

b). Sirius generates energy 20 times faster, so it consumes protons 20 times faster ($7.224 \times 10^{39} \text{ s}^{-1}$).

c). Sirius contains twice as many protons, but consumes them 20 times faster, so it only last about 10% of the lifetime of Sun.

d). $0.1 \times 7.36 \times 10^{10} \text{ yrs} = 7.36 \times 10^9 \text{ yrs}$.