Homework #6

1. a). Rotational speed is $v = \omega \times r$, where ω is the angular speed of an object. In this problem, the angular speed is $\omega_1 = \frac{2\pi}{P} = \frac{2\pi}{10^6 \times 365 \times 24 \times 3600} = 2\times10^{-13}\ radian\cdot s^{-1}$. The radius of the cloud fragment is $r_1 = 0.1\ ly = 0.1\times9.46\times10^{12}\ km = 9.46\times10^{11} km = 6323.5\ AU$. From these, we can get the rotational speed is $v_1 = \omega\times r = 2\times10^{-13} radian\cdot s^{-1}\times9.46\times10^{11}\ km = 0.1892\ km\cdot s^{-1}$.

b). Because the angular momentum is conserved during the contraction, we have $m\times\omega_1\times r_1^2 = m\times\omega_2\times r_2^2$, we can also write this formula as $m\times v_1\times r_1 = m\times v_2\times r_2$ ($v = \omega\times r$). $\frac{v_2}{v_1} = \frac{r_1}{r_2} = \frac{6323.5\ AU}{50\ AU} = 126.47$. The rotational speed is $v_2 = 126.47\times v_1 = 126.47\times0.1892\ km\cdot s^{-1} = 23.93\ km\cdot s^{-1}$. The perimeter of the outer edge circle is $l = 2\pi r = 2\times3.14\times50\ AU = 4.7\times10^{10}\ km$. The period of the outer edge is $P = \frac{l}{v_2} = \frac{4.7\times10^{10}\ km}{23.93\ km\cdot s^{-1}} = 1.9\times10^9\ sec = 62.3\ yrs$.

c). The orbital speed is $v_o = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.67\times10^{-11}\ N\ m^2\ kg^{-2}\times1.99\times10^{30}\ kg}{50\ AU}} = \sqrt{\frac{1.327\times10^{20}\ N\ m^2\ kg^{-1}}{7.48\times10^{12}\ m}} = 4212\ m\cdot s^{-1} = 4.212\ km\cdot s^{-1}$. The escape speed is $v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2}\cdot\sqrt{\frac{GM}{R}} = \sqrt{2}\times v_o = 5.963\ km\cdot s^{-1}$.

d). Astronomers believe that a lot of angular momentum is lost during the contraction.

2. a). The density $\rho_1 = \frac{m}{V_1} = \frac{m}{\frac{4}{3}\pi r_1^3} = \frac{2.0\times10^{30}\ kg}{4.19\times(9.46\times10^{11} km)^3} = 5.64\times10^{-7}\ kg\cdot km^{-3} = 5.64\times10^{-19}\ g\cdot cm^{-3}$. ($1 kg\cdot km^{-3} = 10^3 g\cdot(10^5 cm)^{-3} = 10^3\times10^{-15} g\cdot cm^{-3} = 10^{-12}\ g\cdot cm^{-3}$)

b). One hydrogen molecule is consist of two hydrogen atoms, thus the total number of molecules in the gas is $\frac{5.64\times10^{-19}\ g\cdot cm^{-3}}{2\times1.67\times10^{-24} g} \approx 168858\ cm^{-3}$. It's also called the numberdensity, cause it describes how many particles present in unit volume.

3. a). The luminosity of the Sun is $L_\odot = 3.9\times10^{26} W$. The distance is $d = 1\ AU = 1.496\times10^{11} m$. The solar constant $F_{sun} = \frac{L_\odot}{4\pi d^2} = \frac{3.9\times10^{26} W}{4\times3.14\times(1.496\times10^{11} m)^2} = 1387.4\ W\cdot m^{-2}$.

b). $F_{bulb} = \frac{100\ W}{4\times3.14\times(1m)^2} = 7.96\ W\cdot m^{-2}$. The solar constant is greater.

4. $p_m = \frac{p}{m} = \frac{100W}{66kg} = 1.5\ Watt\ per\ kilogram.$
$P_M = \frac{P}{M} = \frac{3.9\times10^{26}W}{2\times10^{30}kg} = 1.95\times10^{-4}\ Watt\ per\ kilogram.$